

# Calculation of Dynamic Loads Due to Random Vibration Environments in Rocket Engine Systems

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## Abstract

An important part of rocket engine design is the calculation of random dynamic loads resulting from internal engine "self-induced" sources. These loads are random in nature and can greatly influence the weight of many engine components. Several methodologies for calculating random loads are discussed and then compared to test results using a dynamic testbed consisting of a 60K thrust engine. The engine was tested in a free-free condition with known random force inputs from shakers attached to three locations near the main noise sources on the engine. Accelerations and strains were measured at several critical locations on the engines and then compared to the analytical results using two different random response methodologies.

## I. Introduction

An important part of rocket engine design is the calculation of the dynamic loads that act on the engine. These loads can greatly influence the weight of many engine components and thus affect overall engine performance, so it is important to be able to calculate them as accurately as possible. Recent NASA engine programs have indicated the need for improved methodologies for calculating the dynamic loads. For example, the Fastrac engine, pictured in Fig. 1, was a 60,000 lb thrust lox-kerosene engine developed at the Marshall Space Flight Center in the late 1990's<sup>1</sup>. It was designed to be a low-cost reusable engine for small launch vehicles and was test-fired in 1999. Another recent engine was the RS-84<sup>2</sup> which was being developed under the NASA Next Generation Launch Technology program<sup>3</sup>.

The Fastrac dynamic loads analysis is documented in Ref. 4. The first attempt at calculating the dynamic loads used an engine system finite element dynamic model in which the major engine components such as the manifold, main combustion chamber, nozzle, turbopump, gas generator, and major ducts were all modeled. This was the first time that NASA had used a complete system model to calculate engine dynamic loads for a new engine. Previous engine programs had relied on a component by component loads approach, mainly because of computational limitations. Unfortunately, the methodology used in Fastrac for the calculation of the random loads resulted in such large loads that the system model approach had to be abandoned in favor of a combination of system modeling and the more traditional component approach. This approach eventually was made to work, but it was not completely satisfactory since it did not take into account dynamic coupling between components. The dynamic loads were to have been validated during engine hot-fire testing, but the Fastrac program ended before any meaningful hot-fire strain gage data was obtained.

Comparison of engine hot-fire test results to the calculated loads is the ultimate test of the loads calculation methodology. However, the hot-fire environment is so complex that it is sometimes difficult to get meaningful results for comparison. It would be useful to be able to control the inputs to the engine system so that extraneous noise and unrelated sources could be excluded. In order to do this, a surplus Fastrac engine was used to develop a

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vibration testbed for calculation of engine dynamic loads. The engine was tested in a free-free condition with known random and sinusoidal force inputs at well-defined locations and the response was measured at various locations on the engines. These measured responses were then compared to results as determined by several different loads calculation methodologies in order to determine which ones work best.

## II. Calculation of Engine Random Dynamic Loads

The dynamic forces acting on a rocket engine can be divided into two general categories:

1. Forces resulting from external sources
2. Forces resulting from internal engine "self-induced" sources

External sources include forces such as ground transportation loads, acceleration g-loads due to the vehicle trajectory, loads from the engine actuators, and aerodynamic, thermal, and acoustic loads resulting from the motion of the vehicle through the atmosphere. Engine self-induced loads are the result of extremely complex processes inside the engine such as combustion pressures, fluid flow, rotating turbomachinery, etc. Self-induced loads are composed of random, sinusoidal, shock and acoustic components. Random loads are the result of combustion processes, fluid flow and turbulence. Sinusoidal loads are the result of imbalances in rotating turbomachinery. Random

and sinusoidal load are generally most severe during engine steady-state operation. Shock loads occur at engine start-up and shut-down and are due to combustion and flow transients. Acoustic loads are highly dependent upon the launch pad configuration and also occur mainly during start-up.

Most of the operating time of an engine is spent at steady state during which the random and sinusoidal loads are usually the dominant components. Because of their complexity, the random loads cannot currently be quantified with enough precision to allow a true dynamic response analysis to be done. That is, we can't simply take an engine system finite element model, apply these forces as functions of time or frequency, and calculate the response because we don't really know what the forces are to that level of detail. However, it is possible to measure the accelerations at various locations on the engine during a hot-fire test. These accelerations can then be used to define a dynamic environment for the engine. For a new engine design we can scale known accelerations from an existing similar engine since test data for the new engine is obviously not available. The dynamic environment is typically defined as a set of acceleration power spectral density (PSD) functions at specific points in the engine. The problem then becomes one of trying to reproduce the engine acceleration environment by exciting the engine system in some way. Ways of doing this can generally be categorized into one of the following three methodologies:

### 1. Enforced Acceleration Methods

Directly apply enforced accelerations at the points in the engine where the environments are defined. This was the initial system model approach used for the Fastrac program. The most direct and traditional approach of this type is to use enforced acceleration in all three directions simultaneously. This generally gives the most conservative loads. Another approach is to apply the enforced accelerations in one direction at a time and then pick the direction that results in the highest load. A third method is to apply accelerations in all 3 directions at once but to discard the pseudo-static portion of the response. Other variations of these approaches are also possible.

### 2. System Equivalent Applied Force Methods

Determine a set of applied forces that will reproduce the measured environment as closely as possible. Forces are typically applied at points where the environment is defined, although they don't have to be. One approach in this category is to apply one force at a single environment point in one direction at a time so as to match the environment at that point in that direction only. For example, the x-direction turbopump environment is reproduced by a single force applied in the x-direction at the turbopump center of gravity. Then, all the x-direction forces are applied simultaneously at all the environment points (turbopump, combustion chamber, gas generator, etc.) and the resulting loads are calculated. This is done for each direction (x, y, and z), one direction at a time, and the direction resulting in the largest load is the one used. The advantage of this approach is that it avoids large pseudo-static loads acting between adjacent elements. The disadvantage is that it still usually results in highly conservative loads.

### 3. Component Method

Calculate loads on a component by component basis. This was the method ultimately used for the random loads calculation in the Fastrac program. Because this approach is well-known, it was not evaluated in this test program.

The goal of random vibration analysis is to determine how the statistical characteristics of the motion of a randomly excited system depends upon the statistics of the excitation and the properties of the vibrating system (mass, stiffness and damping).

The general equation of motion for a discrete structure with N degrees of freedom is

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{F(t)\} \quad (1)$$

where  $[M]$  = Mass matrix (dimension N x N)  
 $[K]$  = Stiffness matrix (dimension N x N)  
 $[C]$  = Damping matrix (dimension N x N)  
 $\{x(t)\}$  = Displacement vector (dimension N x 1)  
 $\{F(t)\}$  = Applied force vector (dimension (N x 1))

As shown by many vibration textbooks<sup>5,6</sup>, the power spectral density (PSD) of the displacement vector is related to the PSD of the applied forces by

$$[S_{xx}(f)] = [H^*(f)][S_{FF}(f)][H(f)]^T \quad (2)$$

where  $[S_{xx}(f)]$  = Matrix of displacement vector PSD's  
 $[S_{FF}(f)]$  = Matrix of applied force PSD's  
 $[H(f)]$  = Matrix of transfer functions  
 $f$  = Frequency

In Eq(2), the \* indicates the complex conjugate and the transfer functions are functions of the system stiffness and mass. The diagonal terms in the PSD matrices  $[S_{xx}(f)]$  and  $[S_{FF}(f)]$  represent the autospectral densities and the off-diagonal terms represent the cross-spectral density terms. Usually, the cross-spectral density terms in the input  $[S_{FF}(f)]$  matrix are zero. Other quantities of interest, such as element force components, can be determined from equations similar to Eq(2). Once the PSD's are calculated from Eq(2), the RMS values can be calculated by integrating them over frequency and then taking the square root.

In the enforced acceleration methods, the acceleration environments are applied directly to the engine system model at the grid points where the environments are defined. If the environment accelerations are applied at a total of p degrees of freedom (DOF), then partition the displacement vector as follows:

$$\{x(t)\} = \begin{Bmatrix} \{x_f(t)\} \\ \{x_s(t)\} \end{Bmatrix} \quad (3)$$

where  $\{x_f(t)\}$  = Free or unconstrained DOF (dimensions of N-p x 1)  
 $\{x_s(t)\}$  = Constrained DOF where acceleration environments are enforced (p x 1)

Using Eq(3) in the equation of motion Eq(1) results in the following partitioned matrix equation:

$$\begin{bmatrix} [M_{ff}] & [M_{fs}] \\ [M_{fs}]^T & [M_{ss}] \end{bmatrix} \begin{Bmatrix} \{\ddot{x}_f\} \\ \{\ddot{x}_s\} \end{Bmatrix} + \begin{bmatrix} [C_{ff}] & [C_{fs}] \\ [C_{fs}]^T & [C_{ss}] \end{bmatrix} \begin{Bmatrix} \{\dot{x}_f\} \\ \{\dot{x}_s\} \end{Bmatrix} + \begin{bmatrix} [K_{ff}] & [K_{fs}] \\ [K_{fs}]^T & [K_{ss}] \end{bmatrix} \begin{Bmatrix} \{x_f\} \\ \{x_s\} \end{Bmatrix} = \begin{Bmatrix} \{F_f\} \\ \{F_s\} \end{Bmatrix} \quad (4)$$

If we assume that there are no applied forces (i.e.,  $\{F_f(t)\} = 0$ ), the first of the two equations implied by Eq(4) can be written as

$$[M_{ff}]\{\ddot{x}_f\} + [C_{ff}]\{\dot{x}_f\} + [K_{ff}]\{x_f\} = -[M_{fs}]\{\ddot{x}_s\} - [C_{fs}]\{\dot{x}_s\} - [K_{fs}]\{x_s\} = \{F_{eq}\} \quad (5)$$

Equation (5) can be solved by using Eq(2) with  $[S_{F_{eq}F_{eq}}(f)]$  used instead of  $[S_{FF}(f)]$ . The second equation implied by Eq(4) gives an expression that allows us to calculate the force required to enforce the acceleration. Thus  $\{F_s(t)\}$  is the force that must be applied to the constrained points in order for the enforced accelerations to be applied.

In the analysis of structures subjected to multiple support enforced motion, the unconstrained DOF can be considered to be made up of two parts,

$$\{x_f(t)\} = \{x_{fs}(t)\} + \{x_{fd}(t)\} \quad (6)$$

where  $\{x_{fs}(t)\} =$  Pseudo-static component of displacement  
 $\{x_{fd}(t)\} =$  Dynamic component of displacements

The pseudo-static component is the portion of the displacement due to the static application of the prescribed support accelerations at each time instant. It is the response the structure would have if it were massless and undamped and is determined by considering only the static part of Eq(5),

$$[K_{ff}]\{x_{fs}\} = -[K_{fs}]\{x_s\} \quad (7)$$

Solving for  $\{x_{fs}(t)\}$ ,

$$\{x_{fs}\} = -[K_{ff}]^{-1} [K_{fs}]\{x_s\} = [K_I]\{x_s\} \quad (8)$$

where  $[K_I] = -[K_{ff}]^{-1} [K_{fs}] \quad (9)$

The PSD of the pseudo-static displacements is can be calculated as follows:

$$[S_{x_{fs}x_{fs}}(f)] = [K_I][S_{x_sx_s}(f)][K_I]^T = \frac{1}{(2\pi f)^4} [K_I][S_{aa}(f)][K_I] \quad (10)$$

where  $[S_{aa}(f)] =$  The acceleration PSD's at the environment points

Equations similar to Eq(10) can be used to calculate the pseudo-static component of other quantities such as element loads, stresses, and strains. Much of the pseudo-static portion of the response is likely an artifact of the enforced motion of multiple constraints. It is often responsible for a large portion of the loads, especially at low frequencies.

The dynamic portion of the response can be calculated by considering Eq(5) with the free DOF broken down into pseudo-static and dynamic components using Eq(6),

$$\begin{aligned} [M_{ff}] \{ \ddot{x}_{fs} \} + [C_{ff}] \{ \dot{x}_{fd} \} + [K_{ff}] \{ x_{fd} \} = \\ - [M_{fs}] \{ \ddot{x}_s \} - [C_{fs}] \{ \dot{x}_s \} - [K_{fs}] \{ x_s \} \end{aligned} \quad (11)$$

Using Eq(9) in Eq(11) results in

$$\begin{aligned} [M_{ff}] \{ \ddot{x}_{fd} \} + [C_{ff}] \{ \dot{x}_{fd} \} + [K_{ff}] \{ x_{fd} \} = \\ - \left( [M_{fs}] + [M_{ff}] [K_I] \right) \{ \ddot{x}_s \} - \left( [C_{fs}] + [C_{ff}] [K_I] \right) \{ \dot{x}_s \} = \{ F_d \} \end{aligned} \quad (12)$$

Eq(12) is now in the form of Eq(1) and we can solve for  $[S_{x_{fd}x_{fd}}(f)]$  using Eq(2).

In the equivalent applied force methods, we attempt to define a set of equivalent force PSD's that when applied to the model reproduce the acceleration environment as closely as possible. In one such approach (subsequently referred to as the unidirectional approach), we assume that we have a set of p uncorrelated applied force PSD's that are applied to the model at the points where the environments are defined. If this is the case we can write a relationship between the applied force PSD's and the acceleration PSD's as follows:

$$\{ S_{aa}(f) \} = [T(f)] \{ S_{FF}(f) \} \quad (13)$$

where  $\{ S_{aa}(f) \}$  = Vector of known environment acceleration PSD's (dimension 3p x 1)

$\{ S_{FF}(f) \}$  = Vector of equivalent force PSD's (dimension 3p x 1)

$[T(f)]$  = Matrix of PSD transfer functions (dimension 3p x 3p)

p = No. of defined acceleration environment points

The transfer function matrix  $[T(f)]$  can easily be found by applying a unit force PSD at each of the p environment points (one by one) and then calculating the PSD of the resulting acceleration response. Note that all quantities in Eq(13) are positive. Unfortunately, we cannot simply invert the transformation matrix to solve for the equivalent force PSD's because the results are not assured to be positive. In the unidirectional approach we first solve for the force PSD's by setting all the off-diagonal terms in the transformation matrix to zero,

$$(S_{FF}(f))_k = \frac{(S_{aa}(f))_k}{T_{kk}(f)} \quad k = 1, 2, \dots, 3p \quad (14)$$

This is equivalent to assuming that each environment acceleration is applied one at a time and in one direction (X, Y, or Z) only. Next, the force PSD's from Eq(14) are applied at each environment point simultaneously, but only in one direction at a time. For example, the X-direction force PSD's are applied at all p points and the resulting loads are calculated. The same is then done for the Y and Z directions which results in a total of 3 sets of loads which are delivered to the stress analysts. For each engine component, the stress analysts then chooses which of the 3 load cases gives them the largest stress.

This approach can lead to very conservative results because of the cumulative effect of applying the "equivalent" forces at each of the environment points simultaneously. However, note that there are two offsetting effects that occur. Since in reality all the environments in each direction occur simultaneously, applying them one direction at a time independently is non-conservative. This effect somewhat offsets the conservativeness of using the "equivalent" unidirectional forces calculated by Eq(14).

### III. Fastrac Testbed Results

A surplus Fastrac engine has been used as a vibration testbed to test the various methodologies mentioned above. The testbed setup is shown in Fig. 2. The engine was supported in a free-free condition by bungee cords and three large shakers were used to input forces at the injector (in engine axial or z-direction), the gas generator (radial direction), and the turbopump (in xy plane). First, a modal test of the engine was conducted using random input from the 3 shakers. The testbed was then driven from 0-350 Hz by the constant force PSD shown in Fig. 3. Tri-axial acceleration responses were measured at 8 locations and strains were measured at 3 locations.

The finite element model (FEM) used to simulate the testbed is shown in Fig. 4. The model consists of approximately 3662 nodes and 3621 elements. After some "tweaking", the free-free modes calculated by the model correlated quite well with the testbed modal test results as illustrated in Table 1.

The measured accelerations at the shaker drive points were used to construct a simulated acceleration environment for the engine. For the injector and turbopump shakers, accelerometer measurements were taken only in the direction of the shaker force. In order to get accelerations for the lateral components of the acceleration environment, the shaker force PSD inputs were applied to the FEM and the accelerations at the injector and turbopump were calculated. These calculated accelerations were then used to complete the acceleration environments in the directions for which there were no measurements taken. The environments were determined by enveloping the acceleration responses much as would be done in an actual engine design. Resonant peaks are enveloped by an approximately  $\pm 5\%$  frequency band with the magnitude set at the actual magnitude of the response. A total of 9 environments were created: Injector X, Y, and Z; Gas Generator X, Y, and Z; and Turbopump X, Y, and Z. A typical environment is shown in Fig. 5 which is the Gas Generator Y-Direction environment. The creation of an engine acceleration environment is subjective and almost an art in some cases. There is a lot of room for variations in choosing envelope widths and heights, but the process used here is fairly typical.

The following 3 sets of analyses were carried out using the FEM with:

1. A forced response analysis in which the measured force PSD's from Fig. 3 were applied to the FEM and the responses then calculated. This is the method that we would like to use in a real engine design analysis if we had information on the applied force PSD's. The method is included as a reference for the other two methodologies.
2. A direct application of enforced accelerations at the 3 shaker drive points using the acceleration environments that were derived as described in the preceding paragraph. The pseudo-static and dynamic components of the response were calculated as well.
3. A set of equivalent forces was derived from the acceleration environment using Eq(14) and these equivalent forces were then applied to the FEM separately in the x, y, and z directions and the responses were calculated.

The results for acceleration RMS values at all accelerometer locations are shown in Figs. 6-7. Fig. 6 shows the results for all accelerometers and all directions. Fig. 7 shows the root-sum-square of all 3 directions at each of the accelerometers which represents an overall acceleration measurement at that point and is one way of eliminating errors associated with the alignment of the accelerometers. These figures clearly indicate that both enforced accelerations and equivalent forces yield very conservative results. The worst-case for the equivalent force analysis is the x-direction and it gives consistently higher results than the enforced acceleration results for 10 of 17 accelerometers. To get an idea of the frequency distribution of the responses, the acceleration PSD's for two Accelerometer locations are shown in Figs. 9-10. Accelerometer #1 is on the LOX injector duct and Accelerometer #5 is mounted on a flange on the RP feedline. Note that the response using the enveloped environment exceeds the actual measured responses for all frequencies. These PSD's are typical of all the other acceleration PSD's.

The RMS strains are presented in Fig. 8 with the strain PSD's presented in Figs. 11-13. Both methods yield very conservative results here as well. The pseudostatic portion of the enforced acceleration response is clearly noticeable as the curves rapidly increases for low frequencies. This is a direct result of the term in the denominator of Eq(10). If we consider only the dynamic portion of the response, however, the enforced acceleration method actually gives RMS values that are comparable to or smaller than the unidirectional equivalent force method results. This is true

even though the enforced accelerations are applied in all 3 directions simultaneously and the the equivalent force terms are applied in only one direction at a time.

Strain gage 1 is located on the gas generator and the enforced acceleration response for this gage is particularly high. In fact, the bar in Fig. 8 is truncated in order that it not dwarf the other bars. Most of the RMS value for SG1, however, is due to the pseudostatic response as can clearly be seen in Fig. 11. The reason for this is that SG1 is located between the gas generator and the turbopump and both of these points have enforced accelerations applied. The large relative motion between them is what causes the large pseudostatic response. This motion is an artifact of the methodology and is not actually there as is apparent from the test data that rolls off to very low values at the lower frequencies. The second strain gage is located on the turbopump exhaust duct which runs down the side of the nozzle. This gage does not experience as much pseudostatic response as SG1, but there is still a significant amount as can be seen from Figs 8 and 12. Strain gage 3 is located on the RP inlet duct that is attached to the turbopump. This duct is small compared to the other two strain gage locations and is dynamically isolated from the rest of the engine components. Because of this, the pseudostatic response at the SG3 location is very small as is obvious from Figs. 8 and 13.

#### IV. Conclusions

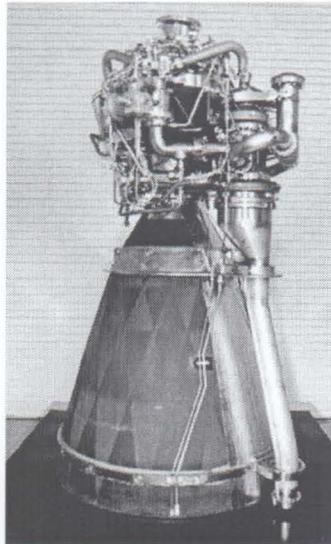
Conclusions based on these results are that when combined with an acceleration environment derived by enveloping accelerometer responses both the enforced acceleration methodology and the equivalent force methodology give very conservative results. Depending on the location, the responses resulting from the use of the enforced acceleration methodology can become very large at low frequencies due to the presence of a significant pseudostatic component. This is especially true for components that are closely coupled to the rest of the system but is less so for components such as ducts that are dynamically isolated from the rest of the engine system. Since the measured strains do show any of this low-frequency response, it is likely that it is an artifact of the methodology used and should be removed from the results. If we neglect this pseudostatic component and keep only the remaining dynamic portion of the response, then the enforced acceleration methodology gives results that are typically closer to the actual measured values than are the results obtained using the unidirectional equivalent force methodology. In addition, the enforced acceleration methodology results in a motion of the structure that exactly matches the defined environment and it is applied in all 3 directions simultaneously, unlike the equivalent force approach which is only applied in one direction at a time. If a system modeling approach is going to be used in the design of an engine, then these results indicate that the enforced acceleration methodology with the pseudostatic component removed should give better results than the equivalent force method.

#### V. References

- <sup>1</sup>Ballard, Richard O. and Olive, Tim, "Development Status of the NASA MC-1 (Fastrac) Engine, AIAA 2000-3898, 36<sup>th</sup> AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit, July 2000.
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- <sup>4</sup>Fraday G., Christensen, E. Mims, K., Harris, D., Parks, R., and Brunty, J. "Engine System Loads Development for the Fastrac 60K Flight Engine", paper AIAA-2000-1612 presented at the 41<sup>st</sup> AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference and Exhibit, Atlanta, GA, Apr. 3-6, 2000.
- <sup>5</sup>Meirovitch, L., Analytical Methods in Vibrations, The MacMillan Co., N.Y., 1989.
- <sup>6</sup>Nigam, N.C., Introduction to Random Vibrations, The MIT Press, Cambridge, MA, 1983.

**Table 1. Comparison of Modal Test and FEM Results**

Mode No.	Frequency (Hz)		Mode Shape Description
	Test	FEM	
1	-	0.0093	Rigid Body
2	-	0.0076	Rigid Body
3	-	0.0034	Rigid Body
4	-	0.0034	Rigid Body
5	-	0.0051	Rigid Body
6	-	0.0066	Rigid Body
7	59.54	58.45	Nozzle 2N
8	60.06	58.60	Nozzle 2N
9	93.43	88.93	1st Bending, TP Rocking Side-to-Side
10	99.39	94.43	1st Bending, TP Rocking Side-to-Side
11	113.82	113.99	Primary 1st Bending
12	141.23	147.41	Nozzle (3N, 0M)
13	146.34	149.48	Nozzle (3N, 0M)
14	147.29	153.70	Nozzle (3N, 0M) Bending, TP/Exhaust Duct Bending
15	167.01	170.52	RP Injector Duct bending side-to-side
16	189.91	193.23	Bending, TP/Exhaust Duct Bending



**Figure 1. The Fastrac (MC-1) Engine**

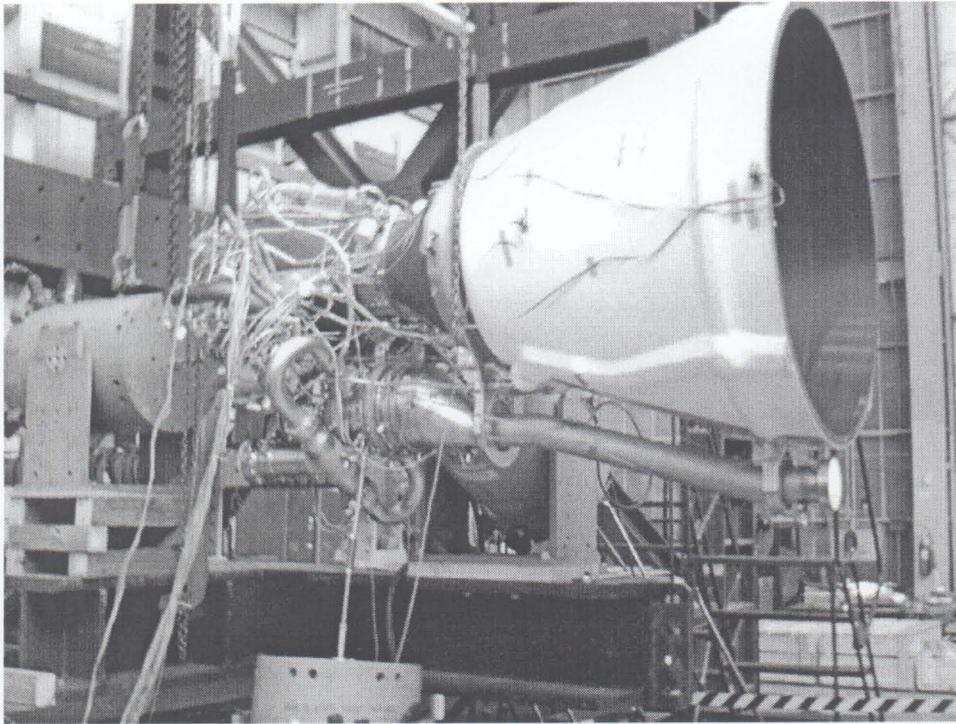


Figure 2. Fastrac Testbed

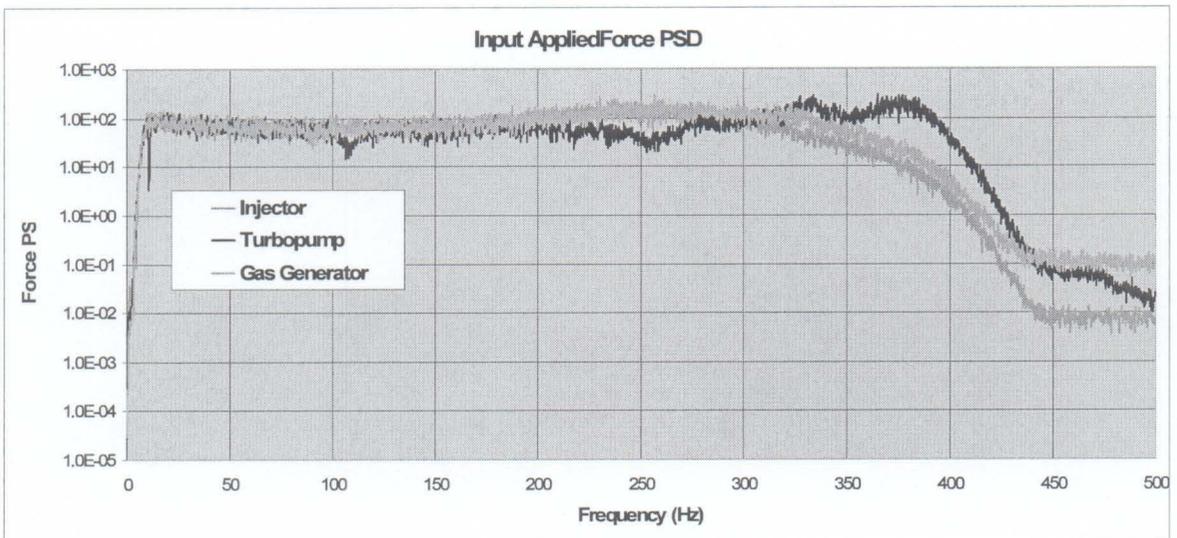


Figure 3. Shaker Force PSD's

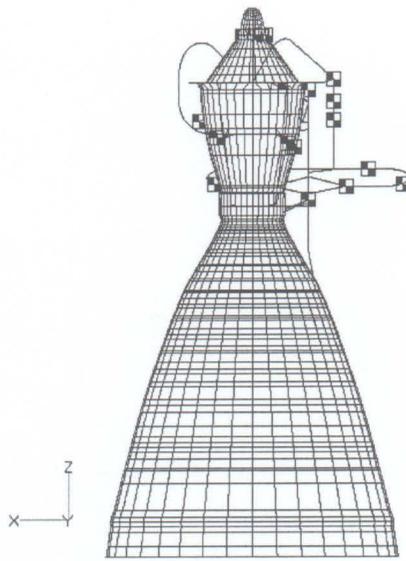


Figure 4. Fastrac Finite Element Model

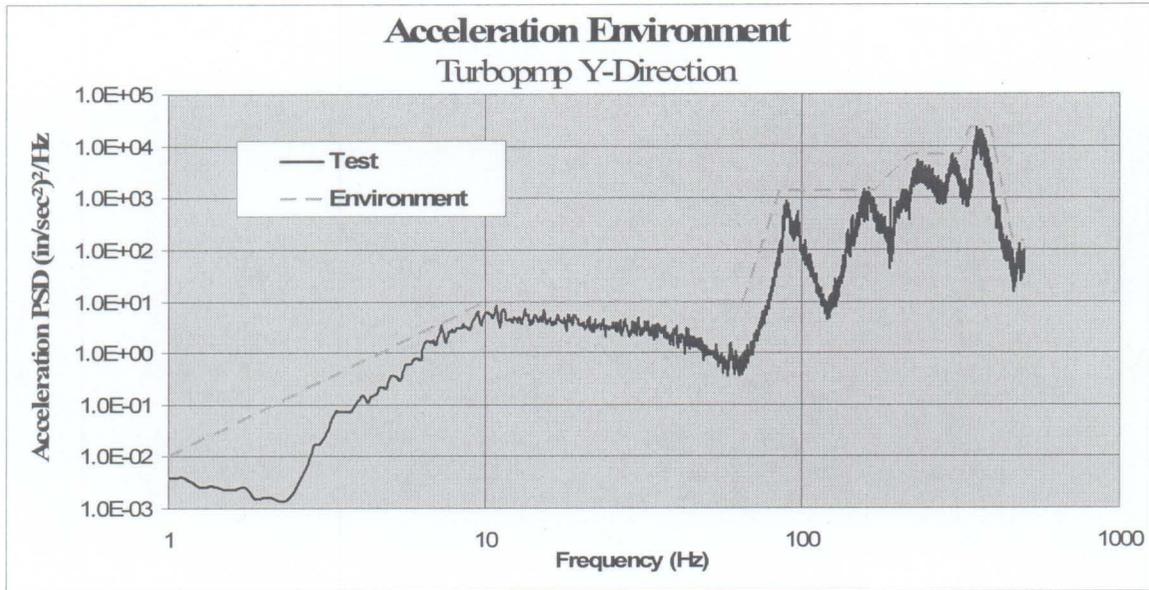


Figure 5. Acceleration Environment – Gas Generator Y-Direction

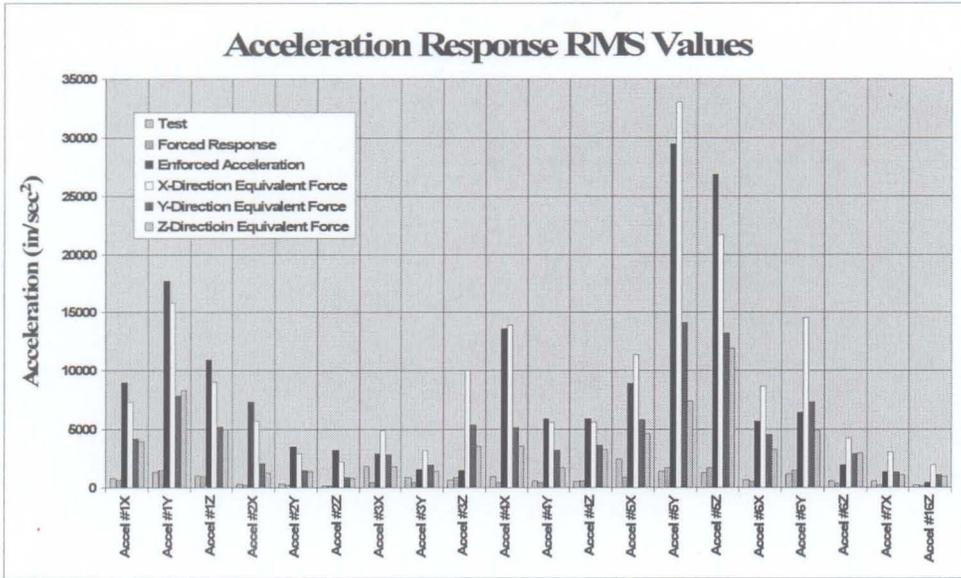


Figure 6. Comparison of Acceleration RMS Values

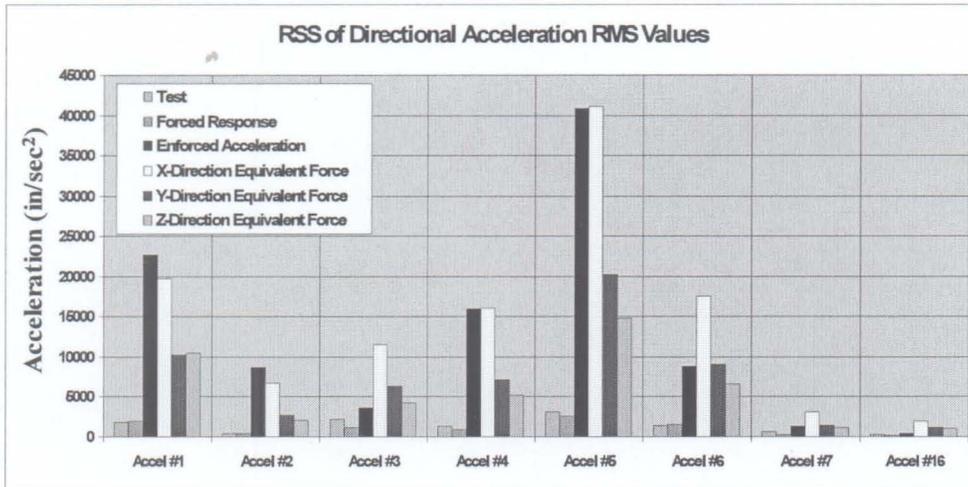


Figure 7. RSS of Directional Acceleration RMS Value

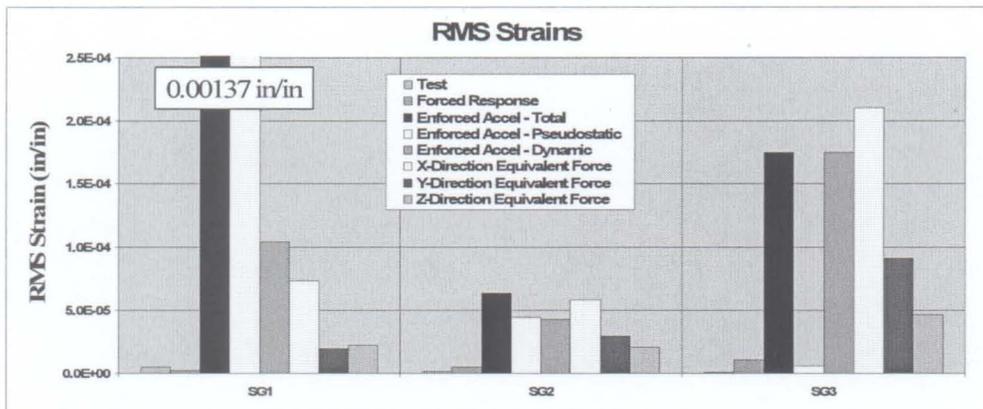
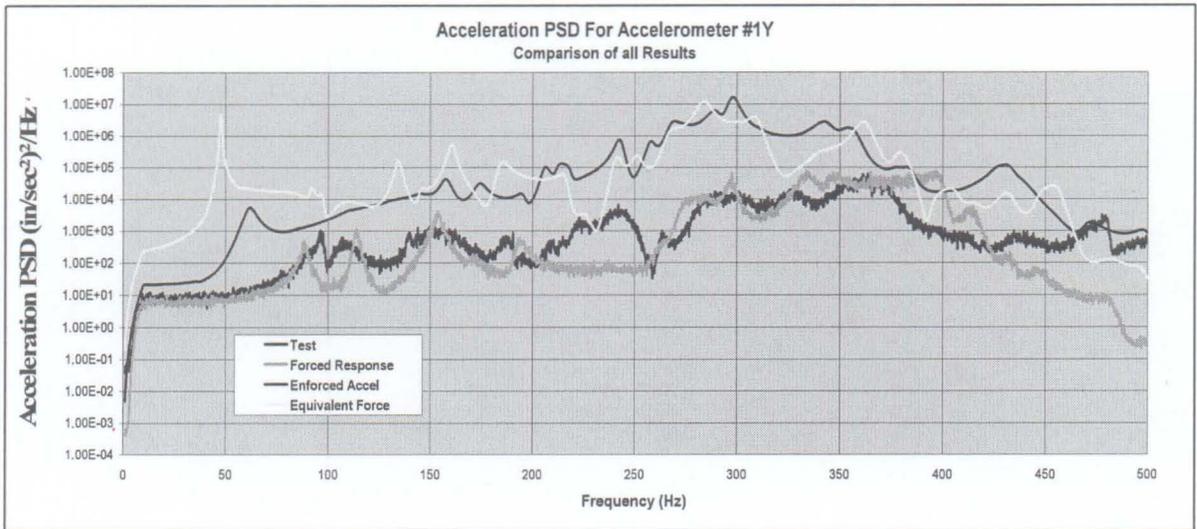
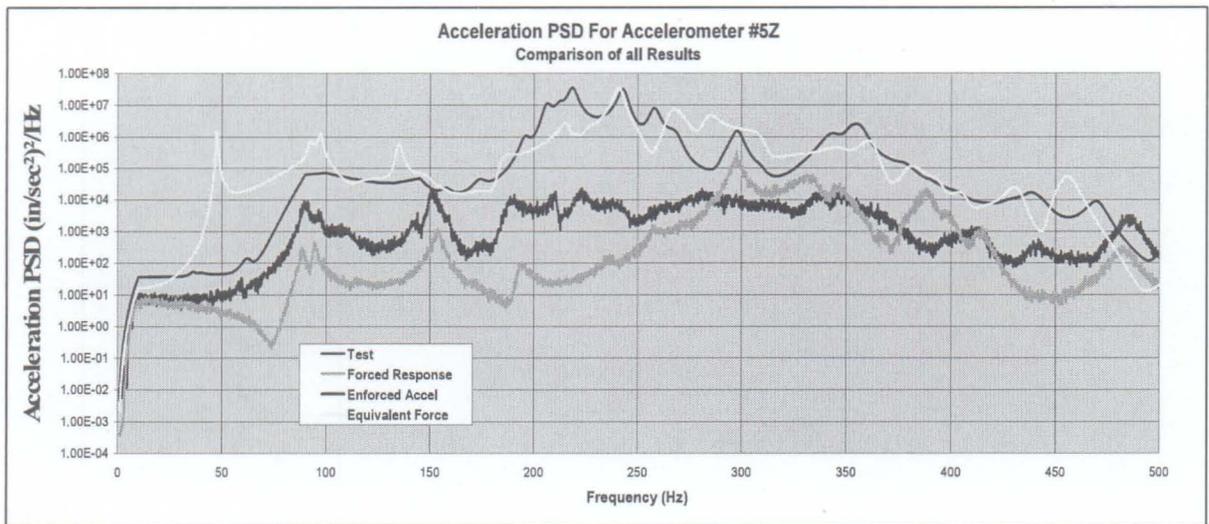


Figure 8. RMS Strains



**Figure 9. Acceleration PSD For Accelerometer #1 Y-Direction**



**Figure 10. Acceleration PSD For Accelerometer #5 Z-Direction**

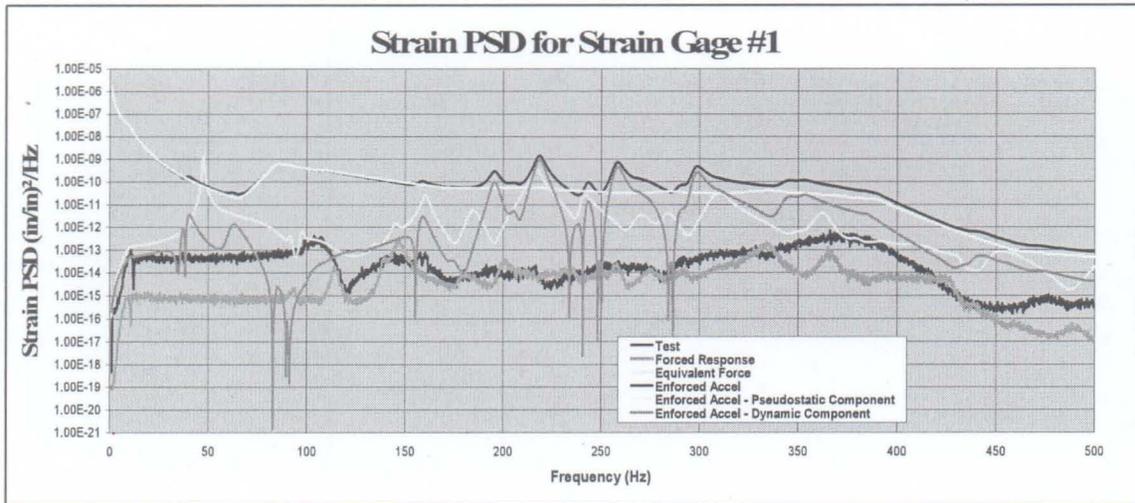


Figure 11. Strain Gage #1 Response PSD

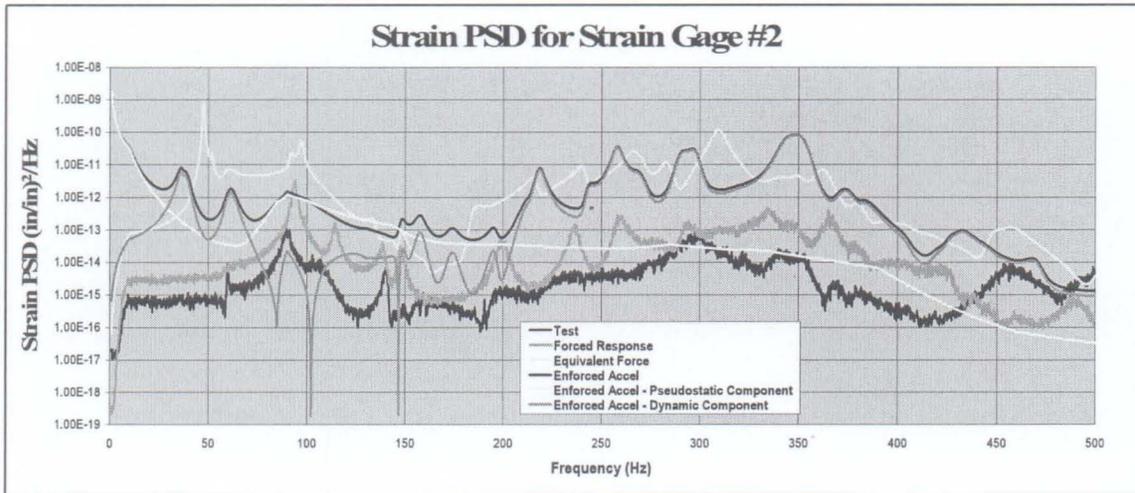


Figure 12. Strain Gage #2 Response PSD

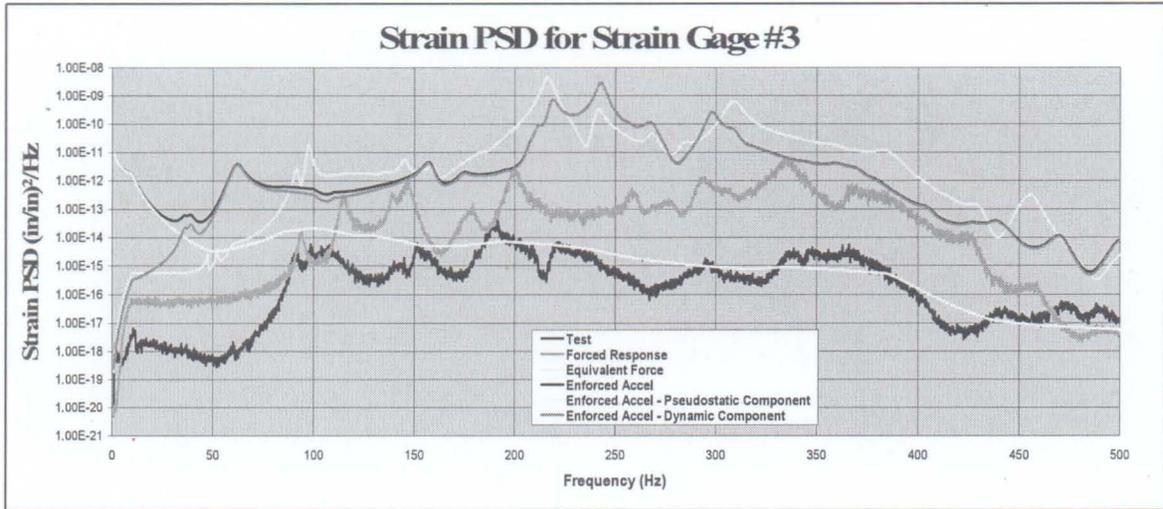


Figure 13. Strain Gage #3 Response PSD



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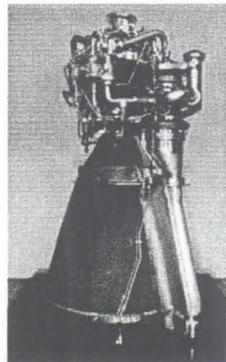
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## Engine System Loads Methodology Development Background

- Calculating rocket engine system loads always has been a quandary; we've always used *measured acceleration responses* as specified *excitations*.
- Before engine system models used, SSME designed conservatively on component level (base excitation of boundaries).
- Fastrac program utilized both engine system model “direct approach” (still applying accel excitation, RSS response from different locations) and component loads approach.
- RS-68 also used engine system direct approach.
- RS-83 planned on using unanchored “response matching method” (back out forces that would cause the accelerations to reduce conservatism).
- J2X planning on using a unidirectional equivalent force approach that generally results in very conservative loads.



Fastrac



SSME

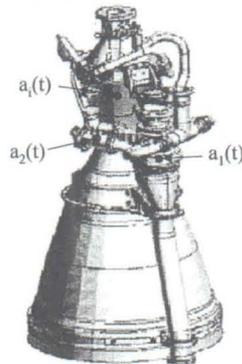
J2X



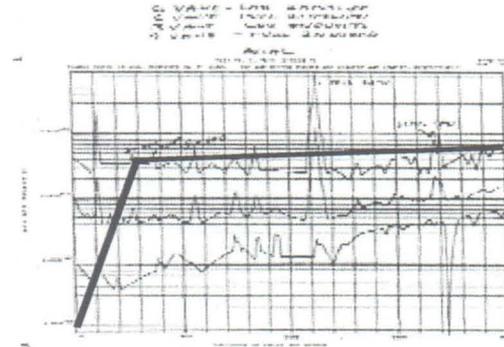
## Engine Dynamic Mechanical Loads

- Engine dynamic loads can be external or internal (self-induced loads)
- External loads include forces from ground transportation, acceleration g-loads, aerodynamic loads, etc.
- Internal Engine Self-Induced Loads
  - Self-induced loads result from extremely complex processes such as combustion, fluid flow, rotating turbomachinery, etc.
  - With the current level of technology, it is impossible to quantify these forces with enough precision to conduct a true transient dynamic analysis.
  - However, we can measure the engine dynamic environment (i.e., accelerations) at key locations in the engine. For a new engine, data from “similar” previous engine designs is scaled to define an engine vibration environment.
  - For steady-state operation there are two types of dynamic environments: sinusoidal (resulting from turbomachinery) and random.

Accelerations are measured at key locations near the primary vibration sources



Acceleration data is enveloped to capture uncertainties thus defining a vibration environment





## Calculating System Random Dynamic Loads

- Try to reproduce the engine environment by forcing engine response to match the measured (enveloped) accelerations
- Several ways this can be done
  - Enforced Accelerations
    - Directly apply an enforced acceleration at the points where environments are defined. This was the initial approach used for the Fastrac.
  - System Equivalent Applied Force Methods
    - Determine a set of applied forces that will reproduce the measured environment. Forces are typically applied at points where the environment is defined.
  - Component Approach
    - Calculate loads on a component basis. More difficult to model interactions between component and other parts of system. This was the method eventually used by Fastrac and by all earlier engine development programs.
- Note that even if we had a “perfect” methodology, the answers would still probably be conservative due to the enveloping of environments.



## System Equations of Motion

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{F(t)\}$$

- where
- $[M]$  = Mass matrix (dimension  $N \times N$ )
  - $[K]$  = Stiffness matrix (dimension  $N \times N$ )
  - $[C]$  = Damping matrix (dimension  $N \times N$ )
  - $\{x(t)\}$  = Displacement vector (dimension  $N \times 1$ )
  - $\{F(t)\}$  = Applied force vector (dimension  $N \times 1$ )

Response PSD can be calculated as follows:

$$[S_{XX}(f)] = [H^*(f)][S_{FF}(f)][H(f)]^T$$

- where
- $[S_{XX}(f)]$  = Matrix of displacement PSD's
  - $[S_{FF}(f)]$  = Matrix of applied force PSD's
  - $[H(f)]$  = Matrix of transfer functions
  - $f$  = Frequency



## Direct Enforced Acceleration Method

- Apply engine acceleration environments directly to the model as enforced accelerations.
- Constrain nodes to have a given acceleration random PSD

$X_f$  = Free DOF

$X_s$  = Support DOF where accelerations are applied

$$\begin{bmatrix} \mathbf{M}_{ff} & \mathbf{M}_{fs} \\ \mathbf{M}_{sf} & \mathbf{M}_{ss} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{X}}_f \\ \ddot{\mathbf{X}}_s \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{ff} & \mathbf{C}_{fs} \\ \mathbf{C}_{sf} & \mathbf{C}_{ss} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{X}}_f \\ \dot{\mathbf{X}}_s \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{ff} & \mathbf{K}_{fs} \\ \mathbf{K}_{sf} & \mathbf{K}_{ss} \end{bmatrix} \begin{Bmatrix} \mathbf{X}_f \\ \mathbf{X}_s \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{F}_s \end{Bmatrix} \quad \text{Eq (1)}$$

$$\mathbf{M}_{ff} \ddot{\mathbf{X}}_f + \mathbf{C}_{ff} \dot{\mathbf{X}}_f + \mathbf{K}_{ff} \mathbf{X}_f = -\mathbf{M}_{fs} \ddot{\mathbf{X}}_s(t) - \mathbf{C}_{fs} \dot{\mathbf{X}}_s(t) - \mathbf{K}_{fs} \mathbf{X}_s(t) = \mathbf{F}_{eq_s}(t) \quad \text{Eq (2)}$$

$$\mathbf{F}_s(t) = \mathbf{M}_{sf} \ddot{\mathbf{X}}_f + \mathbf{M}_{ss} \ddot{\mathbf{X}}_s + \mathbf{C}_{sf} \dot{\mathbf{X}}_f + \mathbf{C}_{ss} \dot{\mathbf{X}}_s + \mathbf{K}_{sf} \mathbf{X}_f + \mathbf{K}_{ss} \mathbf{X}_s \quad \text{Eq (3)}$$

Note: Eq(2) will result in different modes and frequencies than Eq(1)

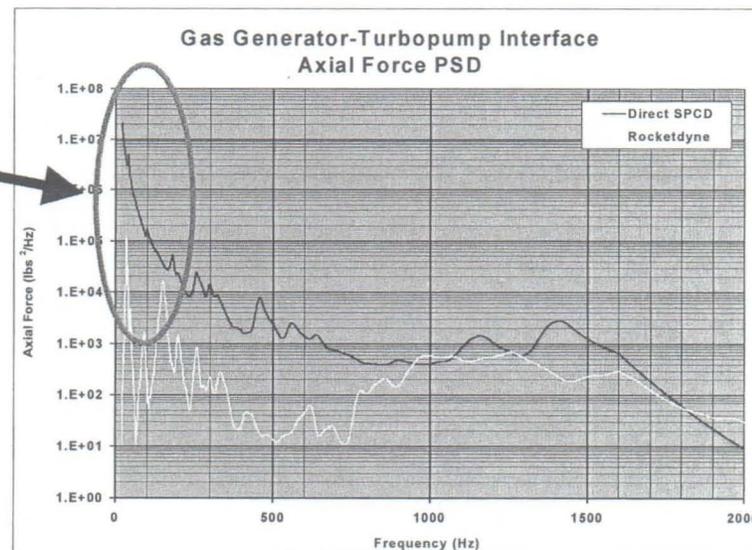
Solve Eq(2) using the NASTRAN random analysis methods (SOL 111)



## Direct Enforced Acceleration Method Pseudo-static Load Component

- Pseudo-static loads are the static forces in the structure due to relative motion between the environment points as they are driven with enforced motion
- Most of the static relative motion is likely an artifact of the methodology and is not present in the actual engine response.
- Can easily calculate the pseudo-static component and remove it from the results, leaving only the dynamic component

Large low-frequency response due to pseudo-static effect





## Direct Enforced Acceleration Method Calculation of Pseudo-static Load Component

Consider Eq(2) again:

$$\mathbf{M}_{ff} \ddot{\mathbf{X}}_f + \mathbf{C}_{ff} \dot{\mathbf{X}}_f + \mathbf{K}_{ff} \mathbf{X}_f = -\mathbf{M}_{fs} \ddot{\mathbf{X}}_s(t) - \mathbf{C}_{fs} \dot{\mathbf{X}}_s(t) - \mathbf{K}_{fs} \mathbf{X}_s(t) = \mathbf{F}_{eq_s}(t)$$

Break  $\mathbf{X}_f$  into two components,

$$\mathbf{X}_f(t) = \mathbf{X}_{fs}(t) + \mathbf{X}_{fd}(t)$$

$\mathbf{X}_{fs}(t)$  = Pseudo-static displacement

$\mathbf{X}_{fd}(t)$  = Dynamic displacement

$\mathbf{X}_{fs}(t)$  is calculated by ignoring the mass and damping terms in Eq(2):

$$\mathbf{K}_{ff} \mathbf{X}_{fs}(t) = -\mathbf{K}_{fs} \mathbf{X}_s(t)$$

$$\mathbf{X}_{fs}(t) = -\mathbf{K}_{ff}^{-1} \mathbf{K}_{fs} \mathbf{X}_s(t) = \mathbf{K}_I \mathbf{X}_s(t) \quad \mathbf{K}_I = \text{Influence Coefficient Matrix}$$

$\mathbf{X}_{fs}(t)$  represents the displacement due to “static” application of the prescribed support accelerations at each time instant. It is essentially the response of the structure if it were massless and undamped.



# Equivalent Applied Force Method

## Unidirectional Approach

- Define a set of equivalent force PSD's that when applied to the model reproduce the acceleration environment as closely as possible
- Unidirectional approach – assume a set of  $p$  uncorrelated applied forced PSD's applied at the model at the points where environments defined
- Can express relationship between applied force PSD's and resulting acceleration PSD's using transfer functions as follows:

$$\{S_{aa}(f)\} = [T(f)]\{S_{FF}(f)\}$$

where  $\{S_{aa}(f)\}$  = Vector of known environment accel PSD's  
 $\{S_{FF}(f)\}$  = Matrix of applied force PSD's  
 $[T(f)]$  = Matrix of transfer functions

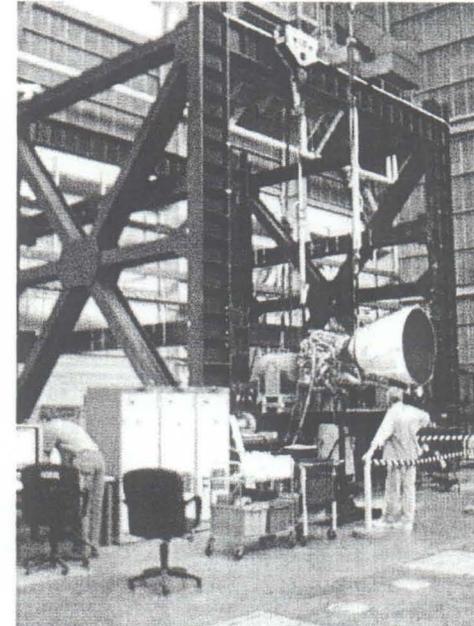
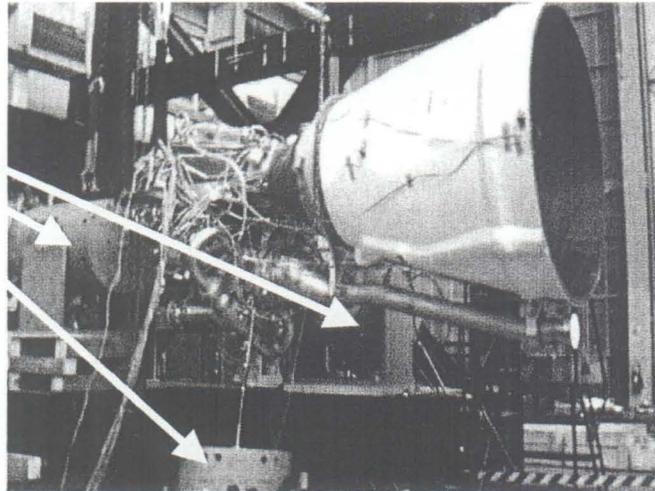
- Can't simply invert  $[T(f)]$  because may not get positive force PSD's
- Neglect off-diagonal terms and solve for force PSD's
- Results in very conservative PSD's, but no pseudo-static loads
- Apply force PSD's to model one direction at a time (unidirectionally) and then choose the direction which gives the worst loads



## Fastrac Engine Testbed

- FASTRAC engine obtained for testing
- Engine supported in free-free condition by bungee cords
- 3 large shakers attached at injector, gas generator, and turbopump locations
- Engine instrumented with 21 accelerometers and 3 strain gages
- Shaker forces applied one at a time and simultaneously

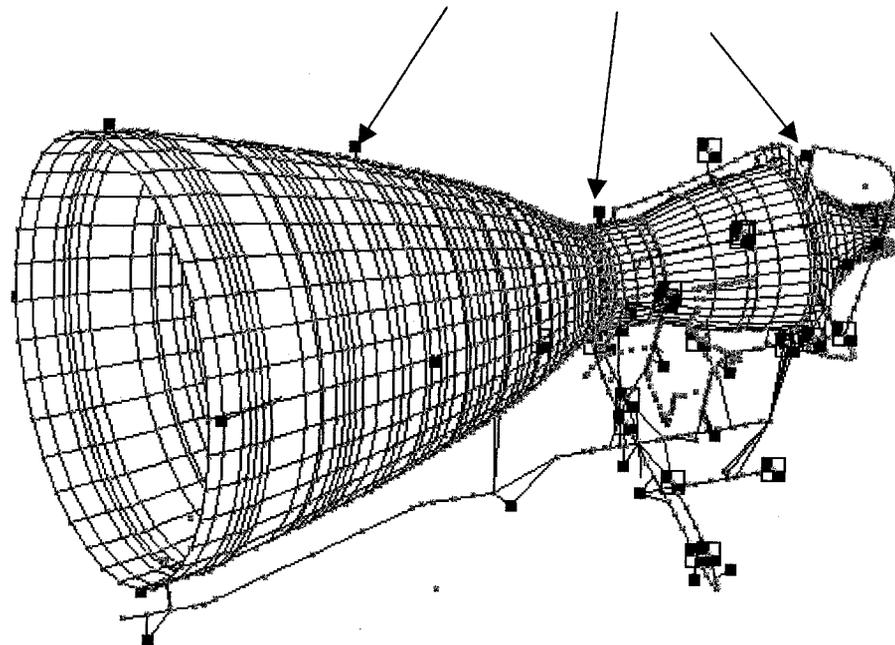
shakers





# Instrumentation

## Accelerometer Locations



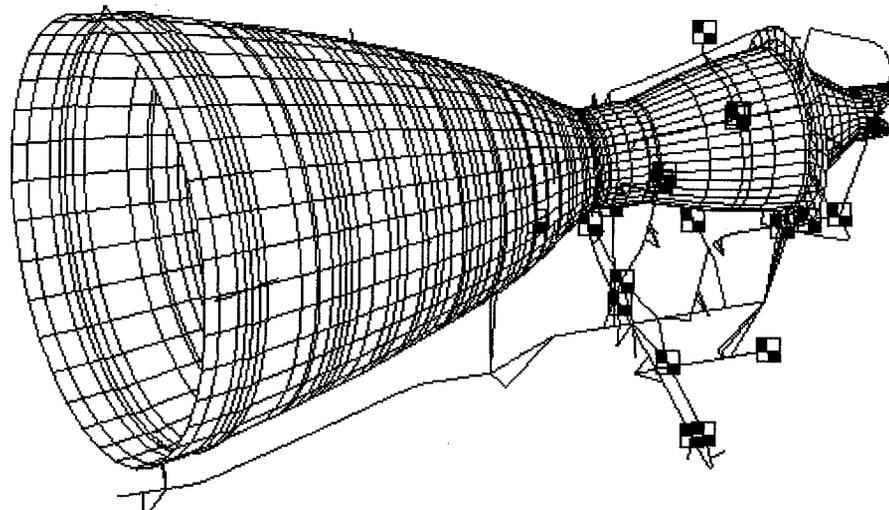
Strain Gage Locations:

Gas Generator, Turbopump Exhaust Duct, RP Discharge Duct



## Testbed Finite Element Model

- Finite element model (FEM) used in previous loads cycles and adapted to testbed configuration
- Model Sizes:
  - 3662 nodes
  - 3621 elements – plates, beams, rods, concentrated masses, rigid
- Correlated to testbed modes and frequencies





## Correlation of FEM to Test Modal Results

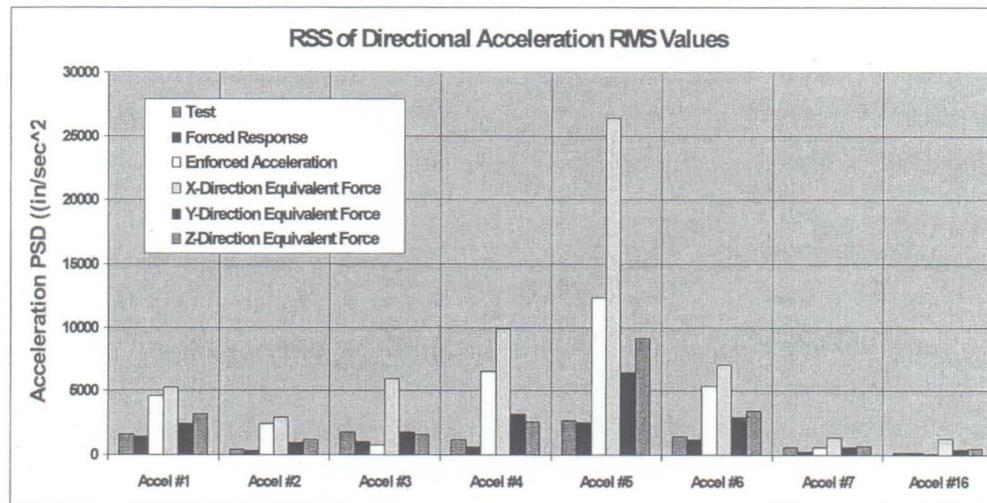
Mode No.	Frequency (Hz)		Mode Shape Description
	Test	FEM	
1	-	0.0093	Rigid Body
2	-	0.0076	Rigid Body
3	-	0.0034	Rigid Body
4	-	0.0034	Rigid Body
5	-	0.0051	Rigid Body
6	-	0.0066	Rigid Body
7	59.54	58.45	Nozzle 2N
8	60.06	58.60	Nozzle 2N
9	93.43	88.93	1st Bending, TP Rocking Side-to-Side
10	99.39	94.43	1st Bending, TP Rocking Side-to-Side
11	113.82	113.99	Primary 1st Bending
12	141.23	147.41	Nozzle (3N, 0M)
13	146.34	149.48	Nozzle (3N, 0M)
14	147.29	153.70	Nozzle (3N, 0M) Bending, TP/Exhaust Duct Bending
15	167.01	170.52	RP Injector Duct bending side-to-side
16	189.91	193.23	Bending, TP/Exhaust Duct Bending



## Test vs. Analysis Results

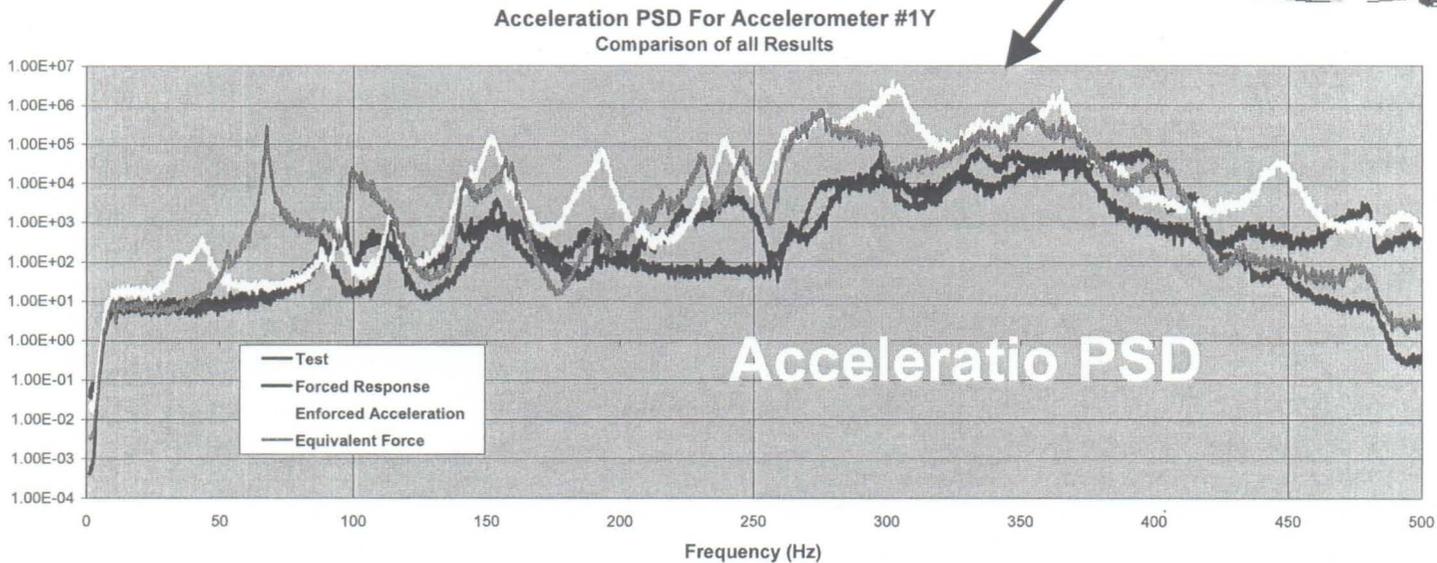
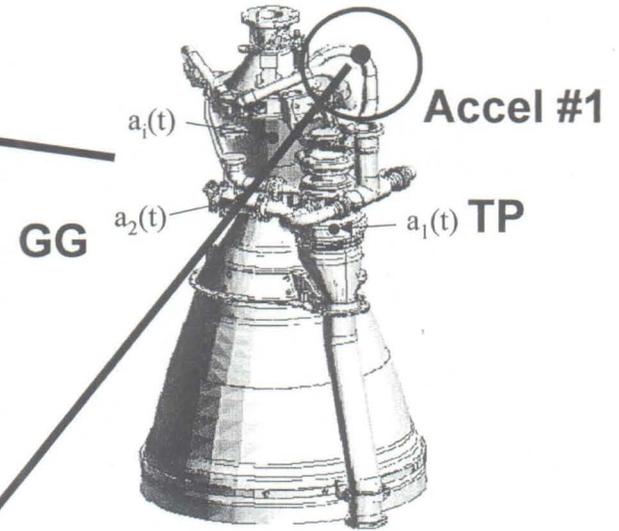
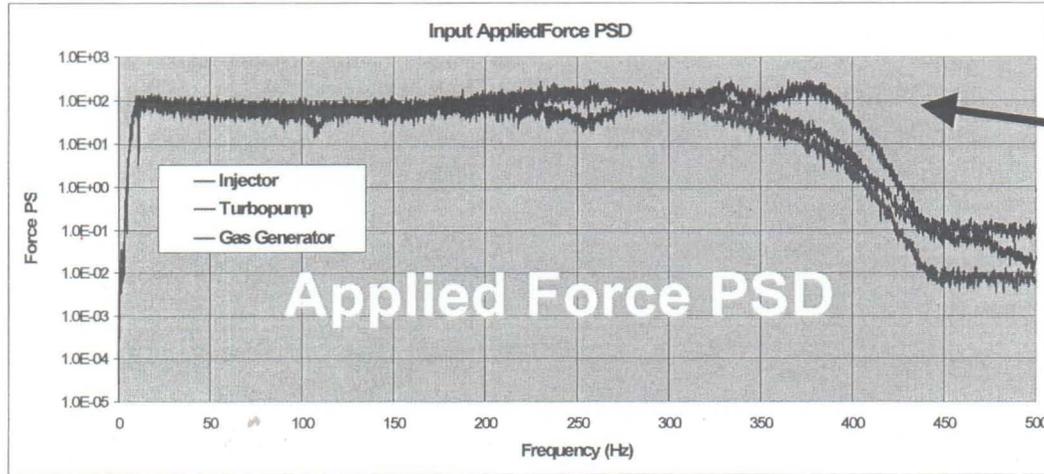
- All shakers applied simultaneously with random force PSD of  $\sim 100 \text{ lb}^2/\text{Hz}$
- Compare test data to following 3 analyses:
  - Forced response where measured force PSD is applied to the FEM
  - Direct enforced accelerations using acceleration environment calculated from measured accelerations
  - Equivalent forces derived from the same acceleration environment

### Accelerometer RMS Values



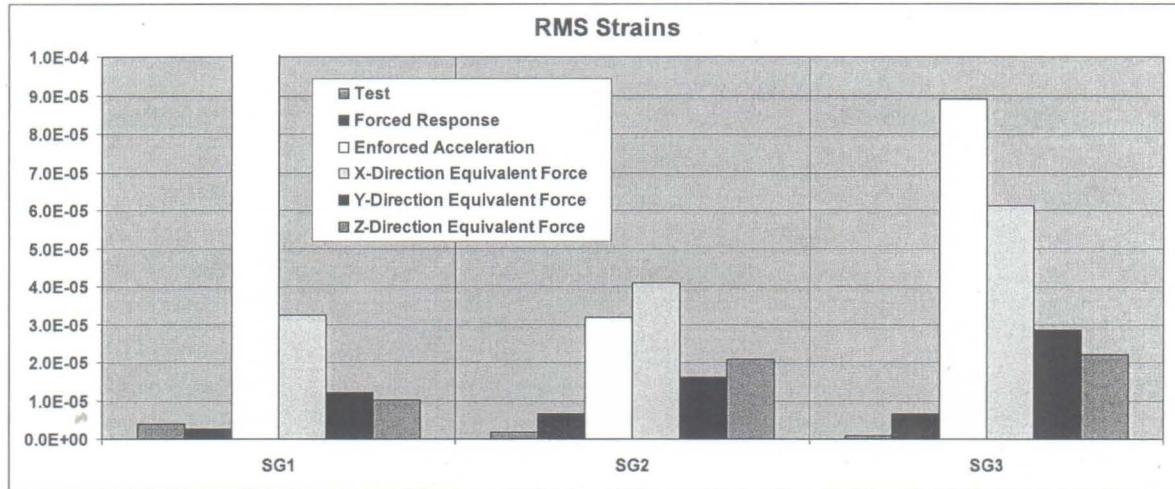


# Response PSD's for Accelerometer #1 at LOX Injector Duct





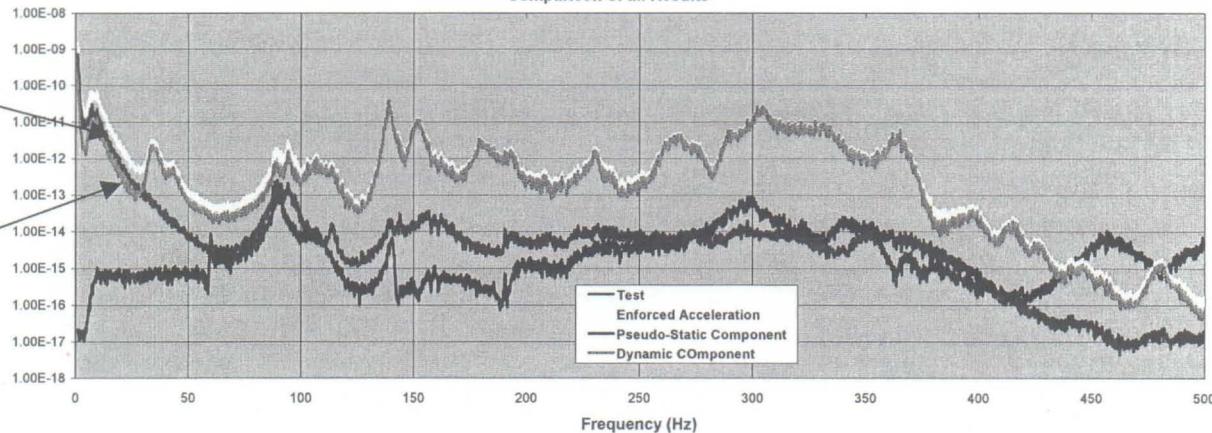
## Response PSD's for Strain Gages



Acceleration PSD For Strain Gage #2  
Comparison of all Results

Pseudostatic  
Effect

Dynamic  
Component





## Conclusions

- Both the enforced acceleration and the unidirectional equivalent force methodologies give conservative results
- Equivalent force gives much higher results for accelerations
- Enforced accelerations gives higher results for strains (and hence loads)
- Removing the pseudo-static portion of the Enforced acceleration methodology loads results in smaller values
- Both methods yield fairly comparable results, but enforced acceleration approach is simpler and more straightforward