On the Minimum Induced Drag of Wings

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Introduction

- The History of Spanload
  Development of the optimum spanload
  Winglets and their implications
- Horten Sailplanes
- Flight Mechanics & Adverse yaw
- Concluding Remarks
History

- Bird Flight as the Model for Flight
- Vortex Model of Lifting Surfaces
- Optimization of Spanload
  Prandtl
  Prandtl/Horten/Jones
  Klein/Viswanathan
- Winglets - Whitcomb
Birds
Bird Flight as a Model

or “Why don’t birds have vertical tails?”

- Propulsion
  - Flapping motion to produce thrust
  - Wings also provide lift
  - Dynamic lift - birds use this all the time (easy for them, hard for us)

- Stability and Control
  - Still not understood in literature
  - Lack of vertical surfaces

- Birds as an Integrated System
  - Structure
  - Propulsion
  - Lift (performance)
  - Stability and control

![Dynamic Lift Chart]

\( \alpha = 0.0358 \)

\( M = 0.225 \)
Early Mechanical Flight

- Otto & Gustav Lilienthal (1891-1896)
- Octave Chanute (1896-1903)
- Samuel P Langley (1896-1903)
- Wilbur & Orville Wright (1899-1905)
Otto Lilienthal

- Glider experiments 1891 - 1896
Dr Samuel Pierpont Langley

- Aerodrome experiments 1887-1903
Octave Chanute

- Gliding experiments 1896 to 1903
Wilbur & Orville Wright

Flying experiments 1899 to 1905
Spanload Development

- **Ludwig Prandtl**
  - Development of the boundary layer concept (1903)
  - Developed the “lifting line” theory
  - Developed the concept of induced drag
  - Calculated the spanload for minimum induced drag (1908?)
  - Published in open literature (1920)

- **Albert Betz**
  - Published calculation of induced drag
  - Published optimum spanload for minimum induced drag (1914)
  - Credited all to Prandtl (circa 1908)
Spanload Development (continued)

- Max Munk
  General solution to multiple airfoils
  Referred to as the “stagger biplane theorem” (1920)
  Munk worked for NACA Langley from 1920 through 1926

- Prandtl (again!)
  “The Minimum Induced Drag of Wings” (1932)
  Introduction of new constraint to spanload
  Considers the bending moment as well as the lift and induced drag
Practical Spanload Developments

Reimar Horten (1945)
Use of Prandtl’s latest spanload work in sailplanes & aircraft
Discovery of induced thrust at wingtips
Discovery of flight mechanics implications
Use of the term “bell shaped” spanload

Robert T Jones
Spanload for minimum induced drag and wing root bending moment
Application of wing root bending moment is less general than Prandtl’s
No prior knowledge of Prandtl’s work, entirely independent (1950)

Armin Klein & Sathy Viswanathan
Minimum induced drag for given structural weight (1975)
Includes bending moment
Includes shear
Prandtl Lifting Line Theory

- Prandtl’s “vortex ribbons”

- Elliptical spanload (1914)

- “the downwash produced by the longitudinal vortices must be uniform at all points on the aerofoils in order that there may be a minimum of drag for a given total lift.” $y = c$
Elliptical Half-Lemniscate

- Minimum induced drag for given control power (roll)
- Dr Richard Eppler: FS-24 Phoenix
Elliptical Spanloads
Minimum Induced Drag & Bending Moment

Prandtl (1932)
Constrain minimum induced drag
Constrain bending moment
22% increase in span with 11% decrease in induced drag
Horten Applies Prandtl’s Theory

- Horten Spanload (1940-1955)
  - induced thrust at tips
  - wing root bending moment
Minimize induced drag (1950)
Constrain wing root bending moment
30% increase in span with 17% decrease in induced drag

“Hence, for a minimum induced drag with a given total lift and a given bending moment the downwash must show a linear variation along the span.” $y = bx + c$
λ. Minimize induced drag (1975)  
Constrain bending moment  
Constrain shear stress  
16% increase in span with 7% decrease in induced drag  

"Hence the required downwash-distribution is parabolic."

\[ y = ax^2 + bx + c \]
Richard Whitcomb’s Winglets
- induced thrust on wingtips
- induced drag decrease is about half of the span “extension”
- reduced wing root bending stress
Winglet Aircraft
Spanload Summary

- Prandtl/Munk (1914)
  Elliptical
  Constrained only by span and lift
  Downwash: $y = c$

- Prandtl/Horton/Jones (1932)
  Bell shaped
  Constrained by lift and bending moment
  Downwash: $y = bx + c$

- Klein/Viswanathan (1975)
  Modified bell shape
  Constrained by lift, moment and shear (minimum structure)
  Downwash: $y = ax + bx + c^2$

- Whitcomb (1975)
  Winglets

- Summarized by Jones (1979)
Early Horten Sailplanes (Germany)

- Horten I - 12m span
- Horten II - 16m span
- Horten III - 20m span
Horten Sailplanes (Germany)

- H IV - 20m span
- H VI - 24m span
Horten Sailplanes (Argentina)

- H I b/c - 12m span
- H XV a/b/c - 18m span
Later Horten Sailplanes (Argentina)

- H Xa/b/c
- 7.5m, 10m, & 15m
Bird Flight Model

- Minimum Structure
- Flight Mechanics Implications
- Empirical evidence
- How do birds fly?
Horten H Xc Example

- Horten H Xc footlaunched ultralight sailplane 1950
Calculation Method

- Taper
- Twist
- Control Surface Deflections
- Central Difference Angle
Dr Edward Udens’ Results

- Spanload and Induced Drag
- Elevon Configurations
- Induced Yawing Moments

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“Mitteleffekt”

- Artifact of spanload approximations
- Effect on spanloads
  - increased load at tips
  - decreased load near centerline
- Upwash due to sweep unaccounted for
Horten H Xc Wing Analysis

- Vortex Lattice Analysis
- Spanloads (longitudinal & lateral-directional) - trim & asymmetrical roll
- Proverse/Adverse Induced Yawing Moments handling qualities
- Force Vectors on Tips - twist, elevon deflections, & upwash
- 320 Panels: 40 spanwise & 8 chordwise
Symmetrical Spanloads

- Elevon Trim
- CG Location
Asymmetrical Spanloads

- $\lambda \ Cl \delta a$ (roll due to aileron)
- $\lambda \ Cn \delta a$ (yaw due to aileron)
- induced component
- profile component
- change with lift
- $\lambda \ Cn \delta a/Cl \delta a$
- $\lambda \ CL$(Lift Coefficient)
  Increased lift:
  increased Cl$\beta$
  increased Cn$\beta^*$
  Decreased lift:
  decreased Cl$\beta$
  decreased Cn$\beta^*$

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Airfoil and Wing Analysis

- Profile code (Dr Richard Eppler)
- Flap Option (elevon deflections)
- Matched Local Lift Coefficients
- Profile Drag
- Integrated Lift Coefficients
  - match Profile results to Vortex Lattice
  - separation differences in lift
- Combined in MatLab
Performance Comparison

- Max L/D: 31.9
- Min sink: 89.1 fpm
- Does not include pilot drag

- Predicted L/D: 30
- Predicted sink: 90 fpm
Horten Spanload Equivalent to Birds

- Horten spanload is equivalent to bird span load (shear not considered in Horten designs)
- Flight mechanics are the same - turn components are the same
- Both attempt to use minimum structure
- Both solve minimum drag, turn performance, and optimal structure with one solution
Concluding Remarks

- Birds as the first model for flight
- Theoretical developments independent of applications
- Applied approach gave immediate solutions, departure from bird flight
- Eventual meeting of theory and applications (applied theory)
- Spanload evolution (Prandtl/Munk, Prandtl/Horten/Jones, Klein & Viswanathan)
- Flight mechanics implications
- Hortens are equivalent to birds

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References

- Prandtl, Ludwig: ”Uber Tragflugel kleinsten induzierten Widerstandes”; Zeitschrift fur Flugtechnik und Motorluftschiffahrt, 28 XII 1932; Munchen, Deustchland.
- Horten, Reimar; and Selinger, Peter; with Scott, Jan (translator): “Nurflugel: the Story of Horten Flying Wings 1933 - 1960”; Weishapt Verlag; Graz, Austria; 1985.
- Horten, Reimar; unpublished personal notes.
- Udens, Edward; unpublished personal notes.
- Klein, Armin and Viswanathan, Sathy; “Approximate Solution for Minimum induced Drag of Wings with a Given Structural Weight”; Journal of Aircraft, Feb 1975, Vol 12 No 2, AIAA.
- Jones, Robert T; “Minimizing induced Drag.”; Soaring, October 1979, Soaring Society of America.
- Koford, Carl; “California Condor”; Audobon Special Report No 4, 1950, Dover, NY.
How do birds fly?