Solar Confocal Interferometers for Sub-Picometer-Resolution Spectral Filters

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ABSTRACT

Aims. The confocal Fabry-Pérot interferometer allows sub-picometer spectral resolution of Fraunhofer line profiles. Such high spectral resolution is needed to keep pace with the higher spatial resolution of the new set of large-aperture solar telescopes. The line-of-sight spatial resolution derived for line profile inversions would then track the improvements of the transverse spatial scale provided by the larger apertures. In particular, profile inversion allows improved velocity and magnetic field gradients to be determined independent of multiple line analysis using different energy levels and ions. The confocal interferometer's unique properties allow a simultaneous increase in both étendue and spectral power. The higher throughput for the interferometer provides significant decrease in the aperture, which is important in spaceflight considerations.

Methods. We have constructed and tested two confocal interferometers. A slow-response thermal-controlled interferometer provides a stable system for laboratory investigation, while a piezoelectric interferometer provides a rapid response for solar observations.

Results. In this paper we provide design parameters, show construction details, and report on the laboratory test for these interferometers. The field of view versus aperture for confocal interferometers is compared with other types of spectral imaging filters. We propose a multiple etalon system for observing with these units using existing planar interferometers as pre-filters. The radiometry for these tests established that high spectral resolution profiles can be achieved with imaging confocal interferometers. These sub-picometer spectral data of the photosphere in both the visible and near-infrared can provide important height variation information. However, at the diffraction-limited spatial resolution of the telescope, the spectral data is photon starved due to the decreased spectral passband.

Key words. Sun: atmosphere – Instrumentation: interferometers – Line: profiles – Sun: magnetic fields

1. Introduction

Solar physicists are beginning, on a regular basis, to spatially resolve below 100 kilometer on the sun by employing optical 1-meter class, and larger, aperture telescopes (Schlichenmaier 2006, Beterovil 2004, Kell et al. 2004, Bernasconi, et al. 1999). It is conceivable to resolve to ~10 km in the next couple of decades with semi-monolithic or interferometric telescopes (Davis et al. 2005, Hammerschlag et al. 2004, Damié 1994). These operational, under-construction, or conceived telescopes include Flare-Genesis (0.8 m), Themis (0.9 m), Swedish Solar Telescope (1.0 m), SunRise (1.0 m), McMath-Pierce (1.5 m), Gregor (1.5 m), Dutch Open Telescope++ (1.5 m), New Big Bear Solar Telescope (1.6 m), Advance Technology Solar Telescope (4.0 m), Magnetic Transition Region Probe (6.0 m) and Giant Solar Optical Telescope (11.0 m). This effort of larger apertures for improved spatial resolution has been conducted in union with superior designs and techniques (e.g., improvements in (i) image stabilization by adaptive optics for ground-based observatories, (ii) thermal designs to reduce locally induced seeing effects, (iii) polarization resolution by lowering instrumental effects, (iv) the size of the CCD arrays to increase the field of view, and (v) imaging processing to allow improved data analysis). Radiative transfer theory has improved with advanced magneto-hydrodynamical numerical simulations at the sub-granular scales to resolve elemental flux tubes. However, visible-infrared, high-spectral-resolution narrow-band imaging filters at a spectral resolution of \( \Delta \lambda / \lambda < 10^{-6} \) have not received much attention. This spectral resolution is currently provided by the non-imaging Fourier Transform Spectrometer (FTS) on the National Solar Observatory (NSO)/McMath-Pierce Telescope. Using the FTS and studying elemental flux tubes, Stenflo et al. (1984) point out that the spectral measurements can be independent of the unresolved spatial elements, and hence high spectral resolution is a key component to understanding the structure of the sun. Mathew et al. (2003) have used the 3 pm full width half maximum (FWHM) spectral resolution Tenerife Infrared Polarimeter (TIP) and inverted the FeI 1.56 \( \mu \)m Stokes profiles using the SPINOR code to obtain an estimate of the magnetic fields gradient in an umbral region. In general, the line-of-sight spatial resolution derived from profile inversions should follow the improvements in the transverse (i.e., image plane) spatial resolution with larger apertures; however this will require sub-picometer spectral resolution. It is clear that the use of several lines with different magnetic, thermal, and Doppler sensitivities will help resolve the vertical information. However this process has its problems associated directly with the sensitivity variation which is used, i.e. different mass, different ions, different transitions, and different Landé factors. Through using a single transition and ex-
ploring the depth by its variation of opacity with position in the line (or response function) we avoid these sensitivity differences. Furthermore we will show that the trade-off between the signal-to-noise ratio and spectral resolution is acceptable. In this paper we present the spherical or confocal interferometer, an unnoticed technique in solar physics, which shows promise in combining high-spectral resolution and imaging, such that vertical gradients in mass advective flows and magnetic field gradients can be derived on a scale compatible with the horizontal gradients.

Normal or planar Fabry-Pérot interferometers (FPP) have a pair of mirrored parallel plates (etalons) and the light passes through the system when the gap between the plates is an integer number of the wavelength (Fabry and Pérot 1902, Vaughan 1989). The FWHM spectral pass-band (ΔλFWHM) is dependent on the separation between the plates. These FPPs are used as standard narrow pass-band imaging filters in solar physics in the US Air Forces Optical Solar Patrol Network (OSPAN, ΔλFWHM ~10 pm), University of Hawaii's Mees Solar Magnetograph (ΔλFWHM ~7 pm), Göttingen Astrophysical Institute's FPI (ΔλFWHM ~6.6 pm), Arcetri Observatory's Interferometric Bimensional Spectrometer (IBIS, ΔλFWHM ~2.3 pm), Kiepenheuer Solar Physics Institute's Triple Etalon Solar Spectrometer (TESSOS, ΔλFWHM ~1.9 pm), and NSO/Sacramento Peak Observatory's dual etalon systems (ΔλFWHM ~1.7 pm). For narrower FWHM filters with larger gap distances, the confocal interferometer should be considered. A confocal (spherical) Fabry-Pérot interferometer (FPS) differs from the parallel-plate interferometer in that the mirrored etalons are identical spherical surfaces where the center of radius is at the surface of the other mirror. (Confocal means having the same foci). Because of the spherical surfaces, the confocal interferometer can increase simultaneously both the spectral resolution and throughput and should be the preferred choice of interferometers for sub-picometer spectral filters (Hercher 1968, Vaughan 1989).

The path of the light through the confocal interferometer is typically a figure-eight shape (Figure 1) with the exit beam re-entering the direction of the initial beam (to within paraxial approximation) (Connes 1958, Hercher 1968, Clark 1972, Vaughan 1989). For the initial incident direction, the interference contributions to an exit ray come after 2nth reflections, where k is a positive integer. The spherical mirrors and their associated thickness affect the final imaging and spectral resolution however we shall show this is not a significant effect. A main point of this paper is to describe the imaging quality of the confocal interferometers and the possible improvements that they could provide for solar physics, since the confocal interferometers have greater throughput and greater alignment tolerances than parallel plate interferometers under certain circumstances. An offset in confocal mirror alignment results in only a change in the optical axis of the system, when the confocal distance, r, is corrected by normal tuning. This results in a necessary alignment of the mirror centers to only the order of ~D/r, where as a planar Fabry-Pérot interferometer, the alignment tolerance for parallelism is on the order of ~λ/D, where λ is the wavelength and D is the aperture.

One concern with increasing the spectral resolution is that this implies a deficit of photons as the spectral passband is reduced, causing longer exposure times. This is obviously problematic if the diffraction-limited spatial resolution is retained. For then the dependency of the size of the telescope is canceled since the light collected is a product of the telescope area and the spatial area sampled. If the aperture is varied it will be found that the product of the telescope’s aperture area and the area of the diffraction-limited resolution element on the sun is constant. At the diffraction limit resolution, the exposure times are independent of the aperture size (Table 1, last column). Therefore, to collect a reasonable number photons for spectral analysis a reduced spatial resolution is required, as we will demonstrate. We will now consider the exposure times for two wavelengths, one in the visible and one in the near infrared.

The exposure time is determined by the time it takes to fill each detector pixel well with electrons. This time is governed by the well capacity of the CCD and the numbers of electrons generated each second by the light incident on each pixel. This later parameter is itself determined by the quantum efficiency of the CCD and the number of photons incident each second on the pixel, n,p, To determine n,p, we consider the total radiant energy incident on a detector pixel per second, P, P, is equal to the solar radiant power per second per unit area per unit wavelength interval at the wavelength of interest, I(λ), multiplied by the wavelength passband, ΔλFWHM, the area of the telescope Aτ, and the effective area of a pixel on the sun, f(Aτ), and corrected for transmission losses, i, : P1 = I(λ)ΔλFWHM Aτ f(Aτ) i. The fraction of the total area collected by the telescope that is imaged on a detector pixel is equal to the ratio of the solid angle of the projected pixel and the solid angle of the sun.

Observing at the telescope diffraction-limited resolution with the detector pixel size set by the Nyquist spatial frequency, the pixel size is half the diffraction-limited size. The projected solid angle of the pixel is then given by Ωpixel = πΔλFWHM 2. The solid angle of the sun is Ωsun = 6.8 × 10⁻⁵ sr, hence f(Aτ) = Ωpixel Ωsun. Assuming λ = λτ = 630.15 nm, a telescope with an effective area of Aτ = 7854 cm², (i.e., Dτ = 1 m), gives Ωpixel = 1.2 × 10⁻¹³ sr, and f(Aτ) = 1.7 × 10⁻⁹. In the continuum, the incident solar flux is I(λτ) = 1645 erg cm⁻² nm⁻¹ sr⁻¹.

![Fig. 1. The confocal optical configuration. In this configuration, the distance, d, between the two spherical mirrors (bold area) is set equal to the confocal distance, r, the radius of the spherical mirrors. To the upper left of the confocal mirrors, a real object is located at S1. The transmitted light paths through the confocal mirrors and a lens are shown with the two resulting real images S1' and S1'' formed by 2nd or 2nd + 2 mirror reflections, respectively. To show the symmetry of the optical path, the virtual object S2 is shown. In this figure, the distance p1 from the optical axis of the entrant ray at mirror 1 and its distance p2 at mirror 2 are approximately equal. The two spherical mirror focal points are F1 and F2. The two reflected beams (oblique dash lines) result when the resonance interferometric condition is not satisfied. (After Johnson 1968)]
In the vertical direction. That is to dissect the line profile structure of investigated the line formation region to isolate the solar physics to a comparable spatial scale in the same scale as the transverse gradients of the flows and fields, the mean height for the photosphere is 1000, the number of electrons needed to be collected is a million. Hence, the exposure times is 18 s. However for photometry we can use 2x2 pixel binning, i.e., a resolution element, and reduce the exposure time to 4.5 s to collect a million photons. In the near infrared at $\lambda_{IR}=1564.85$ nm where the diffraction limit spatial resolution is larger but the solar flux is lower, $I(\lambda_{IR}) = 269$ erg cm$^{-2}$ mm$^{-1}$ s$^{-1}$; the photon flux is $n_{ph} = 1.6 \times 10^5$ photons per second per pixel and the exposure time is 2.9 s with an IR 0.5 QE detector. At both wavelengths it is assumed that the pixel well depth is $W_{p} = 1 \times 10^5$ e$^-$. Assuming Poisson counting noise is the dominate noise, the detector digitization then must be able to resolve $\frac{(W_{p}+n_{ph})}{\sqrt{n_{ph}}}$, e.g. 10 bit A/D. Multiple reads of the CCD is necessary but we have assumed that the readout rate is a fraction of the exposure time, hence the readout is ignored. We also assumed that the dark current can be ignored when compared with the Poisson noise.

For one spectral position the exposure times are acceptable considering the solar dynamics time scales with respect to a diffraction-limited resolution element on the sun. However to obtain a high-spectral-resolution line profile at 1 pm spectral resolution and at the diffraction limit, the exposure time become problematic. Stepping at an interval of 0.5 pm over a spectral scan range of 30 pm, the photoelectrons for this line profile are obtained in 270 s at $\lambda_{I}$ and 171 s for $\lambda_{F}$. Of this, of course is this the full field of view. To obtain the full Stokes profiles, the addition polarimetry requirements increase the total time by a factor of 6. Therefore, to obtain full Stokes profiles scans at a time significantly shorter than the solar oscillation period, the spatial resolution must be larger than the diffraction limit. For extended, horizontal stratified regions (e.g., penumbral Evershed flow regions) where resolving the vertical stratification is important, the employment of larger than diffraction-limited resolution would be acceptable and allow shorter exposure times.

A main emphasis for imaging interferometry is to provide high spatial resolution compatible with the spectral line information to be derived. The larger the gap between the etalon plates in either the parallel or spherical interferometers, the higher the spectral resolution becomes, i.e. the smaller the spectral passband. Since the large ground-based and space-based solar telescopes, which are planned or being built, produce higher transverse spatial resolution on the scale of 10-200 km (Table 1); it is desirable and necessary to also increase the spectral power to isolate the solar physics to a comparable spatial scale in the vertical direction. That is to dissect the line profile structure of the line formation region (e.g., photospheric lines have a height of formation FWHM ~200-500 km). In particular to determine the vertical (or line-of-sight) gradients of flows and fields, to the same scale as the transverse gradients of the flows and fields, and to interrupt the corresponding line asymmetries. The pressure scale height for the photosphere is ~150 km and the photon mean path is also on this scale.

Having high spectral resolution is important in determining the vertical gradients. Balasubramaniam and Uitenbroek (2001) investigated the velocity gradients over the height of formation of the non-magnetic sensitive iron line FeI 557.6 nm (Landé factor of $g=0$). They showed with numerical modeling that a 0.5 pm passband is required to resolve the ~100 m/s advective flows. A second example infers that a sub-picometer passband in the near infrared can determine the vertical magnetic gradient of the normal field. Sanchez Almeida (2006) showed theoretically that the vertical magnetic gradient can be derived from high spectral resolution observations of a single Zeeman sensitive line. As an example of this, the difference between the spectral Stokes $V$ profiles of a line formed within a magnetic field region with a decreasing gradient and of a line formed within a constant magnetic field is plotted in Figure 2. Using the umbra M model of Maltby et al. (1986) with a magnetic field strength of $B = 3000$ G, velocities of $v_{\text{adv}} = 1$ km/s and $v_{\text{adv}} = 0$ km/s, an inclination of 45 deg, and an azimuth of 45 deg, these magnetic field calculations were generated with vertical field gradients of 0, -3 and -6 G/km. Independent of the polarization strength, these differences can be resolved with sub-picometer spectral resolution. Observationally, Mathew et al. (2003) used a profile inversion program to infer a vertical magnetic gradient from the TIP observations ($\Delta \lambda_{FWHM} \approx 3$ pm) and obtained a value of ~4 G/km. With sub-picometer resolution, the magnetic gradient can be improved and compared with values from other Zeeman sensitive lines at higher heights.

Increasing the spatial resolution to well below 100 km has brought about a bogus concern that visible photospheric structure will be limited by photon diffusion to an optical mean-free-path, i.e. about one pressure scale height, due to smoothing by radiative transfer. If this were true, it would also effect the inversion of the line profile and hence set a limit on the useful spectral resolution. The argument was that the photon mean free path is ~200 km for unity optical depth at 500 nm ($\tau_{500} = 1$) (Allen 1973). Countering that argument, visible observations by Rouppe van der Voort et al. (2004, 2005) and Stein and Nordlund's (2006) hydrodynamic simulations have shown that much smaller structures exist and a resolution of less than 100 km is required for the photosphere. Bruls and von der Lühe (2001), in an analysis of radiative transfer effects on the visibility of small-scale structures, performed 2D non-LTE radiative transfer computations for thin flux sheets with widths ranging from 10 to 160 km in the solar photosphere. They demonstrated that such small structures could be observed as small scale variations of intensity and polarization. With the conclusion that the size limit where the photospheric structure cannot be observed (due to smoothing radiative transfer effects) must lie well below 100 km. Synthesis of polarimetry (e.g., AIR) and of a line formed within a constant magnetic field is plotted in Figure 2. Using the umbra M model of Maltby et al. (1986) with a magnetic field strength of $B = 3000$ G, velocities of $v_{\text{adv}} = 1$ km/s and $v_{\text{adv}} = 0$ km/s, an inclination of 45 deg, and an azimuth of 45 deg, these magnetic field calculations were generated with vertical field gradients of 0, -3 and -6 G/km. Independent of the polarization strength, these differences can be resolved with sub-picometer spectral resolution. Observationally, Mathew et al. (2003) used a profile inversion program to infer a vertical magnetic gradient from the TIP observations ($\Delta \lambda_{FWHM} \approx 3$ pm) and obtained a value of ~4 G/km. With sub-picometer resolution, the magnetic gradient can be improved and compared with values from other Zeeman sensitive lines at higher heights.

In the next section we will compare the confocal interferometer with other spectral filters in terms of field of view and aperture size. Then we will provide initial design parameters, explain the construction details for two designs, and report on the laboratory test of these units. In the conclusion, we will propose a multiple etalon system for future testing the piezoelectric-controlled unit to obtain sub-picometer spectral profiles of photospheric in both the visible and near-infrared. These observations will employ existing planar Fabry-Perot interferometers as prefilters to the confocal system.
The diffraction limit resolution for large solar telescopes using values in the text (λ = 630.2 nm / 1564.8 nm)

<table>
<thead>
<tr>
<th>Telescope Aperture D_{tel} (m)</th>
<th>Angular Resolution δ_{off} (arcsec)</th>
<th>Spatial Resolution ΔX (km)</th>
<th>Number of Diffraction Elements per arcsec</th>
<th>Full-Well-Depth Exposure Times at Diffraction Limit with 1 pm FWHM (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.16 / 0.39</td>
<td>115 / 285</td>
<td>39 / 6</td>
<td>0.16 / 0.02</td>
</tr>
<tr>
<td>2</td>
<td>0.08 / 0.20</td>
<td>57 / 142</td>
<td>159 / 26</td>
<td>0.16 / 0.02</td>
</tr>
<tr>
<td>4</td>
<td>0.04 / 0.10</td>
<td>29 / 71</td>
<td>639 / 103</td>
<td>0.16 / 0.02</td>
</tr>
<tr>
<td>6</td>
<td>0.03 / 0.06</td>
<td>19 / 48</td>
<td>1431 / 232</td>
<td>0.16 / 0.02</td>
</tr>
</tbody>
</table>

For the infrared iron line at λ_{Fe}=1564.85 nm, two differential Stokes V-profiles are shown. A differential profile is the difference between two Stokes profiles with one profile originating within the magnetic field and a second profile originating within the magnetic field of a different field direction. The two different gradients plotted are -6 G km^{-1} (thick line) and -3G km^{-1} (thin line). A sub-pixel/micrometer resolution will discriminate between these models, with sufficient photons and polarimetry. For the visible line, \( \Delta \lambda = \lambda_i \) while the incident angle produces an identical wavelength shift at the horizontal line in upper panel) that the off-axis oblique chief ray at the incident angle produces an identical wavelength shift at the central wavelength \( \lambda_c \). Free spectral range FSR, total finesse F, reflectance finesse F_{ext}, integer order number m, and passband (\( \Delta \lambda_{FWHM} \)) are given by the well-known relations: (i) \( \lambda_{ed} = 2 n \frac{d}{\cos(\theta/2)} \), (ii) FSR = \( \lambda_{ed} / 2 \), (iii) \( F_{ext} = \pi R^{(m+1)}/(1-R) \), (iv) \( \Delta \lambda_{FWHM} = \text{FSR}/F, \) and (v) \( \Delta \lambda_{FWHM} = \text{FSR}/F, \) where \( \Delta \lambda_{FWHM} \) is the central wavelength \( \lambda_c \), free spectral range FSR, total finesse F, reflectance finesse F_{ext}, integer order number m, and passband (\( \Delta \lambda_{FWHM} \)).

2. Confocal Interferometer Comparison with Other Filters

We will now compare the confocal interferometer with other imaging filter systems. For a 4-meter aperture telescope and for a given set of field of views (FOVs), Figure 3 compares the optical aperture requirement for the confocal interferometer with an air-gap and solid planar Fabry-Pérot interferometer (FP), a Lyot filter, and a Michelson interferometer. It shows that for a particular on-axis spectral resolution (e.g., 0.5 pm, dash horizontal line in upper panel) that the off-axis oblique chief ray at the incident angle produces an identical wavelength shift at the edge of the FOV of the same size as the on-axis resolution. For the particular filters this spectral shift is dependent on the refractive index of the gap or the particular type of interferometers. This typically sets the spectral resolution requirement for a collimated or telecentric optical system (Gary et al. 2003) and is due to the fact that the oblique chief ray angle in the interferometer sets the limits to the FOV for a given spectral resolution, via the Lagrange invariant or the ratio of the telescope aperture to the interferometer aperture. The confocal interferometer is similar to the Michelson interferometers in that it allows for a small aperture interferometer, compared to the 100 mm plus etalons for the air gap etalon. The solid Michelson interferometer is designed for one wavelength while confocal interferometers can cover a wide range of wavelengths dependent on the reflectivity function of the etalon coatings. The relative wavelength shift, \( \delta \lambda / \lambda \), as a function of the off-axis oblique chief-ray incident angle \( \theta \) is given by \( \delta \lambda / \lambda \approx (K/S)^2 \), where constant K is \( 1, 1/n_2^2, (n_2-n_1)/2n_2n_1, (p_{2}/p_{1}p_{2}/p_{1}, n_{2}p_{1}-n_{2}p_{2}) \), and \( \theta^2/2 \) respectively, for a wide-angle Fabry-Pérot filter, a solid Fabry-Perot filter, a wide field Lyot filter, a solid polarizing Michelson interferometer, and the confocal interferometer, where \( (n_1, n_2, n_{1} - n_{2} ) \) and \( (p_1, p_2) \) are particular refractive indices and thicknesses (Gary et al. 2003, Hercher 1968). From these wavelength relationships, the confocal interferometer is nearly equivalent to the Michelson filters, and these have an advantage over an air-gap Fabry-Pérot interferometer because a larger effective field of view can be obtained, or, equivalently, a smaller aperture can be used with a given field of view. The increase in aperture for an air-gap interferometer is a factor of ~2 for a solid-gap (LiNbO_3) etalon, a factor of ~10 for a wide field, birefringent, Lyot filter with split-calcite elements, and a factor of ~100 for a confocal and a polarizing Michelson interferometer. However, for the ability to make multi-wavelength observations at sub-picometer spectral resolution, the advantage goes to a confocal interferometer.

2.1. On-Axis Modeling the Confocal Interferometer

The characterization of a parallel plate Fabry-Pérot interferometer (FP) for a telecentric beam configuration gives an effective FWHM dependent on the maximum angle of incident \( \theta \), while for a collimated beam configuration there is a variation of line center across the FOV. The single etalon equations for the central wavelength \( \lambda_c \), free spectral range FSR, total finesse F, reflectance finesse F_{ext}, integer order number m, and passband (\( \Delta \lambda_{FWHM} \)).
Each interferometer both performing in the visible and near infrared. For the FPP parameters, a better choice than the FPP when \( \Delta r = 2.12 \delta \), the étendue relation \( U_{FFP} > U_{FP} \) states that the confocal interferometer has greater light collecting property (Vaughan 1989). For large-aperture solar telescopes, the critical result is that a smaller aperture can be used for the same number of photons passing through.

We have designed and built two sub-picometer confocal interferometer both performing in the visible and near infrared. Each FPS was designed with a \( \Delta \lambda_{FWHM} = 0.13 \) pm (FSR=2 pm) at \( \lambda_V = 630.2 \) nm and \( \Delta \lambda_{FWHM} = 0.8 \) pm (FSR=12 pm) at \( \lambda_{IR} = 1564.85 \) nm. This allows the solar FeI Zeeman sensitive lines at \( \lambda_V \) and \( \lambda_{IR} \) to be scanned and allows a frequency stabilized HeNe laser line (\( \lambda_{HeNe} = 632.8 \) nm) to be employed for laboratory test. The relationship between FSR, FWHM, reflectance, and confocal separation is shown in Figure 4 for the visible and near infrared lines. Although the plot of reflectance versus FSR and FWHM is independent of the wavelength, we have separated the line plots into two reflectance groups in order to show their correlation with the confocal separation distances for \( \lambda_V \) and \( \lambda_{IR} \). For the two wavelengths, the largest two squares plot the corresponding design values for \( R=0.92 \) for the same separation distance, \( r=50 \) mm. The set of box symbols at the ordinate FSR=1 pm refer to a separation of \( r=100 \) mm for \( \lambda_V \). The reflectance finesse is \( F_R = \pi R(1-R^2) = 18.8 \) and the resolv-
2.2. Off-Axis Modeling the Confocal Interferometer and Imaging Theory

An optical configuration for the confocal interferometer is shown in Figure 1 within the paraxial approximation. The rays starting from $S_1$ after making quadruple (or $2^4$) reflections are reentrant with the original direction. After making two reflections or $2^2$, another set of rays emerge and form an additional image ($S'_1$). For each of the two beams, the FPS intensity is 50% that of a FPP, however, in principle, the second beam could be recombined or used. This second image appears to be coming from the virtual source at $S_2$. There are two beams that emerge back toward the source side (dashed arrows in Figure 1). Clark (1972) gives the imaging relationships for an imaging confocal optical interferometer with the etalon plates with a radius of curvature given by $r$ and a separation of $d = r + e$, where $e$ is a small displacement. For $e=0$, the separation is the exact confocal distance. The phase difference for the rays is given by

$$\delta_1 = \frac{2\pi}{\lambda}\left(\rho_2^3 - \rho_1^3 + 2\pi(\rho_1^2 + \rho_2^2)\right) + \delta_{\text{on-axis}},$$

where $\rho_1$ and $\rho_2$ are the distances of the ray from the optical axis at the first and second spherical etalon surface, $\delta_{\text{on-axis}} = 8\pi(r+e)/\lambda$ is the on-axis phase difference; and, for $\rho = \rho_1 = \rho_2$, the off-axis phase difference is given by

$$\delta_1 = \delta_{\text{on-axis}} = \frac{2\pi}{\lambda}\left(\alpha^2/16 + \cos^2\theta\right),$$

where $\alpha = 2\rho/r$ (Vaughan 1989, eqns. 5.6 and 5.7). Using Fermat’s principal, Clark (1972) determines the best focus, for a specific $\rho$, using the central diffraction angle $\alpha_c$ for which $\partial(\delta)/\partial \alpha = 0$. This also sets the largest range of $\alpha$ for which the spectral information is passed and the associated spatial resolution. For input intensity $I_0$, the output diffraction intensity for the $S'_1$ is given by

$$I(\alpha) = (I_0/2) [1/(2F/\pi)^2 \sin^2(\delta_1/2)],$$

and $\alpha_c = (8\pi r)/\lambda$. A plus and minus sign denotes if the wavelength is longer or shorter than the central peak. On the two sides of the intensity peak at HWHM (half width at half maximum), the phase difference is $\delta_1 = (2\pi/\lambda)(4\pi r/\lambda F) + \delta_{\text{off-axis}}$, hence the cutoff angles become

$$\alpha_c = \pm(1/(8\pi r))(16/\lambda F)^{1/2},$$

which gives the variation of acceptable angles for which the passband is within FWHM. Hence if $\alpha_c = 0$ then $\alpha_c = (64\pi r F)^{1/2} \sim 2.8(\Delta\lambda_{\text{FWHM}}/\lambda)^{1/2} \sim 5.15$ deg. This is consistent with the angle $2p/\pi \sim 2(\lambda/\pi F)^{1/2} \sim 3.6$ deg, derived from Hercher’s (1968, eqn. 17) spot radius. Therefore the FOV is approximately 4 deg.

3. Construction of a Confocal Interferometer

For testing the image quality of the confocal interferometers, we have constructed two confocal interferometers using IC Optical Systems spherical mirrors of $r=50.065$ mm internal confocal radius and external radius of $r_2=60.30$ mm. These 25 mm diameter fused silica plates (n=1.459) have a 15 mm clear aperture and a plate thickness of 10.235 mm. The measured mirror coating transmittances is 5.13% at 630.98 nm and 4.97% at 1564.2 nm. The exterior surfaces, with anti-reflecting coatings for both wavelengths (R<0.25%), were made concentric with a radius of curvature of 60.25 mm. By employing concave-convex etalon plates and the plate thickness, the optical power of each etalon plate is minimized for imaging quality and spectral resolution. The thick lens formula $f^{-1} = -(n-1)(1/\rho_1 + 1/\rho_2) + (n-1)/n \rho_1 \rho_2$ gives a relative low-power, large focal length of $f \sim 623$ mm (while an optical Zemax code gives $f \sim 925$ mm).

In order to calibrate the etalon we need to be able to scan j-free spectral ranges, i.e., the confocal displacement should be $\delta d_j = j\lambda/4n$, a few microns. The spectral resolution should be $\sim 1/10^{th}$ of the $\Delta\lambda_{\text{FWHM}}$, or about $(\lambda/4n)/(10 \mu m)$. To infinitesimally displace the mirrors by this amount, the two confocal interferometers were designed to have good mechanical and environmental stability (Figure 5). The piezoelectric-controlled interferometer follows the design by Budker et al. (1999) (cf., Vaughan 1989, see 5.5.1). The thermally-controlled FPS design was engineered to allow initial precise displacement near the confocal separation.

3.1. Piezoelectric-Tunable FPS

The piezoelectric controller utilizes a thin-wall hollow cylinder of lead zirconate titanate (Pb(Zr$_{0.53}$Ti$_{0.47}$O$_3$, PZT) made of Channel Industries C-5700 and milled by Boston Piezo Optics.
Fig. 6. Near confocal Fabry-Pérot fringe patterns. The curves give the radii of the circular interference pattern for collimated light in the paraxial approximation.

Fig. 7. The two constructed confocal interferometers with sub-pico meter spectral resolution. For each, the confocal distance is 50 mm and the aperture is 25 mm.

The calibration between voltage and displacement is required to correct for hysteresis and nonlinearity (Figure 8). For a voltage of 60 volts, the linear displacement along the cylindrical axis is given by $\Delta d = \Delta V/\lambda$, where $\Delta V = -0.25$ nm/volt is the piezoelectric constant, $\lambda$ is the length of the cylinder, and $t$ is the wall thickness of the cylinder; then $\Delta d = 2.5 \text{ nm} \cdot \Delta V$ or $\Delta d = 4.5 \times 10^{-3} \text{ A} \text{ V}$. A voltage of 60 volts allows tuning through one free spectral range for the visible line. For a finesse of $F=13$, then the displacement resolution of the tube should be 1.2 nm or $\Delta V(0.5) \approx 0.25$ nm with a voltage resolution of $\Delta V/500$. The coefficient of thermal expansion led to a displacement of $\sim 100 \text{ nm/C}$, hence the temperature needs to be stable or known to within $\sim 0.1 \text{ C}$. The calibration between voltage and displacement is required to correct for hysteresis and nonlinearity (Figure 8).

The construction schematic is shown in Figure 5. The rear fixed mirror (M2) is in a groove attached with cyanacrylate adhesive. The front mirror (M1) is attached to the piezoelectric cylinder (PZT) with epoxy. Between the Invar steel mounting tube and the PZT ceramics cylinder, a thin plastic-shim angular ring is placed to insure that the nickel electrodes of the PZT will not be shorted by touching the steel tube. The shim is attached to the Invar by cyanacrylate adhesive. The PZT cylinder is attached to the shim and to the Invar via epoxy. There is a section of the shim material (1/8" of the ring) that is missing and hence the epoxy over this section is directly attached to the Invar. The wires to the PZT cylinder were attached to the nickel coated walls using Sn-62 solder. The steel parts are milled Freecut Invar 36 Alloy which is a low thermal expansion alloy, and allow a threaded-screw displacement for setting the initial separation of the mirrors.

### 3.2. Thermally-Adaptive FPS

The thermally-adaptive design uses a 50 mm-long glass cylinder between the confocal interferometer mirrors (Figure 7, upper panel). Initially all zero expansion glass cylinder was considered with a thermal coefficient of expansion of $C_{TF}=0.05 \text{ ppm/C}$. Using a thermal system that controls the temperature to $\Delta T=0.1 \text{ C}$, implies controlling the spacing by steps of $\Delta e = e C_{TF} \Delta T = 50 \text{ mm} \times 0.000000005 \times 0.1 = 0.00025 \text{ nm}$. However, for a system working in the IR at $\lambda_{IR}$ (or in the visible at $\lambda_{V}$) this change represents a change in the filter wavelength of $\delta \lambda = 4 \delta e / \lambda$, hence $\delta e = \lambda \delta \lambda / 4 \lambda$.

Using a thermal system that controls the temperature to $\Delta T=0.1 \text{ C}$, implies controlling the spacing by steps of $\Delta e = e C_{TF} \Delta T$. For $\lambda = 630.2 \text{ nm}$, the intensities for $\Delta \lambda = 0.1 \lambda$ are $6.0 \times 10^{-6}$ at $\lambda_{IR}$ and $1.3 \times 10^{-5}$ at $\lambda_{V}$. Hence, for a system working in the IR at $\lambda_{IR}$ (or in the visible at $\lambda_{V}$) this change represents a change in the filter wavelength of $\delta \lambda = 4 \delta e / \lambda$, hence $\delta e = \lambda \delta \lambda / 4 \lambda$.
for reasonable total changes in temperature. Therefore, the thermal control design was refined.

The refinement was produced by employing a dual-glass cylinder (BK7 and Zerodur) separating the spherical mirrors, where the thermal expansion of the glass cylinder controls the inter-etalon spacing. Assuming a set of characteristics for the FWHM and FSR at \( \lambda_u \) and \( \lambda_l \) and assuming that at least 5 points are needed across the FWHM to define the profile sufficiently, then Table 2 was generated assuming that the thermal control resolution is 0.1 C. Using (de/e) = (CTE \( \Delta T \)) = (\( \Delta l/l \)) and with the assumption that \( \Delta l = 0.02 \) pm, i.e. the step size, and \( \Delta l \) = 0.08 pm for a change of \( \Delta T = 0.1 \) C, then (de/e) = (0.02x10^{-6}/630.2) = 3.17x10^{-8}, (de/e) = (0.08x10^{-6}/1564.85) = 5.1x10^{-8} for \( \Delta T = 0.1 \) C. Therefore, with a change of \( \Delta T = 1 \) C, we have the thermal coefficient of expansion requirements for the respective wavelengths: CTEBK = (de/e) = 3.17x10^{-8} and CTEBK = (de/e) = 5.1x10^{-8}. Using a segment of length of L1 of Zerodur (denoted by Z) with its TCE = 5x10^{-8} and a segment of length L2 of BK7 with CTEBK = 7x10^{-8} then we can obtained the correct overall CTE by solving: L1CTEZ + L2CTEBK = (L1 + L2)CTEBK for 630.2 nm. Hence if the overall length is L1 = L2 then L1(CTEZ - CTEBK) = (L - L1)(CTEBK - CTEBK) or L1 = L(CTE - CTEBK)/(CTE - CTEBK). For L1(L2 = (3.17x10^{-8} - 7.1x10^{-8})/5x10^{-8} = 0.962 and for L = 50 mm, then L1 = 48.1 mm and L2 = 1.89 mm for BK7. Therefore by introducing a 1.89 mm segment of BK7, the combination provides the correct over all CTE need for the vis-
cugl

torc transducer (type AD590) with a heater strip. The measured time constant for the constructed interferometer was 360 s. The resulting temporal sensitivity is too long for solar observations; however the dual-glass cylinder has no associated hysteresis.

### Table 2. Initial thermal design requirements with \( R = 0.9 \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \lambda_u = 630.2 ) nm</th>
<th>( \lambda_l = 1564.85 ) nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>FWHM</td>
<td>100 pm</td>
<td>0.40 pm</td>
</tr>
<tr>
<td>FSR</td>
<td>2 pm</td>
<td>15 pm</td>
</tr>
<tr>
<td>Step Size (at 5pts/FWHM)</td>
<td>0.02 pm</td>
<td>0.08 pm</td>
</tr>
<tr>
<td>Number of points to cover the full FSR</td>
<td>100</td>
<td>188</td>
</tr>
<tr>
<td>Change in temperature required to cover FSR</td>
<td>10C</td>
<td>19C</td>
</tr>
</tbody>
</table>

### 4. Alignment

The parallel plates of a FPP must be exactly parallel, i.e., to within \( \Delta D \sim 10^{-8} \) radians. Conversely, because of the spherical shape of the FPP, the alignment of the centers of the mirrors needs to be only roughly aligned, to within about 0.1 D/r = 10^{-2} radians. This allows the axial alignment to be glued in place with no further improvement in tilt. For the axial separation of the confocal mirrors, the precision is more critical than the axial alignment and is given by \( \Delta r = (r \Delta r/32F) \Delta /2 \), hence, for \( r = 50 \) mm, \( \Delta = 1.564 \) mm, \( F = 15 \), \( \Delta r = 13 \) mm \( \sim 8 \) (Vaughan 1989, p. 195). However, setting the initial separation is straight for-

tward (Hercher 1968), and is based on the fringe movement about the confocal separation. The fringe pattern radius for fringe orders \( m = [-10,10] \) is plotted versus the departure from the con-

![Fig. 9. An image of part of an Air Force Resolution Chart imaged at \( \lambda = 630.2 \) nm through the FPS. Both exit images (S1' and S2') of the FPS are seen.](image)

### 5. Test Results

The confocal interferometer sub-picometer test results are given in terms of profile line scanning and imaging. In Figure 8, the results are shown using the piezoelectric-controlled FPS. The scans show the hysteresis effect of the lead zirconate titanate crystal (cf. Crawford 1961). A Powell multiple parameter least-
squares fit of the scans determined that the sub-picometer spectral resolution was FWHM = 0.33 pm with FSR = 1.99 pm, F= 6.0, and effective reflectivity of 0.77. These acceptable values include the effect of complete optical configuration and hence the finesse is lower than the theoretical value.

Figure 9 shows the imaging capability of the confocal interferometer. An U.S. Air Force Resolution Chart is imaged at \( \lambda = 630.2 \) nm. For this large field of view, the two images of the FPS are captured by the CCD camera, and the effects of spherical aberrations can be seen at the outer edge of the image. The center of the optical axis is near the center of the figure. For this magnification, the resolution is limited by the CCD.

Employing larger magnification with a zoom lens, Figure 10 shows the center of USAF resolution charts imaged employing an entrance mask with an aperture of \( 2r = 2r(\Delta/rF) \Delta /2 \) = 3.8 mm were F=6 and \( \lambda = 630.2 \) nm. The image was formed using an negative pattern. 1951 USAF resolution chart (Edmund Optics NT38-256, maximum resolution 228 lp/mm, i.e., Group 7/Element 6) using a 175 mm focal length lens with the image and object distances equal (350 mm). The 4 mm aperture was...
Near Infrared Magnetograph (NTRM) could be employed which using the Dunn Solar Telescope. Also for the NTR, the New Peak’s high-resolution dual Queensgate etalons ET50-1001 (FSR=72.1 pm) and ET50-998 (FSR=3932 pm) could be employed. In the near IR, the etalons ET50-993 (FSR=11.3 pm, FWHM=522.5 pm) and ET50-998 (FSR=168.3 pm, FSR=3932 pm) could be employed as a prefILTER for the FPS. For a S/N of 500, a 15-point \( \Delta R \) spectral profile scan time is estimated to be 40 minutes using the Dunn Solar Telescope. Also for the NIR, the New Jersey Institute of Technology/Big Bear Solar Observatory’s Near Infrared Magnetograph (NIRM) could be employed which has a FPS with FWHM=8.8 pm and FSR=548.7 pm. We plan to employ these systems to obtain sub-picometer spectral resolution.

lution Stokes I and V line profiles in the umbra regions to determine the vertical magnetic field and velocity gradients across the height of formation of the spectral lines Fe I \( \lambda R \) and \( \lambda HR \). However, the final spectral information will be degraded by the relatively long integration times.

With the construction of ground-based solar telescopes with aperture larger than a meter and the planned National Solar Observatory’s 4-meter Advanced Technology Solar Telescope, the observed transverse spatial resolution will be less than the half-width of the vertical height of formation of the observed spectral lines. Using a confocal sub-picometer interferometer, the observations can be made of the same spectral line to produce vertical height information compatible to the transverse scale within a time frame consistent with the resolution. These observations can be supplemented with multiple line observations; however, multiple line analysis relies on the assumption that the height of formation of the atomic levels can be directly related to the height of formation of the observed line. The radiometry calculations given in the introduction confirm that such observations can be made, and we argue that these should be pursued.

With the completion of the future observations listed in the last paragraph, the usefulness of the observations will be tested.

6. Conclusions

The confocal interferometer sub-picometer FWHM spectral filters for visible and near-IR solar magnetography and Dopplergraphy will lead to a better understanding of the magnetic field and advective flow gradients through the solar atmosphere and links observation between different spectral lines at various heights. For high spectral resolution, a confocal (also called a spherical or Connes) interferometer enhances the scientific value of future solar satellite missions by increasing the field of view and throughput of magnetographs, decreases the cost of the mission by allowing smaller optical components, and increases the scientific value of future solar satellite missions by increasing the field of view and throughput of magnetographs, decreases the cost of the mission by allowing smaller optical components, and increases the mechanical tolerances of the interferometer. We have developed, tested, and demonstrated the imaging capabilities of a very narrow-pasband confocal interferometer. The FPO and spatial resolution of a FPS is set by the diffraction limit of the confocal spot size. The spectral resolution is set by the adjustment of the confocal separation. The FPS can be used for imaging and provides sub-picometer spectral resolution. The high throughput of the FPS as compared to the FPS makes this interferometer the choice for obtaining high resolution solar line profiles.

For future studies, there are two imaging FPS instruments that can be used as prefilters for our FPS research on solar phenomena. In the visual and the near IR, the NSO Sacramento Peak’s high-resolution dual Queensgate Fabry-Pérot interferometers can be employed, with the narrowest passband compatible with the FPS’s FSR. At \( \lambda = 630.2 \) nm the NSO’s 50 mm etalons ET50-1001 (FWHM=9.0 pm, FSR=327 pm) and ET50-1046 (FWHM=1.7 pm, FSR=72.1 pm) can be employed. In the near IR, the etalons ET50-993 (FWHM=11.3 pm, FSR=522.5 pm) and ET50-998 (FWHM=168.3 pm, FSR=3932 pm) could be employed as a prefILTER for the FPS. For a S/N of 500, a 15-point \( \Delta R \) spectral profile scan time is estimated to be 40 minutes using the Dunn Solar Telescope. Also for the NIR, the New Jersey Institute of Technology/Big Bear Solar Observatory’s Near Infrared Magnetograph (NIRM) could be employed which has a FPS with FWHM=8.8 pm and FSR=548.7 pm. We plan to employ these systems to obtain sub-picometer spectral resolution.