Solving the AI Planning + Scheduling Problem
Using Model Checking via Automatic Translation
From the Abstract Plan Preparation Language
(APPL) to the Symbolic Analysis Laboratory (SAL)

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## Contents

1 Introduction 2

2 Simple Example 2

3 Constraints Expressed Through Allen Operations 11
   3.1 Approach ................................................................. 11
   3.2 The contained_by Allen Operator: A contained_by B .......... 12
   3.3 The contains Allen Operation: A contains B ..................... 14
   3.4 The meets Allen Operator: A meets B ............................. 16
   3.5 The met_by Allen Operator: A met_by B ........................... 17
   3.6 The starts Allen Operator: A starts B ............................. 19
   3.7 The ends Allen Operator: A ends B ............................... 20
   3.8 The equals Allen Operator: A equals B ............................ 20
   3.9 The before Allen Operator: A before B ........................... 21
   3.10 The after Allen Operator: A after B ............................. 22
   3.11 The overlaps Allen Operator: A overlaps B ..................... 22

4 Transition Parameters 24

5 Constraint Parameters 25
   5.1 Example 1 – Parameter Matching .................................... 25
   5.2 Example 2 – Constant Parameters ................................... 27
   5.3 Example 3 – Constant Parameters ................................... 28
   5.4 Example 4 – Constant Parameters ................................... 29

6 Processing IF THEN ELSE expressions 30

7 Semantic Subtleties 33
   7.1 Non-scheduled Actions and Vacuous Solutions .................. 33
   7.2 Multiple Invocations with After Constraints ................... 34
   7.3 Terminal States ......................................................... 34
   7.4 Parameters ............................................................. 35
      7.4.1 Constant Parameters ............................................. 35
      7.4.2 Matching and Wild Card Parameters .......................... 36
   7.5 Loss of Allen Op Symmetry ........................................... 36

8 Observations and Conclusions 38
1 Introduction

The Abstract Plan Preparation Language (APPL) [2] is a planning language inspired by the New Domain Description Language (NDDL), a powerful planning language developed at NASA Ames [4] which has evolved from the Planning Domain Description Language (PDDL)[5]1. APPL is built around the idea that not all constraints are alike in the specification of an AI planning problem. In particular, the APPL language provides a special notation for the actions which must be mutually exclusive on a particular timeline. This feature enables a more compact specification of actions and states that is similar to the specification of state transition systems.

Because the syntax and semantics of APPL are simpler than those of NDDL, APPL is more suitable for formal verification, static analysis, and automated test generation. However, the language is much less expressive than NDDL.

This paper describes the algorithms and techniques used to translate APPL to SAL[3]. This translator was developed under the Spacecraft Autonomy for Vehicles and Habitats (SAVH) project, whose purpose is to develop verification & validation technology that can enable the application of AI planning to man-rated space applications. The long-term goal of our research is to develop techniques to verify the correctness of the domain knowledge that is typically expressed in planning languages. The key idea is that once the planning domain knowledge has been captured in a SAL model, many different properties of that domain can be analyzed using the SAL model checker. However, at this point in the SAVH program, a target domain has not been developed, so our efforts have centered on the development of basic technology. In particular, we have focused on the mechanisms for expressing and translating constraints, which are central to the NDDL language. To enable the assessment of the power and usefulness of our translators, one verification property is generated. This property states that no plan exists that satisfies all of the constraints. The model checker is then deployed on this property. If a plan does exist, then the model checker will produce a counterexample that is a viable plan that satisfies the constraints. We do not envision that this approach will be nearly as efficient or powerful as EUROPA 2. We do believe that this translation technology could be useful for establishing that key properties of the domain knowledge are true. We think that this approach may one day provide a powerful means for debugging the specifications of domain knowledge that are used in safety-critical planning systems.

2 Simple Example

We will begin our look at the technique for translating APPL to SAL with a very simple two timeline example2:

---

1PDDL is the planning language of the Extensible Universal Remote Operations Architecture 2 (EUROPA 2). EUROPA 2 is a component-based software library for constructing highly-tailored, domain-specific planners.

2We assume a prior knowledge of SAL syntax throughout this document.
PLAN ex1

TIMELINE A
ACTIONS
   A0: [2,\_]  
   A1  
   A2

TRANSITIONS
   A0 -> A1 -> A2

END A

TIMELINE B
ACTIONS
   B0: [2,\_]  
   B1: [1,10]

TRANSITIONS
   B0 -> B1

END B

INITIAL-STATE
   |-> A.A0  
   |-> B.B0

GOALS
   A.A2  
   B.B1

END ex1

Timeline A has three actions A0, A1, and A2 with only one possible sequence. The duration of action A0 is at least 2 time units, while the other two actions have no restrictions on their
duration. Similarly, timeline B has two actions B0 and B1, with only one possible sequence. B0 has a duration of at least 2 time units, while B1 takes between 1 and 10 units.

Corresponding to these actions, the following types are generated

A_actions: TYPE = {A0, A1, A2, A_null};

B_actions: TYPE = {B0, B1, B_null};

In addition to the declared actions, a null state is created for each of the timelines. There are two purposes for these extra states:

- They provide a means for the completion of an action when the action has no successor.
- They provide a convenient mechanism for recording when a goal state has been reached on each timeline. As shown below, a transition from a timeline’s goal state to the null state is generated by the translator.

The generated SAL model consists of three modules:

- Module A_m which corresponds to timeline A.
- Module B_m which corresponds to timeline B.
- Module Clock which advances time.

The structure of the module generated for timeline A is:

A_m : MODULE =
BEGIN
  INPUT
  GLOBAL
  LOCAL
  INITIALIZATION

  TRANSITION
  [
    A0_to_A1: %% A0 -> A1
    []
    A1_to_A2: %% A1 -> A2
    []
    A2_to_A_null: %% A2 -> A_null
  ]
Each of these sections is used in the translation:

- **INPUT** is used to select values for parameters of actions.
- **GLOBAL** is used to hold state values and their current parameter values.
- **LOCAL** is used to hold the start time of the currently scheduled action.
- **INITIALIZATION** is used to set initial values.
- **TRANSITION** specifies rules for transitioning from one action to another.

The three modules will be asynchronously composed. In SAL this is specified as follows:

```plaintext
System: MODULE = A_m [] B_m [] Clock;
```

The SAL tool links the variables named in the **GLOBAL** and **INPUT** sections together. In other words, variables with the same name are equated even though they are specified in different modules.

The SAL model checker is then used to search through all possible sequences of actions on the timelines to find sequences which satisfy all of the constraints specified in the APPL model. These constraints fall into two broad categories:

- Timing constraints that impact durations and start/stop times of actions.
- Allen operations [1] that specify relationships between time intervals of actions.

The search is started at time 0 and proceeds forward in time until the planning horizon is reached. The stop time is specified via a constant as follows:

```plaintext
MAX_TM: int = 150;           %% Always generated
TM_rng: TYPE = [0 .. MAX_TM]; %% Always generated
```

The progression of time is controlled by the `clock` module:

```plaintext
Clock: MODULE =
BEGIN
INPUT
    tm_intv: [1 .. 10]
OUTPUT
    time: TM_rng
INITIALIZATION
    time = 0;
TRANSITION
```
This module advances the clock by a value in the range of \(1 \ldots 10\). The clock is stored in the variable \(\text{time}\). Note that since the modules are asynchronously composed, the clock module can effectively advance time at any point. Both the \(\text{A}_m\) and the \(\text{B}_m\) modules include the variable \(\text{time}\) in their INPUT sections.

The GLOBAL sections of all of the timeline modules contain variables which record the action that is scheduled during the current time:

GLOBAL

\[
\begin{align*}
\text{B}\_\text{state} & : \text{B}\_\text{actions}, \\
\text{A}\_\text{state} & : \text{A}\_\text{actions}
\end{align*}
\]

The durations of the actions are controlled by the use of a \textit{start} variable declared as follows

LOCAL

\[
\begin{align*}
\text{start} & : \text{TM}\_\text{rng} \\
\text{INITIALIZATION} & \\
\text{start} & = 0;
\end{align*}
\]

Whenever an action is initiated, the initiation time is stored in this variable. The durations of an action will be controlled through this variable. This will be explained more carefully when we discuss the transitions section.

The initial actions on a timeline can be specified as follows:

INITIAL-STATE

\[
\begin{align*}
|\rightarrow & \text{A.A0} \\
|\rightarrow & \text{B.B0}
\end{align*}
\]

If a timeline’s initial state is not specified, then the model checker will explore all possible start states.

The APPL TRANSITIONS section is the major focus of the translation process. The SAL TRANSITIONS section is constructed from this part of the APPL model. For example, the following APPL code:

TRANSITIONS

\[
\begin{align*}
\text{A0} & \rightarrow \text{A1} \rightarrow \text{A2}
\end{align*}
\]
is translated into the SAL code:

```
TRANSITION
[
    A0_to_A1:     %% A0 -> A1
    A_state = A0
    AND time >= start + 2
    -->
        A_state' = A1;
        start' = time;
]
A1_to_A2:     %% A1 -> A2
    A_state = A1
    -->
        A_state' = A2;
        start' = time;
]
A2_to_A_null: %% A2 -> A_null
    A_state = A2
    -->
        A_state' = A_null;
]
```

When a transition occurs, an action is completed and another transition is initiated. No empty time slots are allowed. The TRANSITIONS section defines three transitions which are labeled as follows:

- A0_to_A1: %% A0 -> A1
- A1_to_A2: %% A1 -> A2
- A2_to_A_null: %% A2 -> A_null

The first transition is guarded by the following expression:

```
A_state = A0
AND time >= start + 2
```

The first conjunct insures that this transition only applies when the current action on the timeline is A0 and the second conjunct insures that the duration of the action is at least 2 time units. This corresponds to the fact that A0 was declared as A0: [2, _]. The expressions after the --> specify that the new state is A1 and that the start variable is re-initialized.

The APPL GOALS statement

---

3We have adopted a constructive semantics of allowed transitions: only the transitions that are explicitly specified are allowed. The EUROPA 2 system takes the opposite approach. All possible transitions are allowed unless specifically ruled out by a constraint. In future versions of the APPL to SAL translator, we may add an option that allows one to choose between constructive or destructive semantics.
GOALS
A.A2
B.B1

lists the two actions that need to be reached (where the default meaning of the expression
is the logical conjunction of the terms). This statement is translated into the following SAL
specification:

 sched_sys: THEOREM
         System |- AG(NOT(A_state = A_null AND B_state = B_null));

Since the “null” states can only be reached from the goal states (i.e. A2 and B1), these
efficiently record the fact that the appropriate goal has been reached on each timeline. Note
that the APPL goal statement has been negated. Therefore, when the model checker is
instructed to establish the property, any counterexample provided by SAL will serve as a
feasible realization of the plan. If a specific ordering of goal achievement is desired, we can
add a constraint using Allen operations in the CONSTRAINTS section to accomplish this. For
example, if goal A2 must be reached before B1, then the following constraint can be added:

         A.A2 WITH before B.B1

Allen operations will be discussed in greater detail in Section 3.

The complete generated SAL model (ex1.sal) is:

%%% File ex1.sal
%%% Generated from ex1.appl
%%% On Fri Oct 20 14:14:39 EDT 2006
%%% By APPL-2.b (08/14/06)

ex1: CONTEXT =
BEGIN

   INFNTY: int = 1000; % Always generated
   MAX_TM: int = 150; % Always generated
   TM_rng: TYPE = [0 .. MAX TM]; % Always generated

   A_actions: TYPE = {A0, A1, A2};

   B_actions: TYPE = {B0, B1};

   A_m : MODULE =
   BEGIN
      INPUT
      time: TM_rng
      GLOBAL

B_state: B_actions,
A_state: A_actions
LOCAL
    start: TM_rng
INITIALIZATION
    start = 0;
    A_state = A0;
TRANSITION
[
    A0_to_A1: \%
    A_state = A0
    AND time >= start + 2
    -->
    A_state' = A1;
    start' = time;
]

[
    A1_to_A2: \%
    A_state = A1
    -->
    A_state' = A2;
    start' = time;
]

[
    A2_to_A_null: \%
    A_state = A2
    -->
    A_state' = A_null;
]
]
END; \%

B_m : MODULE =
BEGIN
    INPUT
time: TM_rng
GLOBAL
    A_state: A_actions,
    B_state: B_actions
LOCAL
    start: TM_rng
INITIALIZATION
    start = 0;
B\_state = B0;
TRANSITION
[
    B0\_to\_B1: \text{%%} B0 \to B1
    B\_state = B0
    -->
        B\_state' = B1;
        start' = time;
]

B1\_to\_B\_null: \text{%%} B1 \to B\_null
B\_state = B1
AND time >= start + 1
AND time <= start + 10
--> 
    B\_state' = B\_null;
]
END; \text{%%} B\_m

Clock: MODULE =
BEGIN
INPUT
    tm\_intv: [1 .. 10]
OUTPUT
    time: TM\_rng
INITIALIZATION
    time = 0;
TRANSITION
[
    advance\_clock:
        time + tm\_intv <= MAX\_TM --> time' = time + tm\_intv;
]
END;

System: MODULE = A\_m
[
    B\_m
    [] Clock;
]
sched\_sys: THEOREM
    System |- AG(\text{NOT}(A\_state = A\_null

10
AND B_state = B_null
));

END %% ex1

3 Constraints Expressed Through Allen Operations

The allowed transitions on a timeline can be further restricted through use of Allen operations. The following Allen operations are allowed in the APPL language:

- contains
- contained_by
- meets
- met_by
- starts
- equals
- ends
- before
- after
- overlaps

In the generated SAL model, these constraints result in additional guards on the transitions and additional variable updates.

3.1 Approach

In this section, we will explain the translation approach without consideration of action parameters. In a later section, we will describe how the action parameters are handled. In the first pass of the translator, all of the Allen operations are collected in a data structure which stores five key fields:

- tml_A: Timeline of the first action.
- act_A: First action.
- allop: The Allen operation.
- tml_B: Timeline of the second action.
• act_B: Second action.

For example, the Allen operation

**Nav.X2 starts Instr.Y3**

results in `<tml_A> = Nav, act_A = X2, allop = starts, tml_B = Instr, act_B = Y3`. In the second pass of the translator, all of these constraints are matched against the transition that is being generated. We will use the following notation to represent the transition under construction:

```plaintext
<tml>: <current_action> -> <next_action>
```

The current timeline is `<tml>` and we are processing the transition from `<current_action>` to `<next_action>`. We will introduce some notation to indicate when there is a “match.” A constraint will be denoted as follows:

```plaintext
tml_A act_A <op> tml_B act_B
```

where `<op>` is one of the Allen ops (e.g. `meets`, `contains`, etc). We will express the matching requirement by drawing lines between items that must be equal:

```plaintext
<tml>: <current_action> ---> <next_action>

tml_A act_A <op> tml_B act_B
```

This diagram expresses the requirement that `<tml> = tml_A` and `<current_action> = act_A`.

The basic approach is as follows. For each transition being generated, the translator searches through all of the constraints looking for matches against patterns associated with each type of Allen operation. The patterns are used to specify:

- guards on the transition which occur on the left or **pre** side of the `-->`
- variable updates which occur on the right or **post** side of the `-->`

These “patterns” are described in the next subsections. We will not include the routine update of the `time` variables in the following sections, to simplify the presentation.

### 3.2 The contained_by Allen Operator: A contained_by B

There are two matching patterns required for the `contained_by` operation. Suppose we have a constraint

```plaintext
tml_A.act_A contained_by tml_B.act_B
```

which can be illustrated as follows:
While generating any transition of the form \(<current\_action> \rightarrow \text{act}_A\) for timeline \(\text{tml}_A\), a check needs to be made to insure that \(\text{act}_B\) is already executing on the \(\text{tml}_B\) timeline. Therefore, the following guard is added to the \(<current\_action> \rightarrow \text{act}_A\) transition:

\[
\text{tml}_B\_\text{state} = \text{act}_B
\]

The translator searches through the list of all of the constraints while processing a \(<current\_action> \rightarrow <next\_action>\) transition. If it finds a constraint with \(<\text{op}> = \text{contained\_by}, \text{tml}_A = \langle\text{tml}\rangle\) and \(\text{act}_A = <\text{next}\_action>\), then this guard is added and the net result is:

\[
\begin{align*}
\text{tml}_A\_\text{state} &= <\text{current}\_action> ; \\
\text{tml}_B\_\text{state} &= \text{act}_B \\
\rightarrow & \quad \text{tml}_A\_\text{state}' = \text{act}_A ; \\
\%\% &= <\text{next}\_action>
\end{align*}
\]

This particular match is illustrated below:

\[
\begin{array}{c}
\langle\text{tml}\rangle : \langle\text{current}\_action\rangle \rightarrow \langle\text{next}\_action\rangle \\
\text{tml}_A \quad \text{act}_A \quad \text{contained\_by} \quad \text{tml}_B \quad \text{act}_B
\end{array}
\]

It is also necessary to keep \(\text{act}_B\) from terminating while \(\text{act}_A\) is still executing. This is accomplished using another pattern:

\[
\begin{array}{c}
\langle\text{tml}\rangle : \langle\text{current}\_action\rangle \rightarrow \langle\text{next}\_action\rangle \\
\text{tml}_A \quad \text{act}_A \quad \text{contained\_by} \quad \text{tml}_B \quad \text{act}_B
\end{array}
\]

In other words for transitions of the form \(\text{tml}_B: \text{act}_B \rightarrow <\text{next}\_action>\) (i.e. where \(\langle\text{tml}\rangle = \text{tml}_B\) and \(\langle\text{current}\_action\rangle = \text{act}_B\)), we must add the following guard:

\[
\text{AND} \quad \text{tml}_A\_\text{state} /= \text{act}_A
\]

The net result is

\[
\begin{align*}
\text{tml}_B\_\text{state} &= <\text{current}\_action> ; \\
\text{AND} \quad \text{tml}_A\_\text{state} /= \text{act}_A \\
\rightarrow & \quad \text{tml}_B\_\text{state} = \text{act}_B \\
\%\% &= <\text{next}\_action> ;
\end{align*}
\]
3.3 The contains Allen Operation: A contains B

There are three matching patterns required for the contains operation:

<table>
<thead>
<tr>
<th>tml_A.act_A</th>
</tr>
</thead>
<tbody>
<tr>
<td>tml_B.act_B</td>
</tr>
</tbody>
</table>

The first pattern is

\[
\text{tml}_A \text{ act}_A \quad \text{contains} \quad \text{tml}_B \text{ act}_B
\]

Whenever there is a match, shown in this illustration, the following additional guard is added to the \( \text{<current_action> -> <next_action>} \) transition:

\[
\text{tml}_B \text{ state} \neq \text{act}_B
\]

The net result is:

\[
\begin{align*}
\text{tml}_A \text{ state} &= \text{act}_A; \\
\% &= \text{<current_action>}; \\
\text{AND} \text{ tml}_B \text{ state} &= \neq \text{act}_B \\
\rightarrow \quad \text{tml}_A \text{ state}' &= \text{<next_action>}; \\
\text{act}_B \text{ while } \text{act}_A &= \text{false};
\end{align*}
\]

The variable \( \text{act}_B \text{ while } \text{act}_A \) is reset to false in preparation for a future execution of the action. The initial value of this variable is set as:

\[
\text{act}_B \text{ while } \text{act}_A = (\text{tml}_A \text{ state} = \text{act}_A) \text{ AND } \\
(\text{tml}_B \text{ state} = \text{act}_B);
\]
The second pattern is

\[
\text{tml: } <\text{current\_action}> \longrightarrow <\text{next\_action}>
\]

\[
\text{tml A act A contains tml B act B}
\]

When a transition into state \text{act\_B} occurs, we need to update the value of the variable \text{act\_B\_while\_act\_A}. If action \text{act\_A} is executing on the other timeline, then the variable \text{act\_B\_while\_act\_A} is set to true as follows:

\[
\text{IF tml\_A\_state = act\_A THEN}
\text{act\_B\_while\_act\_A' = true;}
\text{ENDIF;}
\]

The net result is

\[
\text{tml\_B\_state = <current\_action>};
\]

\[
\text{tml\_B\_state' = act\_B } \quad % = <\text{next\_action}>;
\]

\[
\text{act\_B\_while\_act\_A' = IF tml\_A\_state = act\_A THEN}
\text{true}
\text{ELSE}
\text{act\_B\_while\_act\_A}
\text{ENDIF;}
\]

where the update syntax is rewritten in a form suitable for SAL.

The third pattern is

\[
\text{tml: } <\text{current\_action}> \longrightarrow <\text{next\_action}>
\]

\[
\text{tml A act A contains tml B act B}
\]

When a transition into state \text{act\_A} occurs, we need to update the value of the variable \text{B\_while\_A} if the time has not advanced. If action \text{act\_B} has just been initiated on timeline \text{tml\_B}, then the variable \text{B\_while\_A} is set to true as follows:

\[
\text{tml\_A\_state = <current\_action>};
\]

\[
\text{tml\_A\_state' = act\_A } \quad % = <\text{next\_action}>;
\]

\[
\text{act\_B\_while\_act\_A' = IF tml\_B\_state = act\_B AND B\_start = time THEN}
\text{true}
\text{ELSE}
\text{act\_B\_while\_act\_A}
\text{ENDIF;}
\]

It should be noted that this allows equal start times for actions \text{A} and \text{B}. If a strict “contains” is desired, one should leave out the code generated from this last pattern.
3.4 The meets Allen Operator: A meets B

There are two matching patterns required for the meets operation:

| tml_A.act_A | tml_B.act_B |

The first one occurs when tml_A.act_A terminates:

Whenever there is a match shown in this figure, the following variable update is added to the <current_action> -> <next_action> transition:

\[
\text{tml}_A \text{ state}' = \text{act}_A; \\
\text{tml}_B \text{ state}' = \text{act}_B;
\]

Thus, whenever act_A terminates, act_B is initiated on timeline tml_B. But, what if the currently executing action on timeline tml_B is not an allowed predecessor of act_B? To handle that case, we need an additional guard on the <current_action> -> <next_action> transition resulting in:

\[
\text{tml}_A \text{ state} = \text{act}_A; \\
( \text{tml}_B \text{ state} = \text{act}_PB1 \text{ OR} \\
\text{tml}_B \text{ state} = \text{act}_PB2 \text{ OR} \\
\vdots \\
\text{tml}_B \text{ state} = \text{act}_PBn ) \\
\rightarrow \\
\text{tml}_A \text{ state}' = \text{next}_action; \\
\text{tml}_B \text{ state}' = \text{act}_B;
\]

where act_PB1, act_PB2, ..., act_PBn are all the allowed predecessor actions of act_B. It is also necessary to check that the duration constraints on all of these predecessor actions are met. These necessitate the use of global start variables: A_start and B_start. If there were two predecessor actions to act_B, say B0 and B1, we would end up with a guard similar to the following:
\begin{align*}
( & ( \text{B\_state} = \text{B0} \\
& \text{AND time} \geq \text{B\_start} + 0 \\
& \text{AND time} \leq \text{B\_start} + 10 \\
& ) \text{ OR } ( \text{B\_state} = \text{B1} \\
& \text{AND time} \geq \text{B\_start} + 5 \\
& ) )
\end{align*}

It is also necessary to make sure that any constraints on the initiated action on the timeline are satisfied. In the above example it is necessary to examine the \text{B0} \rightarrow \text{act\_B} and \text{B1} \rightarrow \text{act\_B} against all of the Allen operators in the database. This is accomplished by a recursive call to the procedure we are describing in this section.

The semantics of the constraint can be strengthened by matching a second pattern:

\[
\begin{array}{c}
\text{<tml>: \<current\_action> \rightarrow \<next\_action>}
\end{array}
\]

\[
\text{tml\_A \ act\_A \ meets \ tml\_B \ act\_B}
\]

When processing the transitions into the \text{act\_B} action on the \text{tml\_B} timeline, we update the \text{tml\_A} timeline as well:

\[
\begin{align*}
\text{tml\_B\_state} &= \<current\_action> \\
\text{AND tml\_A\_state} &= \text{act\_A}; \\
\rightarrow \text{tml\_B\_state}' &= \text{act\_B} \\
\text{tml\_A\_state}' \in \text{act\_NA\_i, \ldots, act\_NA\_n};
\end{align*}
\]

where \text{act\_NA\_i, \ldots, act\_NA\_n} are the successor actions of \text{act\_A}. And, of course, we must check that the durations and constraints are satisfied for this transition as before.

### 3.5 The met\_by Allen Operator: A met\_by B

There are two matching patterns needed for the \text{met\_by} operation:

\[
\begin{array}{c|c}
\text{tml\_A\_act\_B} & \text{tml\_B\_act\_A} \\
\end{array}
\]

Whenever the translator finds the following match:

\[
\begin{array}{c}
\text{<tml>: \<current\_action> \rightarrow \<next\_action>}
\end{array}
\]

\[
\begin{align*}
\text{tml\_A \ act\_A \ met\_by \ tml\_B \ act\_B}
\end{align*}
\]
the following variable update is added to the \texttt{<current\_action> -> <next\_action>} transition:

\begin{verbatim}
tml\_A\_state' = act\_A;
\end{verbatim}

In other words, when \texttt{act\_B} is terminated on timeline \texttt{tml\_B}, action \texttt{act\_A} is started on the other timeline. This of course, necessitates a check that a predecessor action of \texttt{act\_A} is currently executing. The net result is:

\begin{verbatim}
tml\_B\_state = act\_B
( tml\_A\_state = act\_PA1 OR
  tml\_A\_state = act\_PA2 OR
   ...
  tml\_A\_state = act\_PAn )
--> 
  tml\_B\_state' = \texttt{<next\_action>}
  tml\_A\_state' = act\_A;
\end{verbatim}

where \texttt{act\_PA1}, \texttt{act\_PA2}, ..., \texttt{act\_PAn} are the predecessor actions of \texttt{act\_A}.

Also it is necessary to match transitions on the other timeline as well:

\begin{verbatim}
<tml>: \texttt{<current\_action> --\rightarrow <next\_action>}
\end{verbatim}

\begin{verbatim}
  tml\_A   act\_A   met\_by   tml\_B   act\_B
\end{verbatim}

In other words, we can allow entrance into state \texttt{act\_A}, if action \texttt{act\_B} is currently executing. Therefore, the following guard must be added:

\begin{verbatim}
tml\_B\_state = act\_B
\end{verbatim}

Since \texttt{act\_A} is met by \texttt{act\_B}, it is necessary that \texttt{act\_B} be terminated with an update to \texttt{tml\_B\_state}:

\begin{verbatim}
tml\_B\_state' IN \{act\_NB1, ..., act\_NBn\};
\end{verbatim}

where \texttt{NB\_i, ..., NB\_n} are the successor actions of \texttt{act\_B}. The net result is

\begin{verbatim}
tml\_A\_state = act\_A; \quad \% = \texttt{<current\_action>};
tml\_B\_state = act\_B
--> 
  tml\_A\_state' = \texttt{<next\_action> ;}
  tml\_B\_state' IN \{act\_NB1, act\_NB2, ... act\_NBn\};
\end{verbatim}

So whenever \texttt{act\_A} is terminated, \texttt{act\_B} is initiated. And of course we must check that the durations and constraints are satisfied for this transition as in the \texttt{meets} operator.
3.6 The starts Allen Operator: A starts B

There is just one matching pattern needed for the starts operation:

<table>
<thead>
<tr>
<th>tml_A.act_A</th>
</tr>
</thead>
<tbody>
<tr>
<td>tml_B.act_B</td>
</tr>
</tbody>
</table>

Whenever there is a match

\[
\text{tml}: \quad \text{<current_action>} \quad \rightarrow \quad \text{<next_action>}
\]

\[
\text{tml } A \quad \text{act } A \quad \text{starts} \quad \text{tml } B \quad \text{act } B
\]

the following variable update is added to the \( \text{<current_action>} \rightarrow \text{<next_action>} \) transition:

\[
\text{tml}_B \_\text{state}' = \text{act}_B;
\]

The net result is

\[
\begin{align*}
\text{tml}_A \_\text{state} &= \text{<current_action>}; \\
( \text{tml}_B \_\text{state} &= \text{act}_PB1 \text{ OR} \\
& \quad \text{tml}_B \_\text{state} = \text{act}_PB2 \text{ OR} \\
& \quad \quad \vdots \\
& \quad \quad \text{tml}_B \_\text{state} = \text{act}_PBn \\
\) \\
\rightarrow & \\
\text{tml}_A \_\text{state}' &= \text{act}_A; \quad \text{tml}_B \_\text{state}' = \text{act}_B;
\end{align*}
\]

where \( \text{act}_PB1, \text{act}_PB2, \ldots \text{act}_PBn \) are all of the allowed predecessor actions of \( \text{act}_B \).

So, whenever \( \text{act}_A \) is initiated, \( \text{act}_B \) is also initiated, and this transition is guarded with the requirement that a predecessor action of \( \text{act}_B \) is executing.

There is a semantics issue that arises here: whether or not \( \text{act}_B \) can be scheduled without \( \text{act}_A \) being started. For example, does the following timeline trace

\[
\begin{array}{c|c|c|}
\hline
\text{A} & \text{A1} | \text{A3} | \\
\hline
\text{B} & \text{B1} | \text{B2} | \\
\hline
\end{array}
\]

satisfy the constraints
A1 -> (A2 | A3 )

B1 -> B2

A.A2 starts B.B2

In this trace, A2 does not execute. Since the semantics of A op B is FORALL A EXISTS B, the above trace represents a vacuous solution and hence does meet the constraint.

3.7 The ends Allen Operator: A ends B

There is just one matching pattern needed for the ends operation:

```
  | tml_A.act_A
  | | tml_B.act_B
```

Whenever there is a match
```
<tml>:   <current_action>    --->   <next_action>
    tml A     act A   ends    tml B     act B
```

action act_B has to be terminated along with action act_A. The net result is:
```
tml_A_state = act_A;  %\% = <current_action>
tml_B_state = act_B

-->
tml_A_state' = <next_action>
    tml_B_state' IN act_NB1, act_NB2, ..., act_NNm;
```

where act_NB1, act_NB2, ..., act_NNm are all of the allowed successor actions of act_B. So, whenever act_A is terminated act_B is also terminated and a successor action is initiated. If there is no successor state for act_B, then the guard is falsified with the addition of

```
AND FALSE
```

If the guard is falsified, then act_A is never allowed to terminate.

3.8 The equals Allen Operator: A equals B

The equals operator

```
  | tml_A.act_A
  | | tml_B.act_B
```

is implemented as the combination of starts and ends. In other words, the Allen operation A equals B is internally translated into A starts B and A ends B.
3.9 The before Allen Operator: A before B

The remaining three operators – before, after, and overlaps – require that additional variables be introduced in the SAL model in order to enforce the desired behavior. For example, to ensure that action act_1 occurs before act_2, a variable must be used to record the fact that act_1 has occurred some time in the past. One extra variable is needed for each before constraint, because the variable that enforces act_1 before act_2 does not help in establishing act_3 before act_4.

The model generator has a mechanism for creating unique variable names, so that no name collision is possible when dealing with multiple constraints. The operator names and the actions’ names and parameters that are involved are part of this identifier.

There are two matching patterns required for the before operation:

- \( tml_A.act_A \) before \( tml_B.act_B \)
- \( tml_B.act_B \) before \( tml_A.act_A \)

For example, the before constraint

\[ tml_A.act_A \text{ before } tml_B.act_B \]

a Boolean variable \( tml_A.act_A \text{ before } tml_B.act_B \) is declared and initialized as FALSE. Then, whenever there is a match:

\[ \text{<tml>: } \text{<current_action> } \longrightarrow \text{ <next_action>} \]

\[ tml_A \quad \text{act A before } \quad tml_B \quad \text{act B} \]

the fact that \( tml_A.act_A \) has occurred is recorded:

\[ tml_A.state = \text{act A}; \quad \text{%% } = \text{<current_action>} \]

\[ \longrightarrow \]

\[ tml_A.state' = \text{<next_action>}; \]

\[ tml_A.act_A \text{ before } tml_B.act_B' = \text{true} \]

Also, action \( \text{act B} \) cannot be executed unless action \( \text{act A} \) has executed in the past. So whenever there is a match:

\[ \text{<tml>: } \text{<current_action> } \longrightarrow \text{ <next_action>} \]

\[ tml_A \quad \text{act A before } \quad tml_B \quad \text{act B} \]

the following code is generated:

\[ tml_B.state = \text{<current_action>} \]

\[ \text{AND } tml_A.act_A \text{ before } tml_B.act_B \]

\[ \longrightarrow \]

\[ tml_B.state' = \text{act B}; \]
Another semantics issue is raised here. The mechanism for recording the occurrence of \texttt{act\textunderscore A} can be modified so that it is matched repeatedly (by resetting the history variable) or just once (without resets). These two interpretations correspond to two distinct views of the Allen operator semantics. There is no clear indication as to which approach is to be preferred. The current implementation accepts a timeline where action \texttt{act\textunderscore B} never occurs. This is a weaker constraint than requiring that for each \texttt{act\textunderscore A}, there should necessarily be a matching occurrence of \texttt{act\textunderscore B}. The current implementation is also strict. In other words, if \texttt{A meets B}, then \texttt{A before B} is not true.

3.10 The after Allen Operator: A after B

The after operator

\[
\text{tml\textunderscore A.act\textunderscore A} \quad \text{tml\textunderscore B.act\textunderscore B}
\]

is the dual of the before operator, i.e. the code generated for the constraint \texttt{act\textunderscore A before act\textunderscore B} is identical to that for \texttt{act\textunderscore B after act\textunderscore A}. Hence, the code generation is very similar to the above: an additional history variable is needed to record the occurrence of \texttt{act\textunderscore A}, when leaving the state \texttt{act\textunderscore A}, this fact is recorded in the history variable. Any transition into \texttt{act\textunderscore B} is then guarded by the condition that the history variable is \texttt{true}.

3.11 The overlaps Allen Operator: A overlaps B

The constraint \texttt{act\textunderscore A overlaps act\textunderscore B}

\[
\begin{array}{c}
\text{tml\textunderscore A.act\textunderscore A} \\
\text{tml\textunderscore B.act\textunderscore B}
\end{array}
\quad \text{or} \quad
\begin{array}{c}
\text{tml\textunderscore A.act\textunderscore A} \\
\text{tml\textunderscore B.act\textunderscore B}
\end{array}
\]

can be satisfied by two scenarios:

- \texttt{act\textunderscore A} occurs first, then the system needs to traverse the following sequence of states and no other: \texttt{act\textunderscore B} starts, \texttt{act\textunderscore A} ends, and afterwards \texttt{act\textunderscore B} ends.

- \texttt{act\textunderscore B} occurs first, then the only allowed sequence is: \texttt{act\textunderscore A} starts, \texttt{act\textunderscore B} ends, and then \texttt{act\textunderscore A} ends.

Monitoring the correct succession of states can be enforced by using one additional variable that records either one of the above initial events and then the subsequent intermediate states that are traversed. Alternatively, we can use two Boolean variables to record which one of the initial conditions has occurred: has \texttt{act\textunderscore A} happened first or \texttt{act\textunderscore B}? There are four separate match patterns that generate code for an overlaps constraint. More precisely we can be either entering or leaving either \texttt{act\textunderscore A} or \texttt{act\textunderscore B}. The first pattern is
The code generated for the first pattern is:

\[
\begin{align*}
\text{tml}_{A}\text{state} & = \text{act}_{A} \quad \text{%} = \text{<current_action>} \\
\text{AND act}_{A}\text{first_overlaps}\text{act}_{B} \\
\rightarrow \\
\text{tml}_{A}\text{state}' & = \text{<next_action>};
\end{align*}
\]

The second pattern is:

\[
\begin{align*}
\text{tml}_{A}\text{state} & = \text{<current_action>} \\
\rightarrow \\
\text{tml}_{A}\text{state}' & = \text{act}_{A} \quad \text{%} = \text{<next_action>} \\
\text{act}_{A}\text{first_overlaps}\text{act}_{B}' & = \text{NOT act}_{B}\text{first_overlaps}\text{act}_{A}
\end{align*}
\]

The third pattern is

\[
\begin{align*}
\text{tml}_{B}\text{state} & = \text{<current_action>} \\
\rightarrow \\
\text{tml}_{B}\text{state}' & = \text{act}_{B}; \quad \text{%} = \text{<next_action>} \\
\text{act}_{B}\text{first_overlaps}\text{act}_{A}' & = \text{NOT act}_{A}\text{first_overlaps}\text{act}_{B}
\end{align*}
\]

The fourth pattern is

\[
\begin{align*}
\text{tml}_{A}\text{state} & = \text{<current_action>} \\
\rightarrow \\
\text{tml}_{A}\text{state}' & = \text{act}_{A} \quad \text{%} = \text{<next_action>}
\end{align*}
\]
The code generated for the fourth pattern is:

\[
\begin{align*}
\text{tml\_B\_state} &= \text{act\_B} \\
&\quad \text{AND } \text{act\_B\_first\_overlaps\_act\_A} \\
\rightarrow \quad &\text{tml\_B\_state'} = \langle\text{next\_action}\rangle;
\end{align*}
\]

## 4 Transition Parameters

The processing of action parameters results in additional guards and variable updates. Consider the following example transition

\[\text{A1}(xx,yy,_,ww) \rightarrow \text{A2}(_,yy,ww)\]

where the matching of parameter names indicates a constraint on the transition. For each matching argument, it is necessary to generate a variable to hold the value of that matched parameter. The translator creates a variable name by concatenating the parameter identifier to the action name. For example, if the A1 and A2 actions were defined as follows:

\[
\begin{align*}
\text{A1}(p1: \text{type1}, p2: \text{type2}, p3: \text{type3}, p4: \text{type4}) \\
\text{A2}(p1: \text{type1}, p2: \text{type2}, px: \text{type4})
\end{align*}
\]

the translator will generate the following GLOBAL variables:

\[
\begin{align*}
\text{A1\_p2: type2}; \\
\text{A1\_p4: type4}; \\
\text{A2\_p2: type2}; \\
\text{A2\_px: type4};
\end{align*}
\]

Note that variables are only created where needed, namely where there is a match. The transition statement equates these parameters as follows:

\[
\begin{align*}
\text{A1\_p2} &= \text{A2\_p2} \\
\text{A1\_p4} &= \text{A2\_px}
\end{align*}
\]

These parameter matchings can be generated on the pre side, such as

\[
\begin{align*}
\text{A1\_p2} &= \text{A2\_p2'} \text{ AND } \text{A1\_p4} = \text{A2\_px'}
\end{align*}
\]

or the post side

\[
\begin{align*}
\text{A2\_p2'} &= \text{A1\_p2}; \\
\text{A2\_px'} &= \text{A1\_p4};
\end{align*}
\]
The primes indicate that these are the values of the variables after the update. The advantage of the \textbf{post} form is that it directly selects the new values for the parameters after the transition. However, in the presence of multiple constraints it is possible to have multiple updates on the same parameter:

\[
\begin{align*}
A_1'_{p2} &= A_2_{p2}; \\
A_1_{p2}' &= B_3_{p2};
\end{align*}
\]

This construction is not allowed in SAL, while the \textbf{pre} version is allowed:

\[
A_1_{p2}' = A_2_{p2} \text{ AND } A_1_{p2}' = B_3_{p2}
\]

Although it is necessary to actually update the variables, the \textbf{pre} form is deployed as much as possible to avoid illegal updates.

5 Constraint Parameters

The impact of parameters on the Allen operation constraints is very complicated. In fact, many subtle semantics issues arise here. These will not be dealt with in detail in this paper. Instead, we will present a few examples of how parameter processing is done, without attempting to be complete.

For convenience, we will restrict our attention to two timelines A and B with actions $A_1, A_2, \ldots$ and $B_1, B_2, \ldots$ For simplicity, we will not address the processing of the action durations in the following subsections. Also for simplicity we will assume that all actions have the following parameter identifiers: $x: \text{animals}, y: \text{animals}, z: \text{animals}$. For example:

\[
\begin{align*}
A_1(x: \text{animals}, y: \text{animals}, z: \text{animals}) \\
A_2(x: \text{animals}, y: \text{animals}) \\
B_4(x: \text{animals}, y: \text{animals})
\end{align*}
\]

5.1 Example 1 – Parameter Matching

Suppose we have the following transitions with matching parameters:

\[
\begin{align*}
A_1(xx) & \rightarrow A_2(_,xx) \\
B_1(aa,_) & \rightarrow B_2(_,aa) \\
A.A_1(aaa) & \text{ WITH meets } B.B_2(fish,aaa)
\end{align*}
\]

When processing the $A_1(xx) \rightarrow A_2(_,xx)$ transition, we must first handle the matching parameter as follows:

\[
\begin{align*}
\text{A_state} &= A_1 \\
& \rightarrow \\
\text{A_state}' &= A_2; \\
A_2_y' &= A_1_x;
\end{align*}
\]
Next, we need to make sure that the `meets` constraint is satisfied. Therefore, the B2 action must be initiated as well.

\[
\begin{align*}
A\_state &= A1 \\
&\text{AND } B\_state = B1 \\
\rightarrow &\quad A\_state' = A2; \\
&\quad B\_state' = B2; \\
&\quad A2\_y' = A1\_x;
\end{align*}
\]

Notice the presence of the guard `AND B\_state = B1`, which insures that the appropriate predecessor state of B2 is executing. Next, we need to make sure that the parameter matchings of the `meets` constraint are satisfied:

\[
\begin{align*}
A\_state &= A1 \\
&\text{AND } B\_state = B1 \\
&\quad \text{AND } B2\_x' = \text{fish} \\
&\quad \text{AND } B2\_y' = A1\_x \\
\rightarrow &\quad A\_state' = A2; \\
&\quad B\_state' = B2; \\
&\quad A2\_y' = A1\_x;
\end{align*}
\]

These guards are indented for easy recognition. Finally, we must make sure that the `B1(aa,\_ ) -> B2(_,aa)` transition rules are observed:

\[
\begin{align*}
A\_state &= A1 \\
&\text{AND } B\_state = B1 \\
&\quad \text{AND } B2\_x' = \text{fish} \\
&\quad \text{AND } B2\_y' = A1\_x \\
\rightarrow &\quad A\_state' = A2; \\
&\quad B\_state' = B2; \\
&\quad B2\_y' = B1\_x; \\
&\quad A2\_y' = A1\_x;
\end{align*}
\]

The `A\_A1(aaa) WITH meets B\_B2(fish,aaa)` constraint must also be considered when processing the transition from B1 -> B2. Clearly, the system should be able to make this transition when the action on the A timeline is A1. But what if the current action on the A timeline is not A1? Although we can envision solutions where B1 proceeds to B2 and the A timeline remains in state A1 awaiting a future B2 to meet, why not take advantage of this situation and transition A1 to A2 along with the B1 to B2 transition (and thus satisfy this Allen op constraint immediately)?

\[
\text{B1_to_B2: } \quad \% \% \ B1 \rightarrow B2
\]
\[
\begin{align*}
\text{B\_state} &= \text{B1} \\
\text{AND (A\_state} &= \text{A1} \Rightarrow (\text{TRUE} \\
&\quad \text{AND A1}_x' = \text{A2}_y' \\
&\quad \text{AND B2}_x' = \text{fish} \\
&\quad \text{AND B2}_y' = \text{A1}_x) \\

\Rightarrow
\end{align*}
\]

\[
\begin{align*}
\text{B\_state}' &= \text{B2}; \\
\text{IF A\_state} &= \text{A1} \text{ THEN} \\
\text{A2}_y' &= \text{A1}_x; \\
\text{A\_state}' &= \text{A2}; \\
\text{ENDIF} \\
\text{B2}_y' &= \text{B1}_x; \\
\text{B2}_x' &= \{x: \text{animals} | \text{true}\}; \\
\text{A2}_x' &= \{x: \text{animals} | \text{true}\};
\end{align*}
\]

Notice that the guard contains the following conditional (\(A\_state = A1 \Rightarrow \ldots\)). If this conditional is true, then the constraints associated with the transition from \(A1\) to \(A2\) are enforced by some further equalities after the conditional and with some variable updates inside an \(\text{IF-THEN-ELSE}\). Note that if the Allen operation contains some constant parameters in the first action (e.g. \(A.A1(\text{fish}) \text{ WITH meets B.B2}\), this is handled by adding a term to the conditional as follows (\(A\_state = A1 \text{ AND A1}_x = \text{fish}\) => \(\ldots\).

5.2 Example 2 – Constant Parameters

\[
\begin{align*}
A1(xx,_,_) &\rightarrow A2(_,xx) \rightarrow A3 \\
B0 &\rightarrow B1 \rightarrow B2(dog,_,_) \\
A.A2 &\text{ starts B.B2(_,cat,\text{donkey})}
\end{align*}
\]

when processing \(A1 \rightarrow A2\), the following will be generated:

\[
\begin{align*}
\text{A\_state} &= \text{A1} \\
\text{AND B\_state} &= \text{B1} \\
\text{AND B2}_y' &= \text{cat AND B2}_z' = \text{donkey} \\
\Rightarrow
\end{align*}
\]

\[
\begin{align*}
\text{A\_state}' &= \text{A2}; \\
\text{B\_state}' &= \text{B2}; \\
\text{B2}_x' &= \text{dog}; \\
\text{B2}_y' &\in \{x: \text{animals} | \text{true}\} \\
\text{B2}_z' &\in \{x: \text{animals} | \text{true}\} \\
\text{A2}_y' &= \text{A1}_x; \\
\text{A2}_x' &\in \{x: \text{animals} | \text{true}\}
\end{align*}
\]
The guard \( \text{AND } B2_y' = \text{cat} \text{ AND } B2_z' = \text{donkey} \) insures that the constant parameters of \( \text{B.B2} \) in the Allen operation are properly set. The variable update \( B2_x' = \text{dog} \) insures that the B timeline transition constraints are met. The last two variable updates insure that the A timeline constraints are met. Note that the \( A2_x' \text{ IN } \{ x: \text{animals} \mid \text{true} \} \) update occurs because of the "_" in the first parameter position of the \( A2 \) action. Also note that although the \( B2_y \) and \( B2_z \) parameters could be updated as follows:

\[
\begin{align*}
B2_y' &= \text{cat} \\
B2_z' &= \text{donkey}
\end{align*}
\]

If the following additional constraint were present

\[ B1(\text{mm}) \rightarrow B2(\text{dog,mm,_}) \]

we would obtain two different updates of \( B2_y \):

\[
\begin{align*}
B2_y' &= \text{cat} \\
B2_y' &= B1_x
\end{align*}
\]

which is not allowed in SAL.

### 5.3 Example 3 – Constant Parameters

\[ A1(\text{mm}) \rightarrow A2(_,\text{mm}) \rightarrow A3 \]

\[ B0 \rightarrow B1 \rightarrow B2 \rightarrow B3 \]

\[ A.A2(\text{dog,ww}) \text{ WITH contains B.B2(ww,_)} \]

When processing \( A2 \rightarrow A3 \), the following is generated:

\[
\begin{align*}
\text{A_state} &= \text{A2} \\
\text{AND} \ (A2_x &= \text{dog} \\
=> \ (\text{NOT } (B\_\text{state} = B2 \\
\text{AND} \ B2_x = A2_y)) \\
\text{AND} \ B2\_\text{while}_A2 \\
\rightarrow & \\
\text{A_state'} &= \text{A3};
\end{align*}
\]

The first thing to note is that the guard has an implication (=>). Thus, the requirement that \( A2 \) contains \( B2 \) is only enforced when the first parameter of \( A2 \) is \text{dog}. Therefore if an \( A2 \) is executing with a first parameter that is not equal to \text{dog}, there is no constraint that it must contain \( B2 \). If the first parameter of \( A2 \) is equal to \text{dog}, then the transition is not allowed to occur as long as \( B\_\text{state} = B2 \text{ AND B2_x = A2_y} \) is true.

If the A timeline transitions were modified to:
A1(mm) -> A2(dog,mm) -> A3
then the following would be generated

    A_state = A2
    AND A2.x = dog
    AND (A2.x = dog
         => (NOT (B_state = B2
                    AND B2.x = A2.y)
             )
         )
    )
    AND B2_while_A2
    -->
    A_state' = A3;

This is logically equivalent to

    A_state = A2 AND A2.x = dog
    AND (NOT (B_state = B2 AND B2.x = A2.y)
    AND B2_while_A2
    -->
    A_state' = A3;

but the translator does not perform this optimization.

The B1 -> B2 transition is

    B_state = B1
    -->
    B_state' = B2;
    IF A_state = A2 AND A2.x = dog AND B2.x' = A2.y THEN
      B2_while_A2' = true;
    ENDIF;
    B2.x' IN {x: animals | true};

Another issue that arises is whether the contains operation should be strict containment or whether equal starting or end times should be allowed. The mechanisms described previously for contains, have allowed equal start or end times.

5.4 Example 4 – Constant Parameters

A0(xx) -> A1(xx,_) -> A2(_,xx) -> A1(_,xx) -> A2(_,xx) -> A1(_,xx) -> A3
B0 -> B1 -> B0
B1 -> B2 -> B3
A.A2(dog,qq) meets B.B2(qq)
When generating a transition from A2 to A1 we get:

```
A_state = A2
AND (A2_x = dog => (B_state = B1
    AND B2_x' = A2_y))

-->  
A_state' = A1;
A1_y' = A2_y;
A1_x' IN {x: animals | true}
IF A2_x = dog THEN
    B_state' = B2;
    B_start' = time;
ENDIF;
```

This IF THEN ENDIF construct is not supported in SAL. Therefore, it is translated into the following in a third pass of the translator.

```
B_state' = IF A2_x = dog THEN B2 ELSE B_state ENDIF;
```

The IF THEN ENDIF version is available in an auxiliary *.sl1 output file. This file is often easier on human eyes than the final *.sal file.

When generating a transition from B1 to B2 we get:

```
B_state = B1
AND ((A_state = A2 AND A2_x = dog) =>
      ( ( A_state' = A1 => A2_y' = A1_y' )
         AND ( A_state' = A3 => A2_x = dog )
         AND B2_x' = A2_y
      )
  )

-->  
B_state' = B2;
IF A_state = A2 AND A2_x = dog THEN
    IF A_state' = A1 THEN
        A1_y' = A2_y;
    ENDIF
    A_state' IN {A1, A3};
ENDIF
B2_x' IN {x: animals | true};
```

### 6 Processing IF THEN ELSE expressions

The specification of an action allows a WITH clause with an “if then else” expression. In each branch, there can be different constraints. But let’s start with a simple case:
A2(x,y: animals):
  WITH
  if x = gorilla then
    starts B.B2(fish,x,y)
  else
    starts B.B2(cat,horse,fish)
  endif

Now suppose the transitions are constrained as follows:

A1(xx,_,_) -> A2(xx,_) -> A1 -> A3
B0 -> B1 -> B2

Then the following is generated for the A1 to A2 transition

A_state = A1
AND (A2_x = gorilla
    => (B_state = B1 AND B2_x' = fish
        AND B2_y' = A2_x'
        AND B2_z' = A2_y')
    )
AND (NOT A2_x = gorilla
    => (B_state = B1
        AND B2_x' = cat AND B2_y' = horse AND B2_z' = fish)
-->
    A_state' = A2;
start' = time;
B_state' = B2
A2_x' = A1_x;
A2_y' IN {x: animals | true}

Nested “if then else” expressions are a bit more tricky to handle. Consider the following example:

A1(x,y: animals)
  WITH
  if x = cat then
    contains B.B1(_,dog)
  else
    if y = dog then
      contains B.B2(cat,_,fish)
    else
      meets B.B2(_,horse,cat)
    endif
A2(x, y: animals)

with the following transitions

A0 -> A1(dog, y) -> A2(_, y)

B0 -> B1(aa, _) -> B2(dog, aa, aa)

The transition from A1 to A2 is generated (omitting timing) as follows:

```plaintext
A_state = A1
AND A1_x = dog
AND (A1_x = cat
    => (NOT (B_state = B1 AND B1_y = dog )))
AND (A1_y = dog AND NOT A1_x = cat
    => (NOT (B_state = B2 AND B2_x = cat AND B2_z = fish )))
AND (NOT A1_y = dog AND NOT A1_x = cat
    => (B_state = B1 AND
        (A1_y = dog AND NOT A1_x = cat
            => (A_state = A1 AND B2_x' = cat AND B2_z' = fish))
        AND B1_x = B2_y'
        AND B1_x = B2_z'
        AND B2_y' = horse AND B2_z' = cat
    ))

-->
A_state' = A2;
A_start' = time;
IF NOT A1_y = dog AND NOT A1_x = cat THEN
    B_state' = B2;
    B2_x' = dog;
    B2_y' = B1_x;
    B2_z' = B1_x;
ENDIF;
A2_y' = A1_y;
```
7 Semantic Subtleties

In the development of the translator, several subtle issues associated with the exact meaning of the Allen operations were discovered. In this section, we will explore some of these.

7.1 Non-scheduled Actions and Vacuous Solutions

Suppose we have two timelines A and B with the following actions and allowed sequences:

A0 -> (A1 | A2)
A1 -> A2
B0 -> B1 -> B2
B0 -> B2

The two transitions from B0 represent alternatives and are equivalent to B0 -> (B1 | B2) in APPL. Now consider the following Allen operation:

A1 WITH ends B1

The following time sequences clearly satisfy the ends constraint:

--------------------------------------------------
A0 | A1 | A2 |
--------------------------------------------------
B0 | B1 | B2 |
--------------------------------------------------

Now suppose that A1 never executes. Is the Allen operation satisfied in this case? Based upon preliminary experimentation, it appears that the EUROPA 2 planner allows such “vacuous” solutions.

Now suppose that B1 is not executing at the time A1 is executing as in the following timeline sequence:

--------------------------------------------------
A1 | A2 |
--------------------------------------------------
B0 | B2 |
--------------------------------------------------

Does this satisfy the constraint? Based upon preliminary experimentation, the EUROPA 2 planner requires that the B1 action be explicitly scheduled and must end with A1. In fact, if there are multiple A1s, each must end its own B1.

Therefore, given an Allen operation A op B, we can express the meaning as follows:

FORALL A: EXISTS B: A op B
7.2 Multiple Invocations with After Constraints

Consider the following Allen Operation

\[ A_1 \text{ before } B_2 \]

Does the following timeline meet this condition?

\[
\begin{array}{c|c|c|c|c}
| & A_1 & A_2 & & \\
| B_1 & B_2 & B_3 & B_2 & \\
\end{array}
\]

or must there be an \( A_1 \) before every \( B_2 \)?

\[
\begin{array}{c|c|c|c|c|c|c}
| & A_1 & A_2 & A_1 & A_2 & & \\
| B_1 & B_2 & B_3 & B_2 & & \\
\end{array}
\]

Based upon our experimentation with EUROPA 2, the first timeline appears to be adequate. This is consistent with the FORALL-EXISTS semantics given above.

7.3 Terminal States

Suppose we have the following Allen operation:

\[ A_1 \text{ meets } B_2 \]

If \( A_1 \) never transitions to \( A_2 \) (i.e. it is still active at HorizonEnd), is the following an acceptable solution?

\[
\begin{array}{c|c|c|c|c|c|c}
| & A_1(horse) & & & & B_1(horse,\_)& B_2(dog,horse) & \\
| Idle & B_1(horse,\_)& & B_2(dog,horse) & & \\
\end{array}
\]

EUROPA 2 accepts this as a valid plan. It should be noted that this is a violation of the FORALL-EXISTS semantics. Therefore, the user of the Allen operators needs to remember that the FORALL-EXISTS semantics does not apply at the beginning and end of the timelines.
7.4 Parameters

The presence of parameters significantly complicates the semantics of the Allen operators. There are three categories to consider:

- constant parameters, e.g. \( A1(\text{horse}) \)
- matching parameters, e.g. \( A1(\text{mmm}) \) meets \( B4(\text{mmm}) \)
- wild card parameters, e.g. \( A1(_) \)

and combinations of these.

7.4.1 Constant Parameters

Suppose we have the following Allen operation:

\[ A1(\text{cat}) \text{ WITH ends } B1 \]

Does the following timeline satisfy this specification?

\[ \begin{array}{c|c|c|c|}
\hline
A1(\text{horse}) & | & A2 & | \\
\hline
\text{Idle} & | & \text{B1(horse)} & | \\
\hline
\text{B2(dog,horse)} & | & | \\
\hline
\end{array} \]

In other words, is this an allowable vacuous solution? In EUROPA 2, the answer is yes. The constant parameter can be thought of as a name extension, e.g. \( A1_{\text{cat}} \). The semantics is thus:

\[ \text{FORALL } A_{\text{cat}}: \text{ EXISTS } B: A_{\text{cat}} \text{ op } B \]

Therefore, given two actions

\[ A(p1,p2,\ldots,pn: \text{type}) \]
\[ B(q1,q2,\ldots,qm: \text{type}) \]

and the following Allen operation

\[ A(a1,a2,\ldots,an) \text{ op } B(b1,b2,\ldots,bm) \]

where all of the arguments are constant parameters, the meaning is as follows:

\[ \text{FORALL } A: p1 = a1 \text{ AND } p2 = a2 \text{ AND } \ldots \text{ AND } pn = an \]
\[ \rightarrow \text{ EXISTS } B, b1, b2, \ldots, bm: \]
\[ A[p1,\ldots,pn](a1,a2,\ldots,an) \text{ op } B[p1,\ldots,pn](b1,b2,\ldots,bm) \]
7.4.2 Matching and Wild Card Parameters

Matching parameters do not restrict the domain of application of the Allen operation. Instead they constrain the allowed values of an acceptable plan. Therefore given two actions

\[ A(p_1, p_2, ..., p_n: \text{type}) \]
\[ B(q_1, q_2, ..., q_m: \text{type}) \]

and the following Allen operation

\[ A(xx,_,yy_, ... ) \text{ op } B(_,yy,_,_,xx, ...) \]

where the \( xx \) and \( yy \) are matching parameters. The meaning is as follows:

\[
\text{FORALL } A: \exists B, xx, yy:\]
\[
A(xx,_,yy_, ... ) \text{ op } B(_,yy,_,_,xx, ...) \]

7.5 Loss of Allen Op Symmetry

When the definitions of the Allen Operations are presented graphically as in Figure 1, one would expect that there would be a symmetry between many of the Allen ops. For example, \( A \) contains \( B \) if and only if \( B \) is contained by \( A \). While this is true of any single pair of action intervals, the symmetry does not hold when used to describe a domain model. The reason is that there is an implicit quantification over many instances of these actions. For example, consider the following specification:

\[ A_1 \text{ meets } B_2 \]

The following plan satisfies this specification:

<table>
<thead>
<tr>
<th>A1</th>
<th>A2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Idle</td>
<td>B1</td>
</tr>
</tbody>
</table>

But, this plan is not satisfied if the above Allen operation is replaced by its dual:

\[ B_2 \text{ met_by } A_1 \]

This observation is consistent with the FORALL-EXISTS semantics discussed previously.
Figure 1: Graphical Definition of Allen Operations
8 Observations and Conclusions

The goal of this work was to develop a capability to translate planning problems specified in the APPL language into the input language of the SAL model checker. This capability was intended to be used in conjunction with another translator from the APPL language to the input language of the EUROPA planning system. Together these translators should enable the formal verification of key properties of a plan specification, especially the specification of a generic domain model.

During the development of this translator, our understanding of plan specification using Allen Operations has greatly increased. We believe we are now in the position to make some observations about the advantages and disadvantages of specifying planning problems using a language built around Allen Operations.

The first observation is that it is easy to construct plan specifications that have no solution. For example, consider the following constraints

\[
\begin{align*}
    & A \text{ meets } B \\
    & C \text{ met}_\text{by} A
\end{align*}
\]

These constraints require that \( A \) must be followed by both \( B \) and \( C \), which is impossible. This might be obvious if the constraints are relatively close to each other in a plan specification, but what if they are several pages apart? Would this contradiction be detected? What might be intended by the above specification is that in some situations one constraint must hold and in another situation the other must hold. This can be accomplished using an if-then-else structure as follows

\[
\begin{align*}
    & \text{if (condition}_1\text{) then} \\
    & \hspace{1em} A \text{ meets } B \\
    & \text{else} \\
    & \hspace{1em} C \text{ met}_\text{by} A \\
    & \text{endif}
\end{align*}
\]

Of course, this requires bringing these previously separate specifications together in one place. This, in turn, may make the local specifications less clear.

The second observation is that the FORALL-EXISTS semantics is somewhat non-intuitive. When one first encounters the Allen operations, one would naturally expect that \( A \text{ meets } B \) and \( B \text{ met}_\text{by} A \) are duals. In other words, they specify the same thing. But as shown in Section 7, this is not the case. Therefore, one must be very careful when using Allen ops to keep in mind the FORALL-EXIST meaning and not rely on the English language meaning. Also, the fact that a constraint can be satisfied vacuously can be non-intuitive. If you have

\[
A(\text{dog}) \text{ meets } C
\]

you can easily fall into the trap of thinking that a solution plan will have an \( A(\text{dog}) \) action that is followed by a \( C \) action. However, it is perfectly acceptable for the planner to deliver a solution with \( A(\text{cat}) \) and never schedule an \( A(\text{dog}) \). Furthermore, there are some exceptions
to this FORALL-EXISTS semantics, which complicate our understanding. In particular, at the beginning and end of the planning horizon, the EXISTS <constraint> clause need not apply. A good example of this is

\begin{align*}
  \text{At meets Going} \\
  \text{Going meets At}
\end{align*}

The last Going or At on a timeline need not satisfy this constraint. Similarly the first Going or At on a timeline need not satisfy the following constraint:

\begin{align*}
  \text{At met_by Going} \\
  \text{Going met_by At}
\end{align*}

The third observation is that the use of multiple Allen Operations can have complex interactions. For example:

\begin{align*}
  \text{A starts B2} \\
  \text{B2 met_by B1}
\end{align*}

Here we declare that every time an A starts, a B2 must also start. But the second constraint requires that B2 be immediately preceded by B1. So the second constraint interacts with the first one as a kind of guard: A cannot be scheduled unless B1 is already executing, and as soon as A starts, B1 must terminate. Note that the declaration

\begin{align*}
  \text{A starts B2} \\
  \text{B1 meets B2}
\end{align*}

which looks very similar, has a very different kind of interaction. Here, A is free to initiate at any time and if B1 is executing at the time that A starts, then the planner could either satisfy the second constraint at this point (by terminating B1 and starting B2) or by letting B1 continue until a second B2 executes. All of these possibilities must be considered when writing a domain specification.

The fourth observation is that Allen Operations do not provide a good means of designating a precise sequence of actions. Consider the following APPL construct:

\begin{align*}
  \text{A1 -> A2 -> A3 -> A4}
\end{align*}

This declaration provides a concise way of saying exactly what sequence of actions can occur. This can be partially accomplished by the following Allen Operations:

\begin{align*}
  \text{A1 meets A2} \\
  \text{A2 meets A3} \\
  \text{A3 meets A4}
\end{align*}

However, one problem remains. The fact that there can be no successor to A4 is not specified. Furthermore, merely making A4 a goal state does not accomplish this because
is a solution to the above Allen op specification. In EUROPA one can designate this terminal 
state by stating that the \textbf{A4} must always be at the end of the timeline. But note that this is 
not a solution that uses Allen ops alone.

The fifth observation is that Allen Operations do not provide a good means of designating 
a restricted sequence of actions, because they can have non-local interactions and can be 
spread throughout a specification. Consider the following APPL construct:

\begin{verbatim}
A1 -> (A2 | A3 | A4) -> A5
\end{verbatim}

The following Allen ops can accomplish this

\begin{verbatim}
if x = 1 then
  A1 meets A2
elsif x = 2 then
  A1 meets A3
else
  A1 meets A4
endif
A2 meets A5
A3 meets A5
A4 meets A5
\end{verbatim}

Now suppose we specify, in a different place, that the timeline must begin with \textbf{A0} and that 
\textbf{A1} is \texttt{met_by} \textbf{A0}:

\begin{verbatim}
A1 met_by A0
\end{verbatim}

Together, one might expect that this specifies that exactly one of the following sequences 
must occur:

\begin{verbatim}
A0 -> A1 -> A2 -> A5
A0 -> A1 -> A3 -> A5
A0 -> A1 -> A4 -> A5
\end{verbatim}

But this specification does not rule out

\begin{verbatim}
A0 -> A3 -> A5
A0 -> A4 -> A5
A0 -> A5
\end{verbatim}
Of course, the addition of \texttt{A0 meets A1} would solve the problem, but this demonstrates how careful one must be when using Allen ops to specify a restricted sequence of operations.

The development of the \texttt{APPL -> SAL} translator was driven by the need to match the \texttt{APPL -> NDDL} semantics. Several things made this difficult. First, we did not have a formal semantics for NDDL, so we had to infer the meaning of NDDL by executing EUROPA on test problems. The FORALL-EXISTS semantics and end-case exceptions were discovered as we built the translator. Second, we discovered that the EUROPA semantics did not match our original understanding, so there is a structural mis-match between APPL and EUROPA. In particular, we had not originally envisioned that EUROPA employs a kind of \textit{inclusive}\footnote{In other words, everything is included except that which is explicitly ruled out by the constraints.} approach to specifying the allowed set of actions on a timeline. In EUROPA, any action can be followed by any other action, unless it is prohibited from doing so by a constraint (e.g. an Allen Operation). However, APPL was designed around the idea of a state machine where all of the transitions are enumerated. In this \textit{constructive} approach, an action can only be followed by another action if it is specifically included in the specification. In the end, we were able to bring these two translators fairly close together with the addition of null states, but we are not completely satisfied with our current solution. Third, the interaction of Allen Operations was much more complex than we envisioned. These interactions led to far more complicated code than we originally expected. Furthermore, we often had to rework the translation scheme as we discovered how EUROPA handled these interactions. For that reason, we do not have confidence that the translator is sound. At best, we have produced a rapid prototype which can serve as a basis for future developments. The authors hope that the techniques described in this paper will be useful to future developers who seek to connect planning languages to symbolic model checkers.

\textbf{References}


This paper describes a translator from a new planning language named the Abstract Plan Preparation Language (APPL) to the Symbolic Analysis Laboratory (SAL) model checker. This translator has been developed in support of the Spacecraft Autonomy for Vehicles and Habitats (SAVH) project sponsored by the Exploration Technology Development Program, which is seeking to mature autonomy technology for the vehicles and operations centers of Project Constellation.