Blocked Force and Loading Calculations for LaRC THUNDER Actuators

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Foreword

This report represents work completed in the 1997-1998 time frame while working as a NASA contractor for SAIC at NASA Langley Research Center. It was originally submitted as a paper to Smart Materials and Structures in Feb., 1998 (Log Number: 00399) and conditionally accepted in July, 1998 pending submission of a minor revision. This was withheld by the author because it was simultaneously submitted as part of a patent disclosure, LAR-15827, in April, 1998.

Much has changed since the time this report was written. This model has since been improved upon over the past 9 years since it was written. It is being reproduced here for historical purposes and to make it more referenceable for those who may wish to reference it.
Abstract

An analytic approach is developed to predict the performance of LaRC Thunder actuators under load and under blocked conditions. The problem is treated with the Von Karman non-linear analysis combined with a simple Raleigh-Ritz calculation. From this, shape and displacement under load combined with voltage are calculated. A method is found to calculate the blocked force vs voltage and spring force vs distance. It is found that under certain conditions, the blocked force and displacement is almost linear with voltage. It is also found that the spring force is multivalued and has at least one bifurcation point. This bifurcation point is where the device collapses under load and locks to a different bending solution. This occurs at a particular critical load. It is shown this other bending solution has a reduced amplitude and is proportional to the original amplitude times the square of the aspect ratio.

Introduction

Thunder actuators are devices constructed from an isotropic laminate of aluminum, LaRC SI adhesive, piezo-electric PZT, and a metal backing of either steel or brass. The LaRC adhesive is solid thermoplastic and the aluminum layer is indented to provide electrical conductivity after the actuator is cured at 250 deg C. After the curing process they form a bow shape after reaching room temperature. If simply supported or arranged in a clamshell arrangement they produce a linear motion when an external voltage is applied.

A simple method for predicting the performance of these devices has long been sought. As of the time of this writing (2/98), a method for predicting the performance of these devices under loading or blocked conditions has never been attempted. One method that is widely used to measure blocked force is to place a mass on top and measure the displacement under load. This is not true blocked force, however. Blocked force is the change in force when constrained not to move. In particular, displacement under load is not the same as the change in force when constrained not to move. Because of the peculiar non-linear properties of the device, it is easy to be lulled into thinking one is the other when, in fact, they are different things.
If the device were perfectly linear, the delta amplitude with voltage would be completely independent of load in the static limit. This is easy to see by taking a linear spring and applying a static external force to find the change in amplitude due to that external force under different loading conditions. A non-linear device, on the other hand, is quite different and does depend on load. In this case there is actually some critical load where the device switches from one bending solution to another at a far reduced amplitude. The current blocked force procedure is to apply enough load until the amplitude is small. This procedure results in a gross overestimate for the blocked force because it depends more on the structural properties of the device than the true actuation force. In fact, the true blocking force has never been measured for these devices.

The correct method to measure blocked force is to constrain the device such that the bottom is constrained to move in one plane but fixed to that plane and the top is connected to a load cell. In that situation the force will be \( F = F(x,V) \). This will be a combination of the spring force plus the actuation force. In a particular limit this may be approximated by \( F(x,V) = -k(x-x_0) + \alpha V \). Of course, this neglects hysteresis effects.

The incorrect method is to place a mass on top and measure the delta amplitude with respect to voltage. This method represents more what it can support structurally. It is a common method with these devices to place a mass on top, then apply a low frequency sinusoid and measure the delta amplitude. This is usually interpreted as the actuation force being the same as the weight of the mass. In reality, this is anything but the case.

One approach to this problem is to use classical lamination theory [1]. However, this leads to an incorrect qualitative behavior [3]. As a result, a non-linear approach must be used. The type of correction used here is one that is valid for large displacement and small strain. This correction is usually attributed to Von Karman [2] in plate theory. Hyer [4] has applied this to his treatment of orthotropic laminates with good success. Campbell [3] extended this to Thunder actuators by treating each layer as isotropic and including voltage terms.

### Solution by the Von Karman Non-Linear Approximation

The Von Karman approximation assumes large displacement and small strain. This suggests including second order terms in \( w \) in addition to the linear terms for \( u \), \( v \) and \( w \). Expressing the strain to include these terms gives,

\[
\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2,
\]

\[
\varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2,
\]

\[
\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}.
\]
By using the relation,
\[ u = u_0 - z \left( \frac{\partial w_0}{\partial x} \right), \quad (2) \]
\[ v = v_0 - z \left( \frac{\partial w_0}{\partial y} \right), \]
we find,
\[ \varepsilon_x = \frac{\partial u_0}{\partial x} + \frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 - z \left( \frac{\partial^2 w_0}{\partial x^2} \right), \]
\[ \varepsilon_y = \frac{\partial v_0}{\partial y} + \frac{1}{2} \left( \frac{\partial w_0}{\partial y} \right)^2 - z \left( \frac{\partial^2 w_0}{\partial y^2} \right), \quad (3) \]
\[ \gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} - 2 \ z \left( \frac{\partial^2 w_0}{\partial x \partial y} \right) \]
The total energy takes the form,
\[ U = \iiint \left( dU_0 - dW_T - dW_A \right) - \iiint w \ p(x, y) \ dx\, dy, \quad (4) \]

where \( dU_1 \) is the stored elastic energy volume density and \( dW_T \) and \( dW_A \) are the temperature and actuation contributions. The last integral is the work done against the external pressure, \( p \). The elastic energy portion of it for the \( k \)'th layer is,
\[ (dU)_k = \frac{1}{2} \left( \sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy} \gamma_{xy} \right)_k \]
\[ = \left( \frac{1}{2} Q_1 \varepsilon_x^2 + Q_2 \varepsilon_y + \frac{1}{2} \frac{Q_1 \varepsilon_y^2}{\Delta T} + \frac{1}{2} Q_3 \gamma_{xy} \right)_k \quad (5) \]

For the thermal portion we have,
\[ (dW)_k = \left( \sigma_x^T \varepsilon_x + \sigma_y^T \varepsilon_y \right)_k \]
\[ = \left( Q_1 + Q_2 \right) \alpha \Delta T \varepsilon_x + \left( Q_1 + Q_2 \right) \alpha \Delta T \varepsilon_y \quad (6) \]

For the actuation portion we have,
\[(dW_j)_3 = \left( \sigma^x_x \varepsilon_x + \sigma^y_y \varepsilon_y \right)_j \]

\[= \left( Q_1 + Q_2 \right) j d_{31} \frac{V}{T} \varepsilon_x + \left( Q_1 + Q_2 \right) j d_{31} \frac{V}{T} \varepsilon_y \]

(7)

Actuation is assumed to occur on the j'th layer and Q is the reduced stiffness with \(Q_{11}=Q_{22}=Q_1\) and \(Q_{12}=Q_{21}=Q_2\). Now that we have the general form for the energy integral we may try a Raleigh-Ritz type solution. The method involves making a guess for the solution. This guess should look qualitatively like the expected solution yet be flexible enough to allow for adjustment. This adjustment comes in the form of changing the guess based on a finite number of parameters which the guess contains. The parameters are adjusted in such a way as to minimize the total energy of the system. In this situation it would be advantageous to choose a guess that is close enough to the classical lamination result that in the linear limit (small scaling) the solution approaches the classical lamination result. Such a guess takes the form,

\[u^0 = w^0_0 + \frac{1}{2} \left( a x^2 + b y^2 \right)\]

(8)

Classical lamination theory assumes no shearing strain between layers for thermal expansion. If we make the same assumptions here we find \(u^0\) and \(v^0\) must take the form,

\[u^0 = c x - \frac{1}{6} a^2 x^3 - \frac{1}{4} a b x \ y^2\]

\[v^0 = d y - \frac{1}{6} b^2 y^3 - \frac{1}{4} a b x^2 \ y\]

(9)

The ansatz represented by Equation 27 and 28 has been used successfully by Hyer [3] in his treatment of orthotropic laminates and by Campbell [3] with Thunder actuators. It is sufficiently flexible enough to be used with isotropic laminates as well as we will soon see. Equations 3, 8 and 9 give,

\[\varepsilon_x = c - \frac{1}{4} a b \ y^2 - a \ z\]

\[\varepsilon_y = d - \frac{1}{4} a b \ x^2 - b \ z\]

(10)

\[\varepsilon_{xy} = 0\]

If the length in the x direction is \(L_x\) and the length in the y is \(L_y\) then, we may approximate \(w^0_0\) by,
\[ w_0^0 = -\frac{1}{8} \left( a L_x^2 + b L_y^2 \right) \]  \hspace{1cm} (11)

This assumes the device is simply supported. For \( p(x,y) \) we choose a point load of,

\[ p(x, y) = F \delta (x) \delta (y) \]  \hspace{1cm} (12)

Other choices are possible such as a constant distributed pressure. However, a point load is closest to the type of load used with the rectangular devices. The solution is obtained by minimizing the total energy with respect to \( a, b, c \) and \( d \). The result of the minimization is,

\[
\begin{align*}
D_1 a + K_8 b - B_1 c - B_2 d + K_1 b^2 + K_6 a b - K_4 b c - K_3 b d + \\
K_7 a b^2 + M' - \frac{1}{8} \left( \frac{L_x}{L_y} \right) F &= 0 \\
K_8 a + D_1 b - B_2 c - B_1 d + K_2 a^2 + K_4 a b - K_5 a c - K_3 a d + \\
K_7 a^2 b + M' - \frac{1}{8} \left( \frac{L_y}{L_x} \right) F &= 0 \tag{13}
\end{align*}
\]

\[-B_1 a - B_2 b + A_1 c + A_2 d - K_4 a b - N' = 0 \]

\[-B_2 a - B_1 b + A_2 c + A_1 d - K_3 a b - N' = 0,
\]

where,
\[ K_1 = \frac{1}{48} \left( B_1 L_x^2 + B_2 L_y^2 \right) \]
\[ K_2 = \frac{1}{48} \left( B_2 L_x^2 + B_1 L_y^2 \right) \]
\[ K_3 = \frac{1}{48} \left( A_1 L_x^2 + A_2 L_y^2 \right) \]
\[ K_4 = \frac{1}{48} \left( A_2 L_x^2 + A_1 L_y^2 \right) \]
\[ K_5 = \frac{1}{24} \left( B_1 L_x^2 + B_2 L_y^2 \right) \]
\[ K_6 = \frac{1}{24} \left( B_2 L_x^2 + B_1 L_y^2 \right) \]
\[ K_7 = \frac{A_2 L_x^2 L_y^2}{1152} + \frac{A_1 \left( L_x^4 + L_y^4 \right)}{1280} \]
\[ K_8 = D_2 + \frac{N}{48} \left( L_x^2 + L_y^2 \right) \]

(14)

The A's, B's and D's have the usual meanings of extensional, coupling and bending stiffness. The force and moment per unit width, \( N' \) and \( M' \), are given by,

\[
\begin{pmatrix}
  N_x' \\
  N_y' \\
  N_{xy}'
\end{pmatrix} = \begin{pmatrix}
  N' \\
  N_x' + N_y' \\
  0
\end{pmatrix} = \begin{pmatrix}
  N \\
  N'
\end{pmatrix}
\]

\[
\begin{pmatrix}
  M_x' \\
  M_y' \\
  M_{xy}'
\end{pmatrix} = \begin{pmatrix}
  M' \\
  M_x' + M_y' \\
  0
\end{pmatrix} = \begin{pmatrix}
  M \\
  M'
\end{pmatrix}
\]

(15)

where,

\[ N^A = -d_{31} \left(Q_1 + Q_2\right) j V \]

\[ M^A = -\frac{1}{2} d_{31} \left(Q_1 + Q_2\right) \left(z_3 + z_2\right) V \]

(16)
and,

\[ N^T = \Delta T \sum_{k=1}^{N} \alpha_k (Q_1 + Q_2) \left( z_k - z_{k-1} \right) \]

\[ M^T = \frac{\Delta T}{2} \sum_{k=1}^{N} \alpha_k (Q_1 + Q_2) \left( z_k^2 - z_{k-1}^2 \right) \]  \hspace{1cm} (17)

The z's are measured with respect to the middle surface and z0 is the top surface. The lengths L_x and L_y are the length and width of the actuator. It is a simple matter to show that the solution in the linear limit (small scaling - L_x=L_y=0) matches the classical lamination result if we take a=-\kappa_x, b=-\kappa_y, c=\epsilon_x and d=\epsilon_y and F=0.

Unlike the classical lamination result, the non-linear analysis predicts unequal curvatures that depend on the magnitude of the scaling and the aspect ratio. It also predicts multiple possible solutions. The nature of these solutions are dome-like solutions which approach cylinders in the limit of large scaling. Under load, saddles are also possible. Typically, there will be one or two stable solutions and possibly a third unstable solution depending on the size and material properties. There are also unphysical solutions in terms of complex numbers.

As a simple example we take 3 layer actuator constructed from 1 mil brass, 1 mil LaRC Si thermoplastic and 6.8 mil PZT 5A. By solving Equation 13 with V=0 and plotting F as a function of dome height we find the spring force for the actuator. The results of this calculation show the spring force is multivalued. When initially in the short axis mode, the actuator may be depressed until a critical load is reached. At this point, the actuator switches and locks to the long axis solution. This has been verified experimentally and is a general characteristic of all Thunder actuators that have unequal x and y curvatures. When the x and y curvatures are strongly unequal, the spring force is almost linear except for the bifurcation point.
Figure 1. Calculation of spring force for a 3 layer actuator.

Blocked force as a function of voltage may be calculated by constraining the distance by Equation 11. and solving Equation 13 to find the resulting force as a function of voltage at a particular constrained distance. Figure 2. is the result of such a calculation for the same 3 layer actuator. In this case the blocked force appears to be almost linear with distance. In reality there is some hysteresis involved. The horizontal axis is voltage and the vertical is pounds. The constrained distance chosen was close to the equilibrium point and the short axis solution is shown in the plot.

![Blocked Force vs. Voltage](image)

Figure 2. Calculation of blocked force as a function of voltage for a 3 layer actuator.

If the constrained distance is close enough to the bifurcation point it is actually possible to cause the actuator to switch from the short axis solution to the long axis solution. The result is a bifurcation point involving voltage and blocked force.

**Special Limits**

There are two special limits that reduce to analytic forms. The first is the classical lamination limit where the size to thickness ratio is very small. The second is the strongly non-linear limit where the x and y curvatures are strongly unequal. This second limit is the most common type of rectangular actuator encountered. However, other types are possible and it is impossible to say for sure how the actuator will turn out without first doing the non-linear calculation first. It is highly desirable to design an actuator that is as close to a cylinder as possible. This is because it will behave more predictably than one that is close to the curvature vs. size bifurcation point. Actuators that are designed to be valid in the classical lamination theory limit are predictable but have a much reduced amplitude.
To find the classical lamination theory limit we solve Equation 13 in the limit \( L_x = 0, L_y = 0 \) with \( L_x/L_y \) fixed. The result is,

\[
a = - \frac{(A_1 + A_2) \left( M' - \left( \frac{L_x}{L_y} \right) F \right) - (B_1 + B_2) N'}{(A_1 + A_2)(D_1 + D_2) - (B_1 + B_2)^2}
\]

(18)

\[
b = - \frac{(A_1 + A_2) \left( M' - \left( \frac{L_x}{L_y} \right) F \right) - (B_1 + B_2) N'}{(A_1 + A_2)(D_1 + D_2) - (B_1 + B_2)^2}
\]

Using the results of Equation 18 with the results of Equations 15, 16 and 17 to find \( F \) expressed in terms of the dome height, \( \xi \), as,

\[
F = -k \left( \xi - \xi_0 \right) + \beta V
\]

(19)

where,

\[
\xi_0 = \frac{1}{8} \left( L_x^2 + L_y^2 \right) \frac{(A_1 + A_2) M' - (B_1 + B_2) N'}{(A_1 + A_2)(D_1 + D_2) - (B_1 + B_2)^2},
\]

(20)

\[
k = 64 \frac{L_x L_y (A_1 + A_2)(D_1 + D_2) - (B_1 + B_2)^2}{(L_x^2 + L_y^2)(B_1 + B_2)}
\]

\[
\beta = - \frac{L_x L_y}{L_x^2 + L_y^2} \left( z_j + z_{j-1} - \frac{1}{2} \frac{B_1 + B_2}{A_1 + A_2} (Q_1 + Q_2) \right) d_{31}
\]

The other limit is the strongly non-linear, perfect cylinder limit. In this case Equation 13 becomes independent of either \( a \) or \( b \), giving two possibilities. These two possibilities are,

\[
D_1 a - B_1 c - B_2 d + M' - \left( \frac{L_x}{L_y} \right) F = 0,
\]

\[- B_1 a + A_1 c + A_2 d - N' = 0,
\]

(21)

\[- B_2 a + A_2 c + A_1 d - N' = 0,
\]

or,
\[ D_1 b - B_2 c - B_1 d + M' - \left( \frac{L_x}{L_z} \right) F = 0 \]

\[- B_2 b + A_1 c + A_2 d - N' = 0 \]  \hspace{1cm} (22)

\[- B_1 b + A_2 c + A_1 d - N' = 0, \]

Equations 21 and 22 may be solved for a or b so that we may find the force as before. The result is,

\[ F_a = -k_a \left( \xi - \xi_b \right) + \beta_a V, \]

\[ F_b = -k_b \left( \xi - \xi_b \right) + \beta_b V, \]

where,

\[ \xi_a = \frac{1}{8} L_x^2 \left( A_1 - A_2 \right) \left( A_1 + A_2 \right) M' - \left( A_1 - A_2 \right) \left( B_1 + B_2 \right) N' \]

\[ \left( A_1^2 - A_2^2 \right) D_1 - A_1 B_1^2 + 2 A_2 B_1 B_2 - A_1 B_2^2 \]

\[ k_a = 64 \frac{L_y}{L_x} \left( A_1^2 - A_2^2 \right) D_1 - A_1 B_1^2 + 2 A_2 B_1 B_2 - A_1 B_2^2 \]

\[ \frac{\left( A_1 - A_2 \right) \left( A_1 + A_2 \right)}{A_1^2 - A_2^2} \]

\[ \beta_a = - \frac{L_x}{L_y} \left( z_j + z_{j-1} - \frac{1}{2} \frac{B_1 + B_2}{A_1 - A_2} \right) \left( Q_1 + Q_2 \right) j d_{31}, \]

and,

\[ \xi_b = \frac{1}{8} L_y^2 \left( A_1 - A_2 \right) \left( A_1 + A_2 \right) M' - \left( A_1 - A_2 \right) \left( B_1 + B_2 \right) N' \]

\[ \left( A_1^2 - A_2^2 \right) D_1 - A_1 B_1^2 + 2 A_2 B_1 B_2 - A_1 B_2^2 \]

\[ k_b = 64 \frac{L_x}{L_y} \left( A_1^2 - A_2^2 \right) D_1 - A_1 B_1^2 + 2 A_2 B_1 B_2 - A_1 B_2^2 \]

\[ \frac{\left( A_1 - A_2 \right) \left( A_1 + A_2 \right)}{A_1^2 - A_2^2} \]

\[ \beta_b = - \frac{L_y}{L_x} \left( z_j + z_{j-1} - \frac{1}{2} \frac{B_1 + B_2}{A_1 - A_2} \right) \left( Q_1 + Q_2 \right) j d_{31}, \]
With these results we may estimate the critical load represented in Figure 1. To find this we use Equation 23 with \( V=0 \) and write \( F_a = F_b \) so that,

\[
k_a (\xi - \xi_0) = k_b (\xi - \xi_0)
\]

(26)

or,

\[
\xi = \frac{k_a \xi_a - k_b \xi_b}{k_a - k_b}
\]

(27)

By substituting this into Equation 23 we find the critical load is then,

\[
F_a = F_b = -\frac{k_a k_b}{k_a - k_b} (\xi_a - \xi_b)
\]

(28)

We may also find the static amplitude with voltage under load. There are two possibilities,

\[
m g = -k_a (\xi - \xi_0) + \beta_a V,
\]

(29)

\[
m g = -k_b (\xi - \xi_0) + \beta_b V
\]

The amplitudes are,

\[
\xi_1 = \xi_a - \frac{m g}{k_a} + \frac{\beta_a}{k_a} V,
\]

(30)

\[
\xi_2 = \xi_b - \frac{m g}{k_b} + \frac{\beta_b}{k_b} V
\]

If the device is originally in the first bending solution and the critical load is applied, it will switch to the second bending solution. The ratio of the delta amplitude with voltage is,

\[
\frac{\Delta \xi_2}{\Delta \xi_1} = \frac{\beta_b}{k_b} \frac{\Delta V}{\beta_a k_a} = \left(\frac{L_y}{L_a}\right)^2
\]

(31)

This reduction in static amplitude with critical load is a purely a structural effect. The true actuation force, on the other hand, is given by Equation 23 - not Equation 28. They are completely different things.
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