Benefits Derived From Laser Ranging Measurements for Orbit Determination of the GPS Satellite Orbit

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Space Administration

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Cleveland, Ohio 44135

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Abstract

While navigation systems for the determination of the orbit of the Global Position System (GPS) have proven to be very effective, the current research is examining methods to lower the error in the GPS satellite ephemerides below their current level. Two GPS satellites that are currently in orbit carry retro-reflectors onboard. One notion to reduce the error in the satellite ephemerides is to utilize the retro-reflectors via laser ranging measurements taken from multiple Earth ground stations. Analysis has been performed to determine the level of reduction in the semi-major axis covariance of the GPS satellites, when laser ranging measurements are supplemented to the radiometric station keeping, which the satellites undergo.

Six ground tracking systems are studied to estimate the performance of the satellite. The first system is the baseline current system approach which provides pseudo-range and integrated Doppler measurements from six ground stations. The remaining five ground tracking systems utilize all measurements from the current system and laser ranging measurements from the additional ground stations utilized within those systems. Station locations for the additional ground sites were taken from a listing of laser ranging ground stations from the International Laser Ranging Service.

Results show reductions in state covariance estimates when utilizing laser ranging measurements to solve for the satellite’s position component of the state vector. Results also show dependency on the number of ground stations providing laser ranging measurements, orientation of the satellite to the ground stations, and the initial covariance of the satellite’s state vector.

Introduction

The task of the laser ranging analysis effort was to determine the added benefits derived from solving for a spacecraft’s state vector when utilizing laser ranging measurements in addition to the current use of pseudo-range and accumulated delta range (ADR), also known as Integrated Doppler measurements. The methodology to complete this analysis is to perform a covariance study for the Global Positioning System (GPS) orbit. The definition for this study of added benefits was a reduction in the covariance estimate of the GPS orbit state vector. The exact covariance statistic that was examined will be discussed in a later section.

Two methods were utilized in this study for the purposes of estimating benefits of laser ranging measurements being applied in solving for the GPS orbit state vector. The first method was to perform the estimated orbit determination (OD) using pseudo-range and ADR measurements from the six Monitor Stations (MS) for the GPS satellite orbit through an Extended Kalman Filter (EKF) analysis. The second method also used the EKF analysis tool, but included laser ranging measurements from the six MS sites along with various amounts of additional sites. Measurements from both methods were used to form estimates for the GPS satellite’s state vector, which was propagated until new measurements were available. Finally, comparisons were made between the performance of the current system and the modified system.

This analysis is intended to be the baseline study comparing the benefits of adding laser ranging measurements from various numbers of ground stations. This analysis is not meant to reflect on operational scenarios, nor provide a baseline operational concept. This analysis is meant to provide information on the benefits of having multiple ground stations in view while providing laser ranging measurements, in comparison to the current system which uses pseudo-range (PR) and Accumulated Delta Range (a.k.a. Integrated Doppler) (ADR) measurements.

EKF Description

The purpose of an EKF is to estimate the states of a non-linear system. The EKF is an extension of the standard Kalman Filter, in which its purpose is to estimate the states of a linear system. The derivation of the EKF is based on linearizing the non-linear system using the Kalman Filter estimate as the nominal state trajectory. The non-linear system is linearized around the Kalman Filter estimate and the Kalman Filter estimate is based on the linearized system (ref. 1).

The method of the EKF used for these simulations was the discrete time system/discrete time measurement EKF. This was the most appropriate method to simulate the EKF because performing continuous time dynamics on a computer requires an extremely large amount of memory and processor power to be performed efficiently. Also, it is important to note initially that there are multiple runs performed for each scenario. This
is due to the fact that the equations of the EKF dictate that the real noise parameters, instead of the covariance of the noise (as in linear Kalman filter problems), be used to form new estimates of the state estimates. The dictating equations for the EKF process can be found in reference one.

State Dynamics Description

The orbit of the GPS satellite was viewed as a simplified two-body problem for this analysis. Therefore, the differential equation that solves the two-body problem is as follows, in equation (1) (ref. 2).

\[ \dot{r} + \frac{\mu}{R^3} r = a_d \]  

Where:
- \( r \) is the position matrix
- \( \dot{r} \) is the 2nd time derivative of the position matrix
- \( \mu \) is the Earth’s Gravitational constant
- \( R \) is the magnitude of the position matrix
- \( a_d \) is the orbital perturbation

It is important to note that the OD analysis solved for more than just the position matrix. The purpose of the OD analysis was to solve for position, velocity, clock bias, and frequency bias estimates. Therefore, the state equation that governed the OD analysis was an extension of the two-body problem. Keep in consideration that the orbital perturbation was assumed to be zero for this analysis. The state is defined in equation (2), while the state equation is given as follows in equation (3).

\[ x = \begin{bmatrix} r \\ v \\ ct_{bias} \\ ct_{bias} \end{bmatrix} \]  

(2)

\[ \dot{x} = \begin{bmatrix} v \\ -\frac{\mu}{R^3} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1/f \end{bmatrix} \]  

(3)

Where:
- \( r \) is the position matrix
- \( v \) is the velocity matrix
- \( c \) is the speed of light in a vacuum
- \( t_{bias} \) is the clock difference between the satellite and the ground stations
- \( f \) is the GPS L1 frequency
- \( w \) is the state noise

Equation (4) shows the fully expanded form of the discrete state equation. Equation (5) converts the state equation from equation (3) into a discrete time state equation, needed for the discrete time EKF, in Earth-Centered Fixed coordinates.

\[ x_k = \begin{bmatrix} x_{k,1} \\ x_{k,2} \\ x_{k,3} \\ x_{k,4} \\ x_{k,5} \\ x_{k,6} \\ x_{k,7} \\ x_{k,8} \end{bmatrix} = \begin{bmatrix} v_{k,x} \\ v_{k,y} \\ v_{k,z} \\ ct_{bias} \end{bmatrix} \]  

(4)

Equation (5) shows the fully expanded form of the discrete state equation. Equation (5) converts the state equation from equation (3) into a discrete time state equation, needed for the discrete time EKF, in Earth-Centered Fixed coordinates.

\[ x_{k+1} = \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 & \Delta \cos(\phi) & \Delta \sin(\phi) & 0 & 0 & 0 \\ -\sin(\phi) & \cos(\phi) & 0 & -\Delta \sin(\phi) & \Delta \cos(\phi) & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta & 0 & 0 \\ -\frac{\mu}{R^3} \cos(\phi) & -\frac{\mu}{R^3} \sin(\phi) & 0 & \cos(\phi) & \sin(\phi) & 0 & 0 & 0 \\ \frac{\mu}{R^3} \sin(\phi) & -\frac{\mu}{R^3} \cos(\phi) & 0 & -\sin(\phi) & \cos(\phi) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\mu}{R^3} & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1/f \end{bmatrix} w_k \]  

(5)
Measurement Description

There are three different measurement types that were utilized within the trade space of this analysis. The first measurement was the pseudo-range (PR) measurement, which was utilized at the MS locations. The equation for the pseudo-range measurement is given in equation (6) (ref. 3).

\[
PR = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} + PR_{bias} + v_{PR}
\]  

(6)

Where:

- \(PR\) is the pseudo-range measurement
- \((x_1, y_1, z_1)\) was the position of the transmitter (or receiver)
- \((x_2, y_2, z_2)\) was the position of the receiver (or transmitter)
- \(v_{PR}\) was the noise term in the pseudo-range measurement

The second measurement type that was utilized at the six MS locations was the Accumulated Delta Range (ADR) measurement. This measurement, which is also called carrier phase, is the integral of the range-rate measurement used with instantaneous Doppler shift. Equation (7) provides the mathematical description of the ADR measurement (ref. 3).

\[
ADR = \sum_{k=0}^{n} \left[ \left( x_{1,k} - x_{2,k} \right) \left( v_{x 1,k} - v_{x 2,k} \right) + \left( y_{1,k} - y_{2,k} \right) \left( v_{y 1,k} - v_{y 2,k} \right) + \left( z_{1,k} - z_{2,k} \right) \left( v_{z 1,k} - v_{z 2,k} \right) \right] + fc \times i_{bias} + v_{ADR}
\]  

(7)

Where:

- \(ADR\) was the accumulated delta range measurement
- \((v_{x 1}, v_{y 1}, v_{z 1})\) was the velocity of the transmitter (or receiver)
- \((v_{x 2}, v_{y 2}, v_{z 2})\) was the velocity of the receiver (or transmitter)
- \(v_{ADR}\) was the noise term in the accumulated delta range measurement

The final measurement type which was utilized only in the modified systems, but at all ground station sites, was the laser ranging (LR) measurement. This was thought of as the equivalent of a two-way radiometric signal, in terms of the equation governing the measurement. Equation (8) provides the mathematical description of the LR measurement (ref. 3).

\[
LR = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} + v_{LR}
\]  

(8)

Where:

- \(LR\) was the laser ranging measurement
- \(v_{LR}\) was the noise term in the laser ranging measurement

Table 1 provides the standard deviation of the noise terms that are assumed for the three measurement equations provided in equations (6) through (8).

<table>
<thead>
<tr>
<th>Noise term</th>
<th>(v_{PR})</th>
<th>(v_{ADR})</th>
<th>(v_{LR})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma)</td>
<td>2 m</td>
<td>5 mm/s</td>
<td>1 m</td>
</tr>
</tbody>
</table>

Table 1.—MEASUREMENT NOISE STANDARD DEVIATION

Station Locations

The first of the two proposed systems that were analyzed was the current system (CS), which utilized pseudo-range and ADR measurements from the six MS locations. The second proposed system was the modified system, from which there were five versions (MSys1, MSys2, MSys3, MSys4, and MSys5). The modified system utilized all measurements for the current system, plus laser ranging measurements from the six MS locations along with measurements from various additional ground stations present in each system. The locations of the additional ground stations are from a listing of laser ranging sites from the International Laser Ranging Service (ref. 4). Figure 1 illustrates the locations of all of the ground station locations with color coded dots representing which system the ground stations are first utilized within on a Mercator projection of the Earth’s surface. Blue dots represent the six MS ground stations. Red dots represent the two sites first utilized in MSys1. Orange dots represent the two sites first utilized in MSys2. Green dots represent the four sites first utilized in MSys3. Pink dots represent the four sites first utilized in MSys4. Finally, purple dots represent the four sites utilized in MSys5. Table 2 provides an alphabetical listing of the all 22 ground stations locations utilized in the study and which systems they were part of.

Figure 1.—Ground station location map.
### Table 2.—Ground Stations

<table>
<thead>
<tr>
<th>Station</th>
<th>CS</th>
<th>MSys1</th>
<th>MSys2</th>
<th>MSys3</th>
<th>MSys4</th>
<th>MSys5</th>
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<tbody>
<tr>
<td>Arequipa</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Ascension Island</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Cape Canaveral</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Colorado Springs</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Concepcion</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Diego Garcia</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Greenbelt</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Hartebeesthoek</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Hawaii</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Koganei</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Komsomolsk</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kwajalein</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>McDonald Obs.</td>
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<td>X</td>
<td>X</td>
<td>X</td>
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<td>Metsahovi</td>
<td>X</td>
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<td>Mt. Stromlo</td>
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<td>X</td>
<td>X</td>
<td>X</td>
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<td>Riyadh</td>
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<td></td>
<td></td>
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<tr>
<td>San Fernando</td>
<td>X</td>
<td>X</td>
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<td></td>
<td></td>
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</tr>
<tr>
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<td>X</td>
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<td>X</td>
</tr>
<tr>
<td>Yarragadee</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Methodology Description

This analysis was performed using the discrete time/discrete measurement EKF procedure described previously. The state was propagated at a rate of 1 Hz. The total simulation was set to run for 1 day (86400 s). It should be noted however, that measurements were not taken on this same one second time period.

The instant that line-of-sight was available, the pseudo-range measurements were recorded at the ground station and again every 60 s after the initial measurement, until the ground station was no longer visible to the satellite.

For ADR measurements, the integration of the delta-range measurements began the first second that the ground station was visible to the satellite. The measurement was completed once the ground station had been in view for 60 s. When the satellite lost visibility to the ground station, the integration procedure was reset and the measurement integration process had to start over on the next visible pass.

Laser ranging measurements were recorded at the ground station the instant that the ground station was visible to the satellite, and again every 60 s after the initial measurement, until the satellite was no longer visible to the ground station.

Note that in order for the ground station to be visible to the satellite, or for the satellite to be visible to the ground station, the ground station must see the satellite with an elevation angle of 10° minimum. This rule applied to all three measurement types that have been described.

The satellite was modeled in the GPS 12 hr orbit at an inclination of 55°. The orbit was assumed to start on the plane of the Earth’s Equator. However, due to the nature that the ground stations were oriented on the surface, the satellite was modeled on starting longitudes of 0° and 90° E. Figures 2 and 3 show the points over the Earth surface (Mercator projection) in which the orbit for these two starting longitudes pass. Note that the point shown in red was the first point of the orbit in the simulation.

Initial errors on the order of 1 km were added to each Cartesian dimension along with 1 m/s velocity errors in each Cartesian dimension. Clock bias and frequency bias states also begin with a 1 km and 1 m/s error, respectively. The other condition that was varied for the simulation was the initial covariance estimate. Initial conditions for the covariance estimate are formed into nine cases, as listed in Table 3.

Finally, for the purpose of this analysis, there were 10 noise profile runs performed for every starting longitude/initial covariance case simulation. Performance along the multiple runs will be combined to attain overall performance.
TABLE 3.—INITIAL COVARIANCE CONDITIONS

<table>
<thead>
<tr>
<th>Case</th>
<th>Position, m²</th>
<th>Velocity, (m/s)²</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
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<tr>
<td>3</td>
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<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>10</td>
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<tr>
<td>7</td>
<td>10</td>
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<tr>
<td>8</td>
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<td>1</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Results

Results for the analysis of the comparison of the six systems are shown through the use of the covariance estimate of the systems. Even though the variance of the noise terms is constant throughout the simulations, multiple noise profiles are needed to analyze the performance of an EKF simulation. This is due to the fact that the equations of the EKF dictate that the real noise parameters, instead of the covariance of the noise, are used to form new estimates of the state. The metric to compare the performance of the six systems was semi-major axis (SMA) covariance, which was based on the final covariance estimate at the end of the simulation.

Equation (9) was used to compare the systems for the SMA covariance statistic for the covariance estimate.

\[
COV_{SMA} = \frac{1}{n_{long} \cdot n_{run}} \sum_{i=1}^{n_{long}} \sum_{i=1}^{n_{run}} \sqrt{P_{(1,1),kf,i,i}^2 + P_{(2,2),kf,i,i}^2 + P_{(3,3),kf,i,i}^2}
\]  

(9)

Where:

- \( COV_{SMA} \) was the semi-major axis covariance error from the covariance estimate
- \( P_{(1,1),kf,i,i}, P_{(2,2),kf,i,i}, P_{(3,3),kf,i,i} \) were the covariance terms for the individual Cartesian dimension position terms at final time \( kf \) for run \( ii \) at longitude case \( i \)
- \( n_{long} \) was the number of starting longitude scenarios
- \( n_{run} \) was the number of noise profile runs

There will be a bar graph that corresponds to the final time SMA covariance statistic calculated by equation (9). Plots will illustrate the non-averaged SMA covariance statistic over time for all of the noise profiles for each of the systems that were examined, for the nine initial covariance estimates, respectively. Each figure has a top and bottom subplot, corresponding to the 0° and 90° E starting longitude conditions. Line color corresponds to systems in the following manner:

- Blue—CS
- Red—Msys1
- Black—Msys2
- Green—Msys3
- Cyan—Msys4
- Yellow—Msys5

Figures 4 and 5 correspond to the results for the initial covariance Case 1 simulations with initial covariance parameters are on the order of 0.1 m² and 0.1 (m/s)².
Figures 6 and 7 correspond to the results for the initial covariance Case 2 simulations with initial covariance parameters are on the order of 0.1 m$^2$ and 1 (m/s)$^2$.

Figures 8 and 9 correspond to the results for the initial covariance Case 3 simulations with initial covariance parameters are on the order of 0.1 m$^2$ and 10 (m/s)$^2$. 

Figure 6.—Final time SMA covariance—Case 2.

Figure 7.—SMA covariance—Case 2.

Figure 8.—Final time SMA covariance—Case 3.

Figure 9.—SMA covariance—Case 3.
Figures 10 and 11 correspond to the results for the initial covariance Case 4 simulations with initial covariance parameters are on the order of 1 m$^2$ and 0.1 (m/s)$^2$.

Figure 10.—Final time SMA covariance—Case 4.

Figure 11.—SMA covariance—Case 4.

Figures 12 and 13 correspond to the results for the initial covariance Case 5 simulations with initial covariance parameters are on the order of 1 m$^2$ and 1 (m/s)$^2$.

Figure 12.—Final time SMA covariance—Case 5.

Figure 13.—SMA covariance—Case 5.
Figures 14 and 15 correspond to the results for the initial covariance Case 6 simulations with initial covariance parameters are on the order of 1 m$^2$ and 10 (m/s)$^2$.

Figure 14.—Final time SMA covariance—Case 6.

Figure 15.—SMA covariance—Case 6.

Figures 16 and 17 correspond to the results for the initial covariance Case 7 simulations with initial covariance parameters are on the order of 10 m$^2$ and 0.1 (m/s)$^2$.

Figure 16.—Final time SMA covariance—Case 7.

Figure 17.—SMA covariance—Case 7.
Figures 18 and 19 correspond to the results for the initial covariance Case 8 simulations with initial covariance parameters are on the order of $10 \text{ m}^2$ and $1 \text{ (m/s)}^2$.

Figures 20 and 21 correspond to the results for the initial covariance Case 9 simulations with initial covariance parameters are on the order of $10 \text{ m}^2$ and $10 \text{ (m/s)}^2$.  

Figure 18.—Final time SMA covariance—Case 8.

Figure 19.—SMA covariance—Case 8.

Figure 20.—Final time SMA covariance—Case 9.

Figure 21.—SMA covariance—Case 9.
It is important to notice how the starting longitude and initial starting covariance affect the performance for each of the systems. Starting longitude will change which stations are visible, and if there are fewer stations visible, then the covariance estimate will grow at a faster rate between measurements. Initial covariance cases in which the velocity terms have a lower or equal initial covariance than the position terms perform more consistently, compared to the opposite case when the velocity terms have larger initial covariance terms than the position terms. This is due to the fact that the position error grows faster as position is the integral of velocity. Table 4 provides the final time percentage of the SMA covariance of the five systems compared to the SMA covariance of the current system, averaged over all initial covariance/initial longitude/noise profile runs. Note that these values represent the percentage of the CS SMA covariance of the five different systems.

<table>
<thead>
<tr>
<th>System</th>
<th>MSys1</th>
<th>MSys2</th>
<th>MSys3</th>
<th>MSys4</th>
<th>MSys5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Results</td>
<td>14.13%</td>
<td>11.01%</td>
<td>5.03%</td>
<td>3.84%</td>
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**Conclusion**

Results for the comparison between the current tracking system utilizing pseudo-range and ADR measurements from the six MS locations with the five modified systems including laser ranging measurements from the same ground sites plus additional sites have been provided. All of the results show benefits of having laser ranging measurements used to solve for the satellite’s position component of the state vector. The results show an initial dependency on the initial longitude of the orbit. A second parameter that has been shown to affect performance is the initial covariance for the system. However, for both of these parameters, the final SMA covariance is not strongly affected.

The parameter that does strongly affect the final time covariance is the number of laser ranging ground stations used. As seen in the results, it is typical that with an increase in the number of laser ranging ground stations, the final time SMA covariance statistic decreases. It is important to note that as more and more ground stations are added to the scenario, the final time covariance does not keep decreasing forever, but appears to reach a lower bound.

The initial covariance of the state is an estimate for how well the state is understood. Typically, when the state’s covariance is larger, then more emphasis is placed on the measurements when producing the EKF Kalman gain. The covariance of the measurement noise is also an important parameter for how the state covariance is propagated. However, when dealing with an OD type of analysis similar to the one performed, where and when there are few measurements available to the receiver, then covariance parameters can increase quickly.

Results shown from this study include the fact that there are differences in performance between the current system and the modified systems including laser ranging measurements. Performance is dependent on the location of the ground stations and how those stations are viewed by the satellite. Therefore, if the additional ground stations for the modified systems were picked differently, then the results would vary. However, this issue is not viewed to significantly modify the results as laser ranging ground stations were selected to be spatially diverse. It is believed that if constraints (such as range and/or speed) are placed within the EKF, performance of the two systems may be better modeled.

This analysis has shown that laser ranging measurements are beneficial and reduce the steady state system performance. Typically, the more stations that are added to the scenario, the lower the steady state system performance will be. MSys3 provides SMA covariance results of 5.03% of the CS results. MSys4 provides SMA covariance of 3.84% of the CS results. MSys5 provides SMA covariance of 3.03% of the CS results. However, there are smaller reductions in SMA covariance when comparing MSys3 to MSys4 or MSys4 to MSys5. Therefore, given the orientation of the ground stations as such as in this report, it appears that MSys3 would give the most benefit. Therefore, it is believed that if laser ranging measurements would in the future be taken into account for doing orbit determination analysis on a GPS orbit, the recommendations would be have measurements taken from the six MS stations with measurements from an additional eight ground stations around the world.

**References**


**Biography**

Bryan W. Welch is a member of the Communications System Integration Branch of the Communications Technology Division at the National Aeronautics and Space Administration's Glenn Research Center at Lewis Field in Cleveland, Ohio. Mr. Welch earned his Bachelor’s of Science Degree in Electrical Engineering from Cleveland State University, graduating summa cum laude in May 2003. In 2006, Mr. Welch earned his Master’s Degree in Electrical Engineering, also at Cleveland State University, with his thesis topic on the utilization of the Ruze Equation for inflatable aperture antennas. He has coauthored many papers, including some presented at the 4th ICNS Conference in April 2004, the ION National Technical Meeting in January 2006, and the Ka Band Conference in September 2006.
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14. ABSTRACT
   While navigation systems for the determination of the orbit of the Global Position System (GPS) have proven to be very effective, the current research is examining methods to lower the error in the GPS satellite ephemerides below their current level. Two GPS satellites that are currently in orbit carry retro-reflectors onboard. One notion to reduce the error in the satellite ephemerides is to utilize the retro-reflectors via laser ranging measurements taken from multiple Earth ground stations. Analysis has been performed to determine the level of reduction in the semi-major axis covariance of the GPS satellites, when laser ranging measurements are supplemented to the radiometric station keeping, which the satellites undergo. Six ground tracking systems are studied to estimate the performance of the satellite. The first system is the baseline current system approach which provides pseudo-range and integrated Doppler measurements from six ground stations. The remaining five ground tracking systems utilize all measurements from the current system and laser ranging measurements from the additional ground stations utilized within those systems. Station locations for the additional ground sites were taken from a listing of laser ranging ground stations from the International Laser Ranging Service. Results show reductions in state covariance estimates when utilizing laser ranging measurements to solve for the satellite’s position component of the state vector. Results also show dependency on the number of ground stations providing laser ranging measurements, orientation of the satellite to the ground stations, and the initial covariance of the satellite’s state vector.

15. SUBJECT TERMS
   Global positioning system; Navigation; Space navigation; Positioning; Orbit determination; State estimation; Kalman filters; Satellite laser ranging; Laser ranging

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