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Transition Models for Engineering Calculations

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Abstract: While future theoretical and conceptual developments may promote a better understanding of the physical processes involved in the latter stages of boundary layer transition, the designers of rotodynamic machinery and other fluid dynamic devices need effective transition models now. This presentation will therefore centre round the development of some transition models which have been developed as design aids to improve the prediction codes used in the performance evaluation of gas turbine blading. All models are based on Narasimha’s concentrated breakdown and spot growth hypothesis.

The first model uses a correlation of the non-dimensional spot source rate density as a function of the local pressure gradient parameter and the freestream turbulence level at the onset of transition. Even although quite reasonable agreement is observed for a significant number of transitional flows, the rigidity of a correlation based on onset parameters leaves a lot to be desired. For gas turbine blade flows in particular, the pressure gradient may not only change rapidly in magnitude, but can also change in sign during transition. Since it was thought that this could have significant influence on the turbulent spot seeding rates, an alternative transition model was postulated in the differential form of a first order system. In the differential model the transition length parameter, $\lambda$, is left as a limited function of the streamwise distance, $x$.

ie.  \[
d\lambda/dx = \left\{2Ax_t/\lambda_{av}^2\right\}\left[1 - (x_t/\lambda_{av})(d\lambda/dx)\right] \exp\left(-Ax_t^2/\lambda_{av}^2\right)
\]

where $\lambda_{av} = (1/x_t) \int_{x_\lambda = 0}^{x_\lambda = 0.25} \lambda \, dx_t$; $A = 0.411$ and $x_t = (x - x_{tran})$

Preliminary studies have indicated that $d\lambda/dx$ may in some circumstances vary significantly at the start of transition, but usually tends to zero for $\lambda > 0.2$. Equally good agreement with the experimental data is therefore found by simply using the original Narasimha intermittency function with $\lambda_{av}$ as defined above. These two latter empirical forms however, impart the required dynamical flexibility on the intermittency characteristics and uncouples the predicted distributions from the start of transition parameters.

Comparisons of the boundary layer integral parameters using intermittency weighted predictions and measured data are used as a basis for assessment and evaluation of the transition models developed.
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Zero Pressure Gradient Correlations

\[ [R_\theta]_{tran} = 0.85 \left[ 163 + e^{6.91 - \theta} \right] \]

\[ R_\lambda = 2.947 \left[ 10 - e^{(1.7 - \lambda/2)} \right] (R_\theta)^{3/2} \]

also \[ R_\lambda = \sqrt{\frac{0.411 \cdot (R_\theta)^3}{N_o}} \]

\[ \therefore N_o \times 10^3 = 47.324 / \left[ 10 - e^{(1.7 - \lambda/2)} \right]^2 \]
Schubauer & Skramstad, 1948
+ Gostelow & Ramachandran, 1983
△ Gostelow & Blunden, 1988
△ Blair & Werle, 1980
▼ Schubauer & Klebanoff, 1956
× Gardiner, 1987
● Abu-Ghannam & Shaw, 1980

1. \( R_\theta = 163 + \exp(6.91 - t) \)

2. \( R_\theta = 0.85[163 + \exp(6.91 - t)] \)

\[
R_\lambda = 2.947[R_\theta]^{3/2}
\]

\[
R_\lambda = 16.63[R_\theta]^{3/2}
\]

\[
R_\lambda = 2.947[10 - \exp(1.7 - t/2)](R_\theta)^{3/2}
\]
\[
\frac{0.85 - 16.9}{P_0} \left( 1 - \frac{1}{t} \right)
\]
\[
N \times 10^3 = \frac{47.324}{[10 - \exp(1.7 - t/2)]^2}
\]

Non-zero Pressure Gradient Correlations

\[
\begin{align*}
N &= \frac{N_o \cdot e^{m \cdot (\log_{10}A + 3.6 \cdot (\log_{10}M - 86)}}}{300 m^4} \\
\left(\frac{m}{s}\right) &= 2.61 + 3.6 \cdot (\log_{10}A) \\
f(m) &= m \cdot (\log_{10}(M) - 86) \\
N &= N_o \cdot e^{N_o \cdot (\log_{10}M)} \\
\frac{df}{dt} &= 6.91 + 2.48 m - 12.27 m^2 \\
g(m) &= 6.91 + 12.75 m + 63.64 m^2 \\
\text{for } \{m < 0\} \\
[\text{Ref.} \text{ran} &= 0.85 \left(163 + e^{0.001 \cdot (m - 86)}\right)] \\
\text{for } \{m > 0\}
\end{align*}
\]
\[ R_\lambda = \left( \frac{0.411 (R_\theta)^3}{N} \right)^{0.5} \]
Transition Length Reynolds Number
for \( t = 0 \) to \( 12\% \), step \( 1\% \)
and \( m = -0.03 \) to \( +0.83 \), step \( 0.01 \)
Freestream Velocity Distributions
(from Sharma et al)

U (m/s)

0 0.5 1
(x/L)

aft-loaded blade
squared-off blade
L = 950 mm

R* x 1000

C* x 1000

transition

(aft-loaded blade) data - Sharma et al

prediction, t = 19%

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Intermittency Functions

\[ \int_0^{x \lambda} \gamma \, dx \]  
where \( \lambda = \lambda_0 \) usin different form

\[ \int_0^{x \lambda} \gamma \, dx \]  
where \( \lambda = \lambda_0 \) usin different form

Intermittency Distributions

\[ (\gamma/\gamma^1 \lambda x \lambda - \gamma) \exp \left[ (x \lambda / \gamma) \gamma^1 \lambda x \lambda - \gamma \right] - 1 \{ \gamma \} = x \lambda / \gamma \delta p \]

\[ (\gamma/\gamma^1 \lambda x \lambda - \gamma) \exp \left[ (x \lambda / \gamma) \gamma^1 \lambda x \lambda - \gamma \right] - 1 \{ \gamma \} = x \lambda / \gamma \delta p \]

Differential forms

\[ (x \gamma - x) = x \gamma \lambda \text{ and } 1 + \frac{1}{\gamma} = 0 \]

\[ (\gamma/\gamma^1 \lambda x \lambda - \gamma) \exp \left[ (x \lambda / \gamma) \gamma^1 \lambda x \lambda - \gamma \right] - 1 \{ \gamma \} = x \lambda / \gamma \delta p \]
data - Abu-Ghannam & Shaw
1st favourable \( \frac{dp}{dx} \)
\( \frac{dU}{dx} = 4.575 \text{ s}^{-1}, \text{(approx)} \)
\( t = 1.55\% \)
\( U = 24.4 \text{ m/s at } x = 1168 \text{ mm} \)


data - Abu-Ghannam & Shaw
1st adverse \( \frac{dp}{dx} \)
\( \frac{dU}{dx} = -2.19 \text{ s}^{-1}, \text{(approx)} \)
\( t = 1.2\% \)
\( U = 18 \text{ m/s at } x = 1168 \text{ mm} \)
Data - Abu-Ghannam & Shaw
zero pressure gradient
$t = 1.25\%$, $U = 22$ m/s

Data - Dhawan & Narasimha
zero pressure gradient
$t = 1.1\%$, $U = 14.42$ m/s
Superimpose blades y/n ? y
Press return to start computation

Compressor Cascade - Freestream Velocity Distributions
(Inviscid Flow Analysis)

suction surface

pressure surface

U (m/s)

x (mm)