Solar Confocal Interferometers for Sub-Picometer-Resolution Spectral Filters

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ABSTRACT

Aims. The confocal Fabry-Pérot interferometer allows sub-picometer spectral resolution of Fraunhofer line profiles. Such high spectral resolution is needed to keep pace with the higher spatial resolution of the new set of large-aperture solar telescopes. The line-of-sight spatial resolution derived for line profile inversions would then track the improvements of the transverse spatial scale provided by the larger apertures. In particular, profile inversion allows improved velocity and magnetic field gradients to be determined independent of multiple line analysis using different energy levels and ions. The confocal interferometer’s unique properties allow a simultaneous increase in both étendue and spectral power. The higher throughput for the interferometer provides significant decrease in the aperture, which is important in spacecraft considerations.

Methods. We have constructed and tested two confocal interferometers. A slow-response thermal-controlled interferometer provides a stable system for laboratory investigation, while a piezoelectric interferometer provides a rapid response for solar observations.

Results. In this paper we provide design parameters, show construction details, and report on the laboratory test for these interferometers. The field of view versus aperture for confocal interferometers is compared with other types of spectral imaging filters. We propose a multiple etalon system for observing with these units using existing planar interferometers as pre-filters. The radiometry for these tests established that high spectral resolution profiles can be obtained with imaging confocal interferometers. These sub-picometer spectral data of the photosphere in both the visible and near-infrared can provide important height variation information.

However, at the diffraction-limited spatial resolution of the telescope, the spectral data is photon starved due to the decreased spectral passband.

Key words. Sun: atmosphere – Instrumentation: interferometers – Line: profiles – Sun: magnetic fields

1. Introduction

Solar physicists are beginning, on a regular basis, to spatially resolve below 100 kilometer on the sun by employing optical 1-meter class, and larger, aperture telescopes (Schlichenmaier 2006, Bettonvil 2004, Keil et al. 2004, Bernasconi, et al. 1999). It is conceivable to resolve to ~10 km in the next couple of decades with semi-monolithic or interferometric telescopes (Davis et al. 2005, Hammerschlag et al. 2004, Damé 1994). These operational, under-construction, or conceived telescopes include Flare-Genesis (0.8 m), Themis (0.9 m), Swedish Solar Telescope (1.0 m), SunRise (1.0 m), McMath-Pierce (1.5 m), Gregor (1.5 m), Dutch Open Telescope++ (1.5 m), New Big Bear Solar Telescope (1.6 m), Advance Technology Solar Telescope (4.0 m), Magnetic Transition Region Probe (6.0 m) and Giant Solar Optical Telescope (11.0 m). This effort of larger apertures for improved spatial resolution has been conducted in unison with superior designs and techniques (e.g., improvements in (i) image stabilization by adaptive optics for ground-based observatories, (ii) thermal designs to reduce locally induced seeing effects, (iii) polarization resolution by lowering instrumental effects, (iv) the size of the CCD arrays to increase the field of view, and (v) imaging processing to allow improved data analysis). Radiative transfer theory has improved with advanced magnetohydrodynamical numerical simulations at the sub-granular scales to resolve elemental flux tubes. However, visible-infrared, high-spectral-resolution narrow-band imaging filters at a spectral resolution of $\Delta \lambda / \lambda < 10^{-8}$ have not received much attention. This spectral resolution is currently provided by the non-imaging Fourier Transform Spectrometer (FTS) on the National Solar Observatory (NSO)/McMath-Pierce Telescope. Using the FTS and studying elemental flux tubes, Stenflo et al. (1984) point out that the spectral measurements can be independent of the unresolved spatial elements, and hence high spectral resolution is a key component to understanding the structure of the sun. Mathew et al. (2003) have used the 3 pm full width half maximum (FWHM) spectral resolution Tenerife Infrared Polarimeter (TIP) and inverted the FeI 1.56µm Stokes profiles using the SPINOR code to obtain an estimate of the magnetic fields gradient in an umbra region. In general, the line-of-sight spatial resolution derived from profile inversions should follow the improvements in the transverse (i.e., image plane) spatial resolution with larger apertures; however this will require sub-picometer spectral resolution. It is clear that the use of several lines with different magnetic, thermal, and Doppler sensitivities will help resolve the vertical information. However this process has its problems associated directly with the sensitivity variation which is used. i.e. different mass, different ions, different transitions, and different Landé factors. Through using a single transition and ex-

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ploring the depth by its variation of opacity with position in the line (or response function) we avoid these sensitivity differences. Furthermore we will show that the trade-off between the signal-to-noise ratio and spectral resolution is acceptable. In this paper we present the spherical or confocal interferometer, an unnoticed technique in solar physics, which shows promise in combining high-spectral resolution and imaging, such that vertical gradients in mass advective flows and magnetic field gradients can be derived on a scale compatible with the horizontal gradients.

Normal or planar Fabry-Pérot interferometers (FP) have a pair of mirrored parallel plates (etalons) and the light passes through the system when the gap between the plates is an integer number of the wavelength (Fabry and Pérot 1902, Vaughan 1989). The FWHM spectral passband ($\Delta \lambda_{\text{FWHM}}$) is dependent on the separation between the plates. These FPFPs are used as standard narrow passband imaging filters in solar physics in the US Air Forces Optical Solar Patrol Network (OSPAF, $\Delta \lambda_{\text{FWHM}} \sim 10$ pm), University of Hawai`i’s Mees Solar Magnetograph ($\Delta \lambda_{\text{FWHM}} \sim 7$ pm), Göttingen Astrophysical Institute’s FPI ($\Delta \lambda_{\text{FWHM}} \sim 6.6$ pm), Arcetri Observatory’s Interferometric Dimensional Spectrometer (IBIS, $\Delta \lambda_{\text{FWHM}} \sim 2.3$ pm), Kiepenheuer Solar Physics Institute’s Triple Etalon Solar Spectrometer (TSES, $\Delta \lambda_{\text{FWHM}} \sim 1.9$ pm), and NSO/Sacramento Peak Observatory’s dual etalon systems ($\Delta \lambda_{\text{FWHM}} \sim 1.7$ pm). For narrower FWHM filters with larger gap distances, the confocal interferometer should be considered. A confocal (spherical) Fabry-Pérot interferometer (FPS) differs from the parallel-plate interferometer in that the mirrored etalons are identical spherical surfaces where the center of radius is at the surface of the other mirror. (Confocal means having the same foci). Because of the spherical surfaces, the confocal interferometer can increase simultaneously both the spectral resolution and throughput and should be the preferred choice of interferometers for sub-picometer spectral filters (Hercher 1968, Vaughan 1989).

The path of the light through the confocal interferometer is typically a figure-eight shape (Figure 1) with the exit beam re-entering the direction of the initial beam (to within paraxial approximation) (Connes 1958, Hercher 1968, Clark 1972, Vaughan 1989). For the initial incident direction, the interference contributions to an exit ray come after 2$^N$ reflections, where k is a positive integer. The spherical mirrors and their associated thickness affect the final imaging and spectral resolution however we shall show this is not a significant effect. A main point of this paper is to describe the imaging quality of the confocal interferometers and the possible improvements that they could provide for solar physics, since the confocal interferometers have greater throughput and greater alignment tolerances than parallel plate interferometers under certain circumstances. An offset in confocal mirror alignment results in only a change in the optical axis of the system, when the confocal distance, $r$, is corrected by normal tuning. This results in a necessary alignment of the mirror centers to only the order of $\sim 0.1 D/r$; where a planar Fabry-Pérot interferometer, the alignment tolerance for parallelism is on the order of $\sim \lambda/D$, where $\lambda$ is the wavelength and $D$ is the aperture.

One concern with increasing the spectral resolution is that this implies a deficit of photons as the spectral passband is reduced, causing longer exposure times. This is obviously problematic if the diffraction-limited spatial resolution is retained. For then the dependency of the size of the telescope is canceled since the light collected is a product of the telescope area and the spatial area sampled. If the aperture is varied it will be found that the product of the telescope’s aperture area and the area of the diffraction-limited resolution element on the sun is constant. At the diffraction limit resolution, the exposure times are dependent of the aperture size (Table 1, last column). Therefore, to collect a reasonable number photons for spectral analysis a reduced spatial resolution is require, as we will demonstrate. We will now consider the exposure times for two wavelengths, one in the visible and one in the near infrared.

The exposure time is determined by the time it takes to fill each detector pixel well with electrons. This time is governed by the well capacity of the CCD and the number of electrons generated each second by the light incident on each pixel. This later parameter is itself determined by the quantum efficiency of the CCD and the number of photons incident each second on the pixel, $n_p$. To determine $n_p$, we consider the total radiant energy incident on a detector pixel per second, $P_i$, $P_i$ is equal to the solar radiant power per second per unit area per unit wavelength interval at the wavelength of interest, $I(\lambda)$, multiplied by the wavelength passband, $\Delta \lambda_{\text{FWHM}}$, the area of the telescope $A_T$, and the effective area of a pixel on the sun, $f(A_T)$, and corrected for transmission losses, $i$: $P_i = I(\lambda) \Delta \lambda_{\text{FWHM}} A_T f(\lambda) \, i$. The fraction of the total area collected by the telescope that is imaged on a detector pixel is equal to the ratio of solid angle of the projected pixel and the solid angle of the sun.

Observing at the telescope diffraction-limited resolution with the detector pixel size set by the Nyquist spatial frequency, the pixel size is half the diffraction-limited size. The projected solid angle of the pixel is then given by $\Omega_{\text{pixel}} = \pi (D/2)^2$. The solid angle of the sun is $\Omega_{\text{sun}} = 6.8 \times 10^{-3}$ sr, hence $f(\lambda) = \Delta \lambda_{\text{FWHM}} / \Omega_{\text{pixel}}$. Assuming $\lambda = \lambda_T = 630.15$ nm, a telescope with an effective area of $A_T = 7854$ cm$^2$, (i.e., $D = 1$ m), gives $\Omega_{\text{pixel}} = 1.2 \times 10^{-13}$ sr, and $f(\lambda) = 1.7 \times 10^{-7}$. In the continuum, the incident solar flux is $I(\lambda_T) = 1645$ erg cm$^{-2}$ nm$^{-1}$.
s\(^{-1}\) at the wavelength \(\lambda_0\). With optics providing a total optical transmittance of \(t_e = 0.01\) and a filter of \(\Delta \lambda_{FWHM} = 1\) pm, we have:

\[
P_i = I(\lambda) \Delta \lambda_{FWHM} A_{\gamma}\gamma, = \frac{(1645 \text{ erg cm}^{-2} \text{ nm}^{-1} \text{ s}^{-1})(0.001 \text{ nm})}{(7845 \text{ cm}^{-1})} (1.7 \times 10^{-4}) (0.01) = 2.2 \times 10^{-5} \text{ erg s}^{-1}.
\]

At a wavelength of \(\lambda_0 = 650.15\) nm, each photon has an energy of \(3.15 \times 10^{-12}\) ergs, so there is a flux of \(n_p = 6.9 \times 10^4\) photons per second on a pixel. With a 0.8 quantum-efficiency (QE) detector the number of electrons per sec is determined by

\[
I(\lambda) = 269 \text{ erg cm}^{-2} \text{ nm}^{-1} \text{ s}^{-1}; \quad \text{the flux is}\ n_p = 1.6 \times 10^5 \text{ photons per second per pixel}\text{.}
\]

If the exposure time is 18 s, however, with photometry we can use 2x2 pixel binning, i.e., a resolution element, and reduce the exposure time to 4.5 s to collect a million photons. In the near infrared at \(\lambda_{IR} = 1564.85\) nm where the diffraction limit spatial resolution is larger but the solar flux is lower, \(I(\lambda) = 269 \text{ erg cm}^{-2} \text{ nm}^{-1} \text{ s}^{-1}\); the photon flux is \(n_p = 1.6 \times 10^5\) photons per second per pixel and the exposure time is 2.9 s with an IR 0.5 QE detector. At both wavelengths it is assumed that the pixel well depth is \(W_p = 1 \times 10^6\) e\(^{-}\). Assuming Poisson counting noise is the dominate noise, the detector digitization then must be able to resolve \(I(W_p / 255, e)\); e.g., 10 bit A/D.

Increasing the spatial resolution to well below 100 km has brought about a bogus concern that visible photospheric structure will be limited by photon diffusion to an optical mean-free-path, i.e., about one pressure scale height, due to smoothing by radiative transfer. If this were true, it would also affect the inversion of the line profile and hence set a limit on the useful spectral resolution. The argument was that the photon mean free path is \(~\sim 200\text{km}\) for unity optical depth at 500 nm (\(r_p = 1\) (Allen 1973)). Countering that argument, visible observations by Rouppe van der Voort et al. (2004, 2005) and Stein and Nordlund’s (2006) hydrodynamic simulations have shown that much smaller structures exist and a resolution of less than 100 km is required for the photosphere. Bruls and von der Lühe (2001), in an analysis of radiative transfer effects on the visibility of small-scale structures, performed 2D non-LTE radiative transfer computations for thin flux sheets with widths ranging from 10 to 160 km in the solar photosphere. They demonstrated that such small structures could be observed as small scale variations of intensity and polarization. With the conclusion that the size limit where the photospheric structure cannot be observed (due to smoothing radiative transfer effects) must lie well below 10 km. Synthesis of line profiles from dynamical flux tube simulations suggests that visible structure in Stokes V will exist down to size scales of 5 km (Stein and Nordlund 2006). Hence scales much smaller than 100 km should be resolvable with imaging and spectral profile inversions.

In the next section we will compare the confocal interferometer with other optical systems in terms of field of view and aperture size. Then we will provide initial design parameters, explain the construction details for two designs, and report on the laboratory test of these units. In the conclusion, we will propose a multiple etalon system for future testing the piezoelectric-controlled unit to obtain sub-picomter spectral profiles of photospheric in both the visible and near-infrared. These observations will employ existing planar Fabry-Perot interferometers as prefilters to the confocal system.
Table 1. The diffraction limit resolution for large solar telescopes using values in the text (L = 630.2 nm / 1564.8 nm)

<table>
<thead>
<tr>
<th>Telescope aperture D (m)</th>
<th>Angular Resolution ( \theta_{off} ) (arcsec)</th>
<th>Spatial Resolution ( \Delta X ) (km)</th>
<th>Number of Diffraction Elements per arcsec</th>
<th>Full-Well-Depth Exposure Times at Diffraction Limit with 1 pm FWHM (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.16 / 0.39</td>
<td>115 / 285</td>
<td>39 / 6</td>
<td>0.16 / 0.02</td>
</tr>
<tr>
<td>2</td>
<td>0.08 / 0.20</td>
<td>57 / 142</td>
<td>159 / 26</td>
<td>0.16 / 0.02</td>
</tr>
<tr>
<td>4</td>
<td>0.04 / 0.10</td>
<td>29 / 71</td>
<td>639 / 103</td>
<td>0.16 / 0.02</td>
</tr>
<tr>
<td>6</td>
<td>0.03 / 0.06</td>
<td>19 / 48</td>
<td>1431 / 232</td>
<td>0.16 / 0.02</td>
</tr>
</tbody>
</table>

2. Confocal Interferometer Comparison with Other Filters

We will now compare the confocal interferometer with other imaging filter systems. For a 4-meter aperture telescope and for a given set of field of views (FOVs), Figure 3 compares the optical aperture requirement for the confocal interferometer with an air-gap and solid planar Fabry-Pérot interferometer (FPI), a Lyot filter, and a Michelson interferometer. It shows that for a given field of view, the decrease in aperture for an air-gap interferometer is a factor of ~2 for a solid-gap (LiNbO₃) etalon, a factor of ~10 for a wide field, birefringent, Lyot filter with split-calcite elements, and a factor of ~100 for a confocal and a polarizing Michelson interferometer. However, for the ability to make multi-wavelength observations at sub-picometer spectral resolution, the advantage goes to a confocal interferometer.

2.1. On- Axis Modeling the Confocal Interferometer

The characterization of a parallel plate Fabry-Pérot interferometer (FPI) for a telecentric beam configuration gives an effective FWHM dependent on the maximum angle of incident (\( \theta \)), while for a collimated beam configuration there is a variation of line center across the FOV. The single etalon equations for the central wavelength \( \lambda_c \), free spectral range FSR, total finesse F, reflectance finesse Fs, integer order number m, and passband (\( \Delta \lambda_{FWHM} \)) are given by the well-known relations: (i) \( \lambda_c = 2 \pi \bar{d} \sin \theta / m \), (ii) \( \text{FSR} = \lambda_c / 2 \), (iii) \( F_s = \pi R |/(1-R)| = - \pi \ln R \), and (iv) \( \Delta \lambda_{FWHM} = \text{FSR} / \bar{d} \), where \( \bar{d} \) is the etalon gap distance for parallel plates, n is the refractive index of the gap, R is the coating reflectance, and assuming no optical defects, i.e., Fc=Fc (Vaughan 1989). For small angles the wavelength shift is \( \Delta \lambda \approx \lambda_c \theta / 2 \) (Figure 3). These inter-relationships determine the specific requirements for the parallel-plate filter. In particular for the planar Fabry-Pérot the ratio of the spectral resolution \( N_r = \lambda_c / \Delta \lambda_{FWHM} \) to the étendue \( (U_{FP}) = \pi D^2 \lambda / 4 \Delta \lambda_{FWHM} \) is given by \( N_r = \lambda_c / \pi D^2 \) (Cuypers 1989). Increasing the gap distance increases the spectral power \( N_r \) but decreases the étendue \( U_{FP} \approx 1 / \bar{d} \).

Setting the distance between mirror centers, d, to the exact confocal separation, r, the corresponding on-axis, paraxial formulae for the confocal (spherical) Fabry-Pérot interferometer (FPI) are: (i) \( \lambda_c = 2 r / m \pi \), (ii) \( F_s = \lambda_c / 2 r \pi \), (iii) \( F_s = \pi R |/(1-R)| = - \pi \ln R \), and (iv) \( \Delta \lambda_{FWHM} = \text{FSR} / \bar{d} \) (Vaughan 1989). The useable aperture or pupil spot radius is given by \( r = \sqrt{r^2 / (F_s / F_c)} \), assuming the FWHM at this edge is within \( 2 \lambda_c \).
Each interferometer both performing in the visible and near infrared.

For the FPP parameters, \(\Delta L\) can increase simultaneously, both, a better choice than the FPP when \(\Delta L\) is large. For large-aperture solar telescopes, the critical result is that a smaller aperture can be used for the same number of photons passing through.

We have designed and built two sub-picometer confocal interferometers both performing in the visible and near infrared. Each FPS was designed with a \(\Delta \lambda_{FWHM} = 0.13\) pm (FSR = 2 pm) at \(\lambda_v = 630.2\) nm and \(\Delta \lambda_{FWHM} = 0.8\) pm (FSR = 12 pm) at \(\lambda_{IR} = 1564.8\) nm. This allows the solar FeI Zeeman sensitive lines at \(\lambda_v\) and \(\lambda_{IR}\) to be scanned and allows a frequency stabilized HeNe laser line (\(\lambda_{HeNe} = 632.8\) nm) to be employed for laboratory tests. The relationship between FSR, FWHM, reflectance, and confocal separation is shown in Figure 4 for the visible and near infrared lines. Although the plot of reflectance versus FSR and FWHM is independent of the wavelength, we have separated the line plots into two reflectance groups in order to show their correlation with the confocal separation distances for \(\lambda_v\) and \(\lambda_{IR}\). For the two wavelengths, the largest two squares plot the corresponding design values for \(R = 0.92\) for the same separation distance, \(r = 50\) mm. The set of box symbols at the ordinate FSR = 1 pm refer to a separation of \(r = 100\) mm for \(\lambda_v\).

The reflectance finesse is \(\eta = \pi R(1 - R^2)^{1/2}\) and the resolving power is \(N_0 = \Delta \lambda_{FWHM}/\lambda = 4FR\). This implies a confocal separation distance of \(r = 50\) mm for sub-picometer spectral resolution, and for ease of manufacturing the first units for testing have an aperture of 25 mm. Because of the root-mean-square (RMS) irregularities and roughness of the spherical surfaces, the total finesse is a combination of the reflectance finesse \(\eta_0\) and the defect finesse \(F_{defect}\). For a RMS irregularity of \(\delta_{RMS} = \lambda_0\), the defect finesse is \(F_{defect} = \pi/2\) (Hercher 1968). Hence to have \(F_{defect}\) to match the \(F_0(R = 0.92)\) then \(\eta_0 = \pi/2\) and \(\delta_{RMS} = \lambda_0/2\), which is a reasonable smoothness for the spherical etalons. For these values the total finesse \(F\) is then \(F = \pi(1/F_0 + 1/F_{defect})^{1/2} = 2^{-1/2}F_0 = 13.3\). This is an effective reflectivity of 0.88. The FWHM design values become \(\Delta \lambda_{FWHM} = 0.15\) pm and 0.90 pm. Inversely, if we had selected the total finesse to be 18.8, then the reflectance and defect finesse must be 26.5 which give \(R = 0.943\). Hence the final design reflectance value was set to be \(R = 0.95 \pm 0.02\) with surface figure to be better than \(\lambda/40\).
2.2. Off-Axis Modeling the Confocal Interferometer and Imaging Theory

An optical configuration for the confocal interferometer is shown in Figure 1 within the paraxial approximation. The rays starting from $S_1$ after making quadruple (or $2k$) reflections are reentrant with the original direction. After making two reflections or $2k$, another set of rays emerge and form an additional image ($S'_1$). For each of the two beams, the FPS intensity is 50% that of a FPP; however, in principle, the second beam could be recombined or used. This second image appears to be coming from the virtual source at $S_2$. There are two beams that emerge back toward the source side (dashed arrows in Figure 1). Clark (1972) gives the imaging relationships for an imaging confocal optical interferometer with the etalon plates with a radius of curvature given by $r$ and a separation of $d=r+e$, where $e$ is a small displacement. For $e=0$, the separation is the exact confocal distance. The phase difference for the rays is given by

$$\delta_1 = (2\pi/\lambda)(r_1^2 + r_2^2) + 2\pi\left(\frac{r_1^2 + r_2^2}{2}\right) \frac{d_{\text{off-axis}}}{r_1} \delta_{\lambda},$$

where $r_1$ and $r_2$ are the distances of the ray from the optical axis at the first and second spherical etalon surface, $\delta_{\lambda}=\delta_{\lambda}(r+e)/\lambda$ is the on-axis phase difference; and, for $\rho=r_1=r_2$, the off-axis phase difference is given by

$$\delta_1 = \delta_{\text{on-axis}} = (2\pi/\lambda)(\alpha/\gamma + \cos^2 \theta),$$

where $\alpha = 2\pi/\gamma$ (Vaughan 1989, eqns. 5.6 and 5.7). Using Fermat's principle, Clark (1972) determines the best focus, for a specific $\rho$, using the central diffraction angle $\alpha_c$ for which $\delta(\delta_1)/\delta_{\lambda}$-0. This also sets the largest range of $\alpha$ for which the spectral information is passed and the associated spatial resolution. For input intensity $I_0$, the output diffraction intensity for the $S'_1$ is given by

$$I(\alpha) = (I_0/2) (1+(2\pi/\gamma)^2 \sin^2(\delta_1/2)), \quad (3)$$

and $\alpha_c = (8\gamma\pi)/\gamma$. A plus and minus sign denotes if the wavelength is longer or shorter than the central peak. On the two sides of the intensity peak at HWHM (half width at half maximum), the phase difference is $\theta = -(2\pi/\gamma)(-4\alpha^2 r + \lambda \lambda^2) + \delta_{\text{off-axis}}$, hence the cutoff angles become

$$\alpha_c = \pm(-1)^{m}(8\pi r) \pm (16\pi^2 r^2)^{1/2}, \quad (4)$$

which gives the variation of acceptable angles for which the passband is within FWHM. Hence if $\alpha_c=0$ then $\alpha_c=(64\pi^2 r)^{1/2} \sim 2.8 (\Delta \lambda_{\text{FWHM}}/\lambda)^{1/2} = 5.15$ deg. This is consistent with the angle $2\alpha/\gamma \sim 2(\lambda/4\pi)^{1/2} = 3.6$ deg, derived from Herrcher's (1968, eqn. 17) spot radius. Therefore the FOV is approximately 4 deg.

3. Construction of a Confocal Interferometer

For testing the image quality of the confocal interferometers, we have constructed two confocal interferometers using IC Optical Systems spherical mirrors of $r_1=50.065$ mm internal confocal radius and external radius of $r_2=60.30$ mm. These 25 mm diameter fused silica plates ($n=1.459$) have a 15 mm clear aperture and a plate thickness of 10.235 mm. The measured mirror coating transmittances is 51.3% and 60.3% at 1500 nm. The exterior surfaces, with anti-reflecting coatings for both wavelengths ($R<0.25\%$), were made concave with a radius of curvature of 60.25 mm. By employing concave-convex etalon plates and the plate thickness, the optical power of each etalon plate is minimized for imaging quality and spectral resolution. The thick lens formula $f^{-1}=(n-1)(1/r_1-1/r_2) + (n-1)/n (r_1 r_2)$ gives a relative low-power, large focal length of $f \sim 623$ mm (while an optical Zemax code gives $f \sim 925$ mm).

In order to calibrate the etalon we need to be able to scan j-free spectral ranges, i.e., the confocal displacement should be $\delta_d = j(\lambda/4\pi)$, a few microns. The spectral resolution should be $\sim 1/10^6$ of the $\Delta \lambda_{\text{FWHM}}$, or about $(\lambda/4\pi)/(10 \text{ F})$. To infinitesimally displace the mirrors by this amount, the two confocal interferometers were designed to have good mechanical and environmental stability (Figure 5). The piezoelectric-controlled interferometer follows the design by Budker et al. (1999) (cf., Vaughan 1989, see 5.5.1). The thermally-controlled FPS design was engineered to allow initial precise displacement near the confocal separation.

3.1. Piezoelectric-Tunable FPS

The piezoelectric controller utilizes a thin-wall hallow cylinder of lead zirconate titanate (PbZr$_{0.53}$Ti$_{0.47}$O$_3$, PZT) made of Channel Industries C-3700 and milled by Boston Piezo Optics.
Fig. 6. Near confocal Fabry-Pérot fringe patterns. The curves give the radii of the circular interference pattern for collimated light in the paraxial approximation.

![Thermally Controlled](image1)

![Piezoelectric Controlled](image2)

Fig. 7. The two constructed confocal interferometers with subpicometer spectral resolution. For each, the confocal distance is 50 mm and the aperture is 25 mm.

The calibration between voltage and displacement is required to correct for hysteresis and nonlinearity (Figure 8).

Fig. 8. Piezoelectric-controlled FPS scans of the HeNe laser line. The data is partially corrected for thermal drift. The solid lines are scans with the PZT voltage increasing and the dotted lines are with voltage decreasing. The separation of profiles set shows the hysteresis effect which can be corrected.

3.2. Thermally-Tunable FPS

The thermally-controlled design uses a 50 mm-long glass cylinder between the confocal interferometer mirrors (Figure 7, upper panel). Initially all Zerodur glass cylinder was considered with a thermal coefficient of expansion of $C_{TE}=0.05$ ppm/$^\circ$C. Using a thermal system that controls the temperature to $\Delta T=0.1$ C, implies controlling the spacing by steps of $\Delta e=c C_{TE} \Delta T = 50 \text{ mm} \times 0.05 \times 10^{-6} \times 0.1 = 0.0025$ mm. However, for a system working in the IR at $\lambda_{IR}$ (or in the visible at $\lambda_{VIS}$) this change represents a change in the filter wavelength of $\delta \lambda = \frac{\Delta e}{n}$; note that $\delta e$ refers to a small displacement associated with a wavelength change $\delta \lambda$ and $\Delta e$ is a small displacement associated with a temperature change $\Delta T$. The offset order $m$ is given by $m = \frac{\delta e}{\lambda}$, hence $\delta \lambda = \frac{(\delta e/m)}{\lambda} = 5 \times 10^{-5} \lambda$ and for $\lambda_{IR}$ (or $\lambda_{VIS}$), then $\delta \lambda_{IR} = 7.8 \times 10^{-7}$ pm (or $\lambda_{VIS} = 3.1 \times 10^{-5}$ pm) which is quite small and implies prohibitively large changes in temperature. The FWHM is given by FSR, $\lambda^2/(4 e)$, divided by the finesse $F$. Hence, with the refractive index $n=1$, the intensity profile FWHM is $\Delta \lambda_{FWHM} = \lambda^2/(4 \pi F)$. For $R=0.90$, the confocal finesse is $F=15$; $\Delta \lambda_{FWHM} \approx 0.8$ pm for $\lambda_{IR}$; and $\Delta \lambda_{FWHM} \approx 0.13$ pm for $\lambda_{VIS}$. Again, $\delta \lambda$ needs to be some fraction of the FWHM in order to measure the line profile, i.e., $\delta \lambda/\Delta \lambda_{FWHM} < 1$, but $\delta \lambda/\Delta \lambda_{FWHM} \approx 7.8/0.8 = 9.7$ for $\lambda_{IR}$ (or $\lambda_{VIS}$). Hence, with Zerodur, the spectral line profile is unresolved by thermal tuning.
for reasonable total changes in temperature. Therefore, the thermal control design was refined.

The refinement was produced by employing a dual-glass cylinder (BK7 and Zerodur) separating the spherical mirrors, where the thermal expansion of the glass cylinder controls the inter-etalon spacing. Assuming a set of characteristics for the FWHM and FSR at £AV and £AR and assuming that at least 5 points are needed across the FWHM to define the profile sufficiently, then Table 2 was generated assuming that the thermal control resolution is 0.1 C. Using (£e/o) = (CTE ΔT) = (ΔZ/λ) and with the assumption that £AV = 0.02 pm, i.e. the step size, and £AR = 0.08 pm for a change of ΔT=0.1 C, then (£e/o)AV = (0.02×10^-3/630.2 nm) = 3.17×10^-9 and (£e/o)AR = (0.08×10^-8/1564.85) = 5.1×10^-8 for ΔT=0.1 C. Therefore, with a change of ΔT=1.0 C, we have the thermal coefficient of expansion requirements for the respective wavelengths: CTEV = (£e/o)V = 3.17×10^-7 and CTEK = (£e/o)AR = 5.1×10^-7. Using a segment of length of L of Zerodur (denoted by Z) with its TCE CTEZ=5×10^-8 and a segment of length L of BK7 with CTEBK7=7.1×10^-8 then we can obtain the correct overall CTE by solving: LZCTEZ+ L2CTE2 = (L1+L2)CTEV for 630.2 nm. Hence if the overall length is L=L1+L2 then L1(CTEZ − CTES)+(L2)CTEV = (L1+L2)(CTEV − CTEK) or L1L2 = L (CTEV − CTEK)/(CTEV − CTEK). For L1/L2 = (3.17×10^-7×7.1×10^-7)/(5.0×10^-8+7.1×10^-7) = 0.962 and for L=50 mm, then L1=48.1 mm for Zerodur and L2=1.89 mm for BK7. Therefore by introducing a 1.89 mm segment of BK7, the combination provides the correct over all CTE need for the visible. The values to meet the correct CTE for the IR needs to be L1=46.74 mm and L2=3.26 mm which was the final design specifications, since they also satisfy the visible requirement. The electric requirement for 20 C temperature range and controlling the individual tripod of legs to 0.1 C is provided by a temperature transducer (type AD590) with a heater strip. The measured time constant for the constructed interferometer was 360 s. The resulting temporal sensitivity is too long for solar observations; however the dual-glass cylinder has no associated hysteresis.

Table 2. Initial thermal design requirements with R=0.9

<table>
<thead>
<tr>
<th>Parameter</th>
<th>£AV = 630.2 nm</th>
<th>£AR = 1564.85 nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>FWHM</td>
<td>0.10 pm</td>
<td>0.40 pm</td>
</tr>
<tr>
<td>FSR</td>
<td>2 pm</td>
<td>15 pm</td>
</tr>
<tr>
<td>Step Size</td>
<td>0.02 pm</td>
<td>0.08 pm</td>
</tr>
<tr>
<td>(at pts/FWHM)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of points to</td>
<td></td>
<td></td>
</tr>
<tr>
<td>cover the full FSR</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Change in temperature</td>
<td>10C</td>
<td></td>
</tr>
<tr>
<td>required to cover FSR</td>
<td>19C</td>
<td></td>
</tr>
</tbody>
</table>

4. Alignment

The parallel plates of a FPP must be exactly parallel, i.e., to within 4/D ~10^-6 radians. Conversely, because of the spherical shape of the FPS, the alignment of the centers of the mirrors needs to be only roughly aligned, to within about 0.1D=10^-2 radians. This allows the axial alignment to be glued in place with no further improvement in tilt. For the axial separation of the confocal mirrors, the precision is more critical than the axial alignment and is given by |£e| = (1 r/32F)^1/2, hence, for r = 50 mm, |£e| = 1.564 µm, F=15, |£e| = 13 µm ± 8.1 (Vaughan 1989, p. 195). However, setting the initial separation is straightforward (Hercher 1968), and is based on the fringe movement about the confocal separation. The fringe pattern radius for fringe orders |m|=[-10,10] is plotted versus the departure from the confocal position in Figure 6. The thick solid line is for |m|=0. The dotted line is the maximum radial dispersion curve (dp/dl). For scale, a rectangle is defined by the spot radius of ρs and departure of w±100 µm. The wavelength is for the photospheric visible Zeeman sensitive line (λ = 630.2 nm). Spot radius ρs is defined when the actual FWHM is ΔλFWHM=1.42 ΔλFWHM. If the confocal departure is less than the confocal distance (w<0), as |e| is decreased, the low order fringe pattern moves outward and for |e|>0, as |e| is increased the fringes move inward. For the piezoelectric tuned FPS, a fine screw adjustment allowed the confocal separation adjustment. For the thermally tuned FPS, the glass tube was carefully polished and checked for the proper separation and then optically contacted to the mirrors.

5. Test Results

The confocal interferometer sub-picometer test results are given in terms of profile line scanning and imaging. In Figure 8, the results are shown using the piezoelectric-controlled FPS. The scans show the hysteresis effect of the lead zirconate titanate cylinder (cf. Crawford 1961). A Powell multiple parameter least-squares fit of the scans determined that the sub-picometer spectral resolution was FWHM=0.33 pm with FSR=1.99 pm, F=6.0, and effective reflectivity of 0.77. These acceptable values include the effect of complete optical configuration and hence the finesse is lower than the theoretical value.

Figure 9 shows the imaging capability of the confocal interferometer. An U.S. Air Force Resolution Chart is imaged at λ=630.2 nm through the FPS. Both exit images (S1 and S2) of the FPS are seen.
Near Infrared Magnetograph (NTRM) could be employed as a prefilter for the FPS. For a Peak’s high-resolution dual Queensgate near-TR can be employed, with the narrowest FWHM =30 nm) was employed. Multiple orders were passed to have minimum laboratory exposure times. The image through the FPS (right) has straylight as a result of the longer exposure time.

slightly offset from the optical axis. The FOV is 2.5 deg. With this optical configuration, the spatial resolution is only slightly degraded, ~11%, as seen by comparing the two images and by comparing the images with the diffraction limit of a 4 mm aperture, which is at the frequency giving by Group 4/Element 4 of the resolution chart. These test result show that a FPS allows imaging interferometry at high spectral resolution.

6. Conclusions

The confocal interferometer sub-picometer FWHM spectral filters for visible and near-IR solar magnetography and Dopplergraphy will lead to a better understanding of the magnetic field and advective flow gradients through the solar atmosphere and links observation between different spectral lines at various heights. For high spectral resolution, a confocal (also called a spherical or Connes) interferometer enhances the scientific value of future solar satellite missions by increasing the field of view and throughput of magnetographs, decreases the cost of the mission by allowing smaller optical components, and increases the mission probability of success by relaxing optical tolerances of the interferometer. We have developed, tested, and demonstrated the imaging capabilities of a very narrow-passband confocal interferometer. The FOV and spatial resolution of a FPS is set by the diffraction limit of the confocal spot size. The spectral resolution is set by the adjustment of the confocal separation. The FPS can be use for imaging and provides sub-picometer spectral resolution. The high throughput of the FPS as compared to the FPP makes this interferometer the choice for obtaining high resolution solar line profiles.

For future studies, there are two imaging FPP instruments that can be used as prefilters for our FPS research on solar phenomena. In the visual and the near IR, the NSO Sacramento Peak’s high-resolution dual Queensgate Fabry-Pérot interferometers can be employed, with the narrowest passband compatible with the FPS’s FSR. At $\lambda_p=630.2$ nm the NSO’s 50 mm etalons ET50-1001 (FWHM=9.0 pm, FSR=327 pm) and ET50-1046 (FWHM=1.7 pm, FSR=72.1 pm) can be employed. In the near IR, the etalons ET50-993 (FWHM=11.3 pm, FSR=522.5 pm) and ET50-998 (FWHM=168.3 pm, FSR=3932 pm) could be employed as a prefilter for the FPS. For a $SNR$ of 500, a 15-point $\lambda_p$ spectral profile scan time is estimated to be 40 minutes using the Dunn Solar Telescope. Also for the NIR, the New Jersey Institute of Technology/Big Bear Solar Observatory’s Near Infrared Magnetograph (NIRM) could be employed which has a FPS with FWHM=8.8 pm and FSR=548.7 pm. We plan to employ these systems to obtain sub-picometer spectral resolution Stokes I and V line profiles in the umbra regions to determine the vertical magnetic field and velocity gradients across the height of formation of the spectral lines FeI $\lambda_5$ and $\lambda_6$. However, the final spectral information will be degraded by the relatively long integration times.

With the construction of ground-based solar telescopes with aperture larger than a meter and the planned National Solar Observatory’s 4-meter Advanced Technology Solar Telescope, the observed transverse spatial resolution will be less than the half-width of the vertical height of formation of the observed spectral lines. Using a confocal sub-picometer interferometer, the observations can be made of the same spectral line to produce vertical height information compatible to the transverse scale within a time frame consistent with the resolution. These observations can be supplemented with multiple line observations; however, multiple line analysis relies on the assumption that the height of formation of the atomic levels can be directly related to the height of formation of the observed line. The radiometry calculations given in the introduction confirm that such observations can be made, and we argue that these should be pursued. With the completion of the future observations listed in the last paragraph, the usefulness of the observations will be tested.

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