DUAL-ARM MANIPULATORS WITH ADAPTIVE CONTROL

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Abstract

The described and improved multi-arm invention of this application presents three strategies for adaptive control of cooperative multi-arm robots which coordinate control over a common load. In the position-position control strategy, the adaptive controllers ensure that the end-effector positions of both arms track desired trajectories in Cartesian space despite unknown time-varying interaction forces exerted through a load. In the position-hybrid control strategy, the adaptive controller of one arm controls end-effector motions in the free directions and applied forces in the constraint directions; while the adaptive controller of the other arm ensures that the end-effector tracks desired position trajectories. In the hybrid-hybrid control strategy, the adaptive controllers ensure that both end-effectors track reference position trajectories while simultaneously applying desired forces on the load. In all three control strategies, the cross-coupling effects between the arms are treated as "disturbances" which are compensated for by the adaptive controllers while following desired commands in a common frame of reference. The adaptive controllers do not require the complex mathematical model of the arm dynamics or any knowledge of the arm dynamic parameters or the load parameters such as mass and stiffness. Circuits in the adaptive feedback and feedforward controllers are varied by novel adaptation laws.

28 Claims, 14 Drawing Sheets
HIGH LEVEL

TASK PLANNING
AND DECOMPOSITION

INTERMEDIATE
LEVEL

TRAJECTORY
GENERATOR
FOR LEFT ARM

COORDINATION

TRAJECTORY
GENERATOR
FOR RIGHT ARM

LOW LEVEL

ADAPTIVE
CONTROLLER

LEFT
ARM

ADAPTIVE
CONTROLLER

RIGHT
ARM

LOAD

FEEDBACK

Fig. 5
**FIG. 9a**

- Graph showing the force function $f_m(t)$ in Newtons as a function of time $t$ (seconds).

**FIG. 9b**

- Graph showing the displacement function $x_{la}(t)$ in centimeters as a function of time $t$ (seconds).
FIG. IIb

$y(t), y_{ld}(t)$

(CENTIMETERS)

FIG. IIc

$y_r(t), y_{rd}(t)$

(CENTIMETERS)
FIG. 11d

X (CENTIMETERS)

Y (CENTIMETERS)
DUAL-ARM MANIPULATORS WITH ADAPTIVE CONTROL

ORIGIN OF THE INVENTION

The invention described herein was made in the performance of work under a NASA contract and is subject to the provisions of Public Law 96517 (35 USC 202) in which the contractor has elected to retain title.

This application is a continuation-in-part of a pending application assigned to the same assignee and identified as a METHOD AND APPARATUS FOR ADAPTIVE FORCE AND POSITION CONTROL OF MANIPULATORS, filed on Apr. 6, 1987 having Ser. No. 07/035,061 now U.S. Pat. No. 4,860,215.

BACKGROUND OF THE INVENTION

1. Field of the Invention

This invention relates to control systems for robotic manipulators and more particularly relates to adaptive control for a plurality of robotic arms, e.g., a multi-arm system in which the cross-coupling between robotic arms through a common load is treated as though it were an externally caused disturbance.

2. Description of the Prior Art

An adaptive control system invented by this inventor is described and claimed in the above-identified application and a considerable number of publications are set forth therein, which publications are incorporated herein by reference as though they were set forth expressly herein. The prior art and references cited therein are called to the attention of the Patent Office as being of background relevance to this invention. As of this filing date, the identified application has received an Office Action and the following patents have been cited; Horack U.S. Pat. No. 4,547,858, Koyama et al U.S. Pat. No. 4,580,229; Sugimoto et al U.S. Pat. No. 4,621,332; and Osaka U.S. Pat. No. 4,725,942. These patents do not have any significant relevance to the invention herein described and claimed.

The system of the identified application is improved upon by being extended from a single arm approach to a dual-arm (or higher) approach with a novel method and apparatus being described for overcoming the cross-coupling that exists in a common load being manipulated by independently controlled manipulators, or arms, in a multi-arm system. Additional background material relevant to the development and a fuller understanding of this invention is given in the following paragraphs.

During the past decade, robot manipulators ("arms") have been utilized in industry for performing simple tasks, and it is foreseen that in the near future anthropomorphic robots will replace human operators in carrying out various complex tasks both in industry and in hazardous environments. Nevertheless, present-day robots can be considered at best as "handicapped" operators due to their single-arm structure. It is evident that a multiplicity of robot arms yields greater dexterity and increased efficiency and provides capability of handling larger loads. Dual-arm robots will therefore have capabilities which may match those of ambidextrous human operators in dexterity and efficiency.


The above-identified articles, to the extent that they are properly considered prior art, do not teach or suggest a dual-arm adaptive control system, nor such a system having adaptive hybrid control of each arm independently. Moreover, only in this application is it taught that a multi-arm adaptive control system is reliably operable so long as the load's inter-arm cross-coupling is treated as though that cross-coupling were an externally caused disturbance. The adaptive hybrid control system of this invention can compensate for that cross-coupling because of the novel force and/or position control laws as herein defined.

There are certain key differences between my single-arm invention's approach and the conventional hybrid control approach of Raibert and Craig, referred to above. Firstly, in my single-arm invention, the force or...
position control problems are formulated in the Carte-
sian space with the end-effector Cartesian forces as the
manipulated variables; whereas in Raibert and Craig,
the problems are formulated in the joint space. The
single-arm invention's formulation results in computa-
tional improvement since inverse Jacobians are not
required for the controllers' operation. Secondly, the
single-arm invention's hybrid system operates on the
measured variables so as to produce the position and
force variables that need to be controlled; whereas in
Raibert and Craig, a selection matrix and its com-
plement are used after formulation of tracking errors. In
summary, the Raibert and Craig disclosure simply does
not teach or suggest the novel concepts of my inven-
tion.

SUMMARY OF THE INVENTION

My invention of the parent application ("single-arm"
 invention) discloses a novel method and apparatus for
the design of adaptive force and position controllers
within a hybrid control architecture. The hybrid con-
troller includes an adaptive force controller which
achieves tracking of desired force setpoints, and an
adaptive position controller which accomplishes track-
ing of desired position trajectories. Force and position
controller gains are varied by novel adaptation laws.
These controllers are computationally fast and suitable
for on-line implementation with high sampling rates and
such adaptive gains are implemented in feedback and
feedforward controllers. These controllers are capable
of compensating for dynamic cross-couplings that exist
between position and force control loops.

A key feature of my single-arm invention is that my
adaptive hybrid controller architecture does not require
knowledge of the complex dynamic model or parameter
values of the manipulator or the environment. The
force and position controllers are linear and stable and
generate real-time signals which vary and compensate
for system non-linearities in order to achieve a desired
position/force response.

An additionally attractive feature of my single-arm
adaptive controllers is an ability to compensate for dy-
namic cross-couplings that exist between the position
and force control loops in the hybrid control architec-
ture. Furthermore, the adaptive force and position con-
trollers have "learning capabilities" to cope with unpre-
dictable changes in the manipulator or environment
parameters such as environment or robotic arm stiff-
ness. This is due to the fact that the controller gains are
adapted rapidly on the basis of the manipulator's Carte-
sian space performance. Low computational require-
ments make the control loops of the single-arm inven-
tion suitable for implementation in on-line hybrid con-
trol with high sampling rates.

The described and improved multi-arm invention of
this application presents three strategies for adaptive
control of cooperative dual-arm robots. In the position-
position control strategy, the adaptive controllers en-
sure that the end-effector positions of both arms track
desired trajectories in Cartesian space despite unknown
time-varying interaction forces exerted through the
load. In the position-hybrid control strategy, the adapt-
ive controller of one arm controls end-effector motions
in the free directions and applied forces in the constraint
directions; while the adaptive controller of the other
arm ensures that the end-effector tracks desired position
trajectories. In the hybrid-hybrid control strategy, the
adaptive controllers ensure that both end-effectors
track reference position trajectories while simulta-
nously applying desired forces on the load.

In all three control strategies, the cross-coupling
effects between the arms are treated as "disturbances"
which are rejected, or compensated for, by the adaptive
controllers while following desired commands in a
common frame of reference. The adaptive controllers
do not require the complex mathematical model of the
arm dynamics or any knowledge of the arm dynamic
parameters or the load parameters such as mass and
stiffness. The controllers have simple structures and are
computationally fast for on-line implementation with
high sampling rates. Simulation results are given to
illustrate the proposed adaptive control strategies.

BRIEF DESCRIPTION OF THE DRAWING

FIG. 1 depicts a block diagram of an improved hy-
brid control architecture in keeping with this invention;
FIG. 2 is a simplified block diagram of a controller
for performing adaptive force control in accordance
with the invention;
FIG. 3 is a simplified block diagram of a controller
for performing adaptive position control in accordance
with the invention;
FIG. 4 is a block diagram of a basic module which is
repeated as necessary for implementation of the control-
er functions of adaptive force and position control in
accordance with this invention;
FIG. 5 depicts a tri-level hierarchal control for two,
or higher-numbered arm, robotic systems;
FIGS. 6 and 7 depict, respectively a single-arm and a
cooperative dual-arm manipulator;
FIG. 8 depicts a simplified load diagram that is useful
in developing a better understanding of the invention;
FIG. 9, including FIGS. 9a, 9b, 9c, 9d, 9e, and 9f,
depict the results of a computer simulation in adaptive
position tracking which are useful in performance eval-
uation of the invention;
FIG. 10, including FIGS. 10a, 10b, 10c, 10d, and 10e,
depict the results of a computer simulation in adaptive
position-hybrid control which are useful in perform-
ance evaluation of the invention; and
FIG. 11, including FIGS. 11a, 11b, 11c and 11d, depicts the
results of a computer simulation in adaptive hybrid-
hybrid control which are useful in performance evalua-
tion of the invention.

DESCRIPTION OF THE PREFERRED
EMBODIMENT

The description of the invention claimed in this appli-
cation will first include a summary of the novel single-
arm force and position control laws and then those laws
will be developed and applied to a multi-arm system of
this invention. Incorporated herein by reference, in
accordance with §608.01(p) of the Manual of Patent
Examining Procedure, from the above-identified single-
arm application, is the mathematical formulation of the
adaptive force and/or position control laws as set forth
in section 1.1, page 10 through section 3, page 28. More
particularly such incorporation includes Equations (21)
through (24) for the gains of the PID controller of FIG.
3 hereof in the implementation of the linear adaptive
force control law given by Equations (25) and (26); and
likewise the formulation and implementation of the
linear adaptive position control law of Equation (37)
and the controller gains of Equations (38) through (43).

To avoid confusion, the Equations of this continua-
tion-inpart application are numbered starting with
Equation no. 101, etc. The exact same force and position control laws apply to multi-arm hybrid adaptive control as apply to single-arm hybrid adaptive control except the mathematical notation is changed slightly. It will be readily apparent to those of ordinary skill in this art, however, that the Equations are the same and one of the keys to the innovative features of this application is noting that the cross-coupling terms, \( C_p \) Equation (25) and \( C_y \) Equation (37) in the adaptive position and force loops of a single-arm system, also compensates for the cross-coupling that the independently controlled arm of one controller introduces into another independently controlled arm in a dual or higher-numbered multi-arm system. Stated in a slightly different way the cross-coupling term which results from an environment in the single-arm disclosure is replaced by the coupling through the load that is being held by a second arm in the multi-arm disclosure. Otherwise, the two systems are the same and the analysis for one system is fully and completely applicable to the other system.

The blanket generality noted above may be appreciated by a brief reference to FIG. 6 in comparison to FIG. 7. FIG. 6 depicts a common way of showing a robot manipulator 600 having at least two joint-connected sections 600A, 600B and an end effector 605. The manipulator 600's end-effector 605 is moveable in a Cartesian space 630 and may come into contact with another object or a wall, such as environment 610. In FIG. 7, two arms 600 and 660 are shown and the environment in this instance is replaced by common load 610 that is being manipulated by and between the two arms, 600 and 660, respectively.

In order to show the correspondence with my earlier invention, I have identified the common load by number 610 so that its correspondence to the wall, or other object 610 will be more readily apparent. Although not depicted, common reference systems, independent controllers with synchronization at the trajectory generator level and common coordination (at the INTERMEDIATE LEVEL, FIG. 5) is to be understood throughout the disclosure as will become readily apparent by the following detailed description.

The presentation of the detailed description given hereinafter in this application is structured as follows. In Section 1 ("position-position"), the position-position control strategy is discussed and documented by simulation results. In Section 2 ("position-hybrid"), the position-hybrid, e.g. position-force control strategy is developed and demonstrated by a numerical example. The hybrid-hybrid, force/position, or vice-versa, control strategy is addressed in Section 3 ("hybrid-hybrid") and an illustrative example is given. The remainder of this application discusses the results of the foregoing multiarm presentation and draws some conclusions.

**SECTION 1-POSITION-POSITION**

In this section, we shall investigate the first control strategy for dual-arm manipulators in which both arms are in pure position control, as shown in FIG. 2. In other words, the positions and orientations of both end-effectors are required to track desired trajectories in a common frame of reference. In this situation, uncontrolled forces and torques will be exerted on the common load 610 held by the end effectors 605,665 of the dual arms such as those represented in the LOW LEVEL section of FIG. 5. Investigated herein is the performance of the adaptive position control law of my system in the face of the interaction forces and torques exerted through the load 610.

The dynamic model of each manipulator arm (arm 600 or 660, FIG. 7) can be represented by a differential equation in Cartesian space as was observed by O. Khatib, in *Dynamic Control of Manipulators in Cartesian Space*. Proc. 6th IFToMM Congress on Theory of Machines and Mechanisms, pp. 1128-1131, New Delhi, India, 1983.

\[
M(x) \dot{\dot{x}} + N(x, \dot{x}) + G(x) + H(x) = f - F
\]

where the above terms are defined as:

\[X, x, \dot{x} = n \times 1 \text{ vectors of end-effector position, velocity and acceleration in a fixed task-related Cartesian frame of reference} \]

\[F = n \times 1 \text{ vector of "virtual" Cartesian forces applied to the end-effector as the control input} \]

\[M(x) = n \times n \text{ symmetric positive-definite Cartesian mass matrix} \]

\[N(x, \dot{x}) = n \times 1 \text{ Cartesian Coriolis and centrifugal force vector} \]

\[G(x) = n \times 1 \text{ Cartesian gravity loading vector} \]

\[H(x) = n \times 1 \text{ Cartesian friction force vector} \]

\[f = n \times 1 \text{ vector of forces and torques exerted by the end-effector on the load.} \]

In the above-noted example the load is the shared common load 610 between the dual-arm system of FIGS. 5 and 7.

The force/torque vector \( f \) both imparts motion to and applies force/torque on the load 610 and acts as the coupling element between the two arms 600 and 660. In the following analysis, the force/torque vector \( f \) will be considered as a "disturbance input" (analogous to my earlier-described auxiliary input signal \( d(t) \)) to the position control system. The function of the control system is to ensure that the end-effector position vector \( X \) tracks the \( n \times 1 \) vector of desired trajectory \( X_d \) despite the disturbance force \( f \). For each manipulator arm, let us apply the linear adaptive position control law described in my earlier application or its corresponding paper which is identified as H. Seraji, *Direct Adaptive Control of Manipulators in Cartesian Space*, Journal of Robotic Systems, February, 1987. Such a position control law is:

\[F(t) = d(t) + [K_p \dot{e}(t) + K_d e(t)] + [G(t) + H(t) + B(t)] \]

\[B(t) = X_d(t) + A(t) \dot{X}_d(t) \]

as shown in FIG. 3, where \( E(t) = X_d(t) - X(t) \) is the \( n \times 1 \) position tracking-error vector. In the control law (102), the \( n \times 1 \) vector \( d(t) \) is an auxiliary signal to be synthesized by the adaptation scheme, while \([K_p E + K_d E] \) and \([C X_d + B X_d + CX_d] \) are the contributions due to the feedback and feedforward controllers respectively. Following my earlier-described method, the required auxiliary signal and controller gains are updated according to the following adaptation laws:

\[d(t) = d(0) + a \int_0^t \dot{e}(t) dt + b e(t) \]

\[K_p(t) = K_p(0) + a \int_0^t \dot{e}(t) dt + a_2 \epsilon(t) \]

\[K_d(t) = K_d(0) + a \int_0^t \dot{e}(t) dt + a_2 \epsilon(t) \]
is an n×1 vector, \{δ1,α1,β1,γ1,λ1\} are positive scalars, \{δ2,α2,β2,γ2,λ2\} are positive or zero scalars, and the prim denotes transposition. In equation (109), \(W_p\) and \(W_r\) are n×n constant weighting matrices chosen by the designer to reflect the relative significance of the position and velocity errors \(E\) and \(\dot{E}\). It must be noted that since we cannot physically apply the Cartesian control force \(F\) to the end-effector, we instead compute the n×1 equivalent joint torque vector \(T\) to effectively cause this force. Thus, for each manipulator arm (arm 600 or 660, FIG. 7, for example), the control law in joint space is given by

\[
T(\theta) = J(\theta)FL + K_d(\theta)\dot{E}(\theta) + K_s(\theta)\dot{E}(\theta) + (110)
\]

where \(\theta\) is the n×1 vector of joint angular positions and \(J(\theta)\) is the n×n Jacobian matrix of the manipulator arm. Because of the simplicity of the adaptation laws (103) through (108), the robot control algorithm can be implemented using high sampling rates (typically 1 KHz). In each sampling period (-1 msec), the controller gains can change significantly whereas the terms \(M, N, G, H\), and \(f\) in the robot model (101) cannot change noticeably. As a result, in deriving equations (103) through (108), it was assumed that these terms are unknown and "slowly time-varying" relative to the gain variations as such are changed by these adaptation laws. It is seen that the inclusion of the disturbance force \(f\) in the robot model (101) does not affect the controller adaptation laws since the change in \(f\) over one sampling period is relatively small.

The above-noted observation likewise suggests that when both manipulator arms are controlled using the two independent adaptive position controllers as implemented by my adaptation laws, we observe that the end-effectors will track the desired position trajectories despite the interaction forces and torques exerted through the load. It must be noted that since the force on the load is not a controlled variable in this invention, this strategy can lead to undesirable load forces when the position trajectories are not planned in coordination or are not tracked closely. The position-position control strategy is illustrated by the following example.

Consider the planar dual-arm manipulator in a horizontal plane (g=0) shown in FIG. 7, where the two 65 arms 600 and 660 are identical and each arm has two links and an end-effector with negligible inertia. Suppose that the load is a linear spring of natural length \(l_0\) and coefficient of stiffness (elasticity) \(K_p\). In the joint space, the dynamic equations of motion for each arm which relates the joint torque vector

\[
T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}
\]

to the joint angle vector \(\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}\) is given by \(H\).


\[
T(\theta) = M(\theta)\dot{\theta} + N(\theta, \dot{\theta}) + H(\dot{\theta}) + J(\theta)\dot{\theta}
\]

where the above terms are defined as:

- Inertia matrix

\[
M(\theta) = \begin{bmatrix} a_1 + 2z_s\cos\theta_2 & 2z_s(2z_s\cos\theta_2) \\ 2z_s(2z_s\cos\theta_2) & 2z_s + (2z_s\cos\theta_2) \end{bmatrix}
\]

- Coriolis and centrifugal torque vector

\[
N(\theta, \dot{\theta}) = \begin{bmatrix} -2z_s\sin\theta_2 \dot{\theta}_2 - \dot{\theta}_1 \dot{\theta}_2 \\ -2z_s\sin\theta_2 \dot{\theta}_2 + \dot{\theta}_1 \dot{\theta}_2 \end{bmatrix}
\]

- Coulomb and viscous friction torque vector

\[
H(\dot{\theta}) = \begin{bmatrix} \dot{V}_1\dot{\theta}_1 + V_s\sin(\dot{\theta}_1) \\ \dot{V}_2\dot{\theta}_2 + V_s\sin(\dot{\theta}_2) \end{bmatrix}
\]

- Jacobian matrix

\[
J(\theta) = \begin{bmatrix} \dot{V}_1\dot{\theta}_1 - I_1\sin(\theta_1 + \theta_2) - I_2\sin(\theta_1 + \theta_2) \\ -I_1\sin(\theta_1 + \theta_2) + I_2\cos(\theta_1 + \theta_2) + \dot{V}_2\dot{\theta}_2 \end{bmatrix}
\]

In the above expressions, \(a_1, \ldots, a_5\) are constant parameters obtained from the masses \(m_1, m_2\) and the lengths \(l_1, l_2\) of the links, \(V_1, V_2\) and \(V_1, V_2\) are coefficients of viscous and Coulomb frictions respectively, and the links are uniform with centers-of-gravity located at mid-lengths. The interaction force vector \(f\) is modelled as

\[
f = \begin{bmatrix} f_x \\ f_y \end{bmatrix} = \begin{bmatrix} K_d\dot{E} - I_0\cos\alpha \\ K_d\dot{E} - I_0\sin\alpha \end{bmatrix}
\]

where

\[
l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

is the instantaneous length of the coordinates of each end-effector given by

\[
x = d_0 + l_1\cos\theta_1 + l_2\cos(\theta_1 + \theta_2)
\]

\[
y = l_1\sin\theta_1 + l_2\sin(\theta_1 + \theta_2)
\]
and \( d_0 \) is the base distance from the origin and \( \alpha \) is the angle between the spring and the x-axis, that is

\[
\tan \alpha = \frac{y_2 - y_1}{x_2 - x_1}.
\]

The mass of the spring is assumed to be negligible compared to the masses of the links and hence the force in the spring dominates over the inertial force required to move the spring.

For the particular system under study, the numerical values of the parameters are chosen as

\[
\begin{align*}
m_1 &= 15.91 \text{ kg; } m_2 = 11.36 \text{ kg; } l_1 = l_2 = -0.432 \text{ m} \\
V_1 &= 6.0 \text{ Nt.m/rad.sec}^{-1}; \quad V_3 = 4.0 \text{ Nt.m/rad.sec}^{-1} \\
V_2 &= V_4 = 1.0 \text{ Nt.m; } K_o = 10000.0 \text{ Nt./m} \\
\end{align*}
\]

The natural length of the spring is set equal to the initial distance between the end-effectors; i.e.

\[
l_0 \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

so that there will be no initial force in the spring. The masses and lengths of the links are those of links 2 and 3 of the Unimation PUMA 560 arm. It must be emphasized that the mathematical model and the parameter values of the arms and the spring are used merely to simulate the system behavior and are not used to generate the control action. This invention does not depend upon knowledge of the complex mathematical model of the arm dynamics or any knowledge of the arm dynamic parameters or the load parameters such as mass or stiffness. As was noted hereinbefore, the lack of requirement of such knowledge must be acknowledged as being one of the key features that contributes to the simplified nature of this adaptive hybrid system invention and its novel force and/or position control laws, or algorithms.

Suppose that the end-effector of the right arm is required to track the desired position trajectories

\[
x_d(t) = 0.864[1 + 3\exp(-t/0.3) - 4\exp(-t/0.4)]m,
\]

\[
y_d(t) = 0.864[3\exp(-t/0.3) - 4\exp(-t/0.4)]m.
\]

to move from the initial point \((0,0,-0.864)\) to the final point \((0.864,0,0)\) on the straight line \(y_d(t) = x_d(t) - 0.864\). The end-effector of the left arm is required to track the desired position trajectories

\[
x_l(t) = 2.0 + 0.432[1 + 3\exp(-t/0.3) - 4\exp(-t/0.4)]m,
\]

\[
y_l(t) = 0.432[-1 + 3\exp(-t/0.3) - 4\exp(-t/0.4)]m.
\]

so that \( f_m \) changes from the initial value of zero to the final value of 373 Newtons in 2 seconds. FIGS. 9(b) through 9(f) show that the end-effector coordinates \( x(t) \) and \( y(t) \) of both arms track their corresponding reference trajectories \( x_d(t) \) and \( y_d(t) \) very closely despite the large interaction force \( f_m \). The paths followed by the end-effectors in the horizontal plane are shown in FIG 9(f). Note that both end-effectors track the straight lines as desired.

We conclude that the adaptive controller for each arm performs remarkably well in faithful position tracking despite very large unknown and time-varying interaction forces between the arms.

**SECTION 2: POSITION-HYBRID**

In this section, the position-hybrid control strategy for dual-arm manipulators will be studied in which the left arm is in pure position control and the right arm is in hybrid position/force control, as shown in one em-
bodiment of FIG. 3. In other words, for the left arm 600, FIG. 7, the end-effector position is required to track a desired trajectory in a frame of reference. For the right arm 660, FIG. 7, in the same reference frame, the contact force between its end-effector 665 and the load 610 must be controlled in the directions constrained by the load 610, while the end-effector 665's position is to be controlled simultaneously in the free directions. This control strategy is also applicable when one robot arm is confined for operation only in a position control mode whereas, simultaneously, the other arm can be controlled in hybrid control mode.

For the left arm 600, the interaction forces and torque exerted through the load 610 are considered as "disturbances," and the adaptive position control system can ensure tracking of the desired position trajectories despite such disturbances, as outlined in Section 1. The adaptive position control law for the left arm shown in FIG. 7 is given by Equation (37) in my earlier application and may be applied here as

\[
T(t) = J(q(t))\tau(t) + K_p(q(t))E(q(t)) + K_d(q(t))E(q(t)) + C(q(t))X(q(t)) + B(q(t))\dot{X}(q(t)) + (\alpha(q(t)))
\]

where \( T(t) \) is the \( n \times 1 \) joint torque vector, \( \theta(t) \) is the \( n \times 1 \) joint angle vector, \( J(q(t)) \) is the \( n \times n \) Jacobian matrix, \( X(q(t)) - X(t) \) the \( n \times 1 \) position tracking-error vector and the terms in Equation (113) are adapted as follows:

\[
\tilde{a}(t) = \tilde{a}(0) + b_1 \int_0^t \eta(t) dt + b_2 \gamma(t)
\]

\[
\tilde{k}_p(t) = \tilde{k}_p(0) + a_1 \int_0^t \eta(t) E(p(t)) dt + a_2 \gamma(t) E(p(t))
\]

\[
\tilde{k}_d(t) = \tilde{k}_d(0) + b_1 \int_0^t \eta(t) E(p(t)) dt + b_2 \gamma(t) E(p(t))
\]

\[
\tilde{c}(t) = \tilde{c}(0) + c_1 \int_0^t \eta(t) X_d(q(t)) dt + c_2 \eta(t) X_d(q(t))
\]

\[
\tilde{b}(t) = \tilde{b}(0) + \gamma_1 \int_0^t \eta(t) X_d(q(t)) dt + \gamma_2 \eta(t) X_d(q(t))
\]

\[
\tilde{a}(t) = \tilde{a}(0) + \lambda_1 \int_0^t \eta(t) X_d(q(t)) dt + \lambda_2 \gamma(t) X_d(q(t))
\]

where

\[
\eta(t) = W_p E(t) + W_e \dot{E}(t)
\]

and the symbols are defined in Section 1.

We shall now discuss the hybrid position/force controller for the right arm 660, FIG. 7. Consider a task-related "constraint frame" (coordinate system) which is defined by the particular contact situation occurring between the right end-effector 665 and the load 610. In 60 this frame, the \( n \) degrees-of-freedom (or directions) is the Cartesian space \( \{X\} \) can be partitioned into two orthogonal sets; the \( m \) constraint directions in subspace \( \{Z\} \) and the \( l \) free directions in subspace \( \{Y\} \), with \( n = m + l \). If the \( m \) constraint directions, the end-effector must be controlled in the \( l \) freed directions. The end-effector is free to move and the end-effector position is to be controlled. In the hybrid control architecture (as noted in the aforesaid Raibert and Craig and Mason articles), two separate controllers may be employed for simultaneous force and position control. The "force controller" achieves tracking of desired force setpoints in the constraint directions; while the "position controller" accomplishes tracking of desired position trajectories in the free directions.

The dynamic model of the right arm in the constraint directions can be written as

\[
A_C(q) \ddot{q}(t) + B_C(q) \dot{q}(t) + C(q)g(t) = F(t)
\]

where \( F(t) \) is the \( m \times 1 \) "virtual" Cartesian force vector applied to the end-effector in the constraint directions, \( Z \) is the \( m \times 1 \) vector of end-effector position, the \( m \times m \) matrices \( A \) and \( B \) are highly complex nonlinear functions of the end-effector position \( X \), \( C \) is the cross-coupling from the position loop into the force loop and \( F(t) \) is the component of the force exerted on the end-effector by the load 610 in the constraint directions. The term \( F(t) \) represents the cross-coupling that exists between the arms 600, 660 through the load 610 and is considered as a "disturbance" to the hybrid controller.

An adaptive force control scheme is developed within the hybrid control architecture as claimed herein. For the right arm 660, the linear adaptive force control law in the constraint directions is given by Equation 122 below as

\[
F(t) = P(t) + d(t) + K_p(q(t))E(t) + \int_0^t E(t) dt - K_d(q(t))\dot{E}(t)
\]

where \( P(t) \) is the desired contact force on the load 610 used as a feedforward term, \( d(t) \) is an auxiliary signal, \( E(t) = P(t) - P(t) \) is the deviation of the actual force \( P(t) \) from the desired value, and \( \{K_p(t), K_d(t), K_d(t)\} \) are adaptive gains of the PID controller. The terms in the force control law, Equation (122), are adapted as follows:

\[
d(t) = d(0) + \delta_1 \int_0^t q(t) dt + \delta_2 \gamma(t)
\]

\[
K_p(t) = K_p(0) + a_1 \int_0^t q(t) E(t) dt + a_2 \gamma(t) E(t)
\]

\[
K_d(t) = K_d(0) + b_1 \int_0^t q(t) E(t) dt + b_2 \gamma(t) E(t)
\]

\[
K_d(t) = K_d(0) + \gamma_1 \int_0^t q(t) \dot{E}(t) dt + \gamma_2 \dot{E}(t)
\]

\[
\dot{\delta}_1 = W_p E(t) + W_e \dot{E}(t)
\]

and the symbols are defined in Section 1.
\{W_1, W_2, W_3\} are constant weighting matrices chosen by the designer to reflect the relative significance of $E^*$, $E$ and $Z$.

The dynamic model of the right arm in the free directions can be written as

$$A_4(x, \dot{x}) \ddot{y}(\dot{y}) + B_4(x, \dot{x}) \dot{y}(\dot{y}) + C_4(x, \dot{x}) y(\dot{y}) + C_5\dot{y} = f_3 = f_{3r}(t)$$  \hspace{1cm} (128)

where $f_3$ is the component of the end-effector force in the free directions, $C_f$ is the cross-coupling from the free loop, $A_\alpha$, $B_\alpha$, $C_\alpha$ are complex nonlinear matrices, $Y$ is the end-effector position vector and $F$ is the "virtual" end-effector control force. For the right arm, the linear adaptive position control law in the free directions is given by

$$F_{r3}(t) = \ddot{y}(t) + K_r(E_r(t) + \ddot{y}(t) + \dot{y}(t) + \dot{y}(t) + \dot{y}(t))$$  \hspace{1cm} (129)

as in Section 2, where $R$ is the desired position trajectory $E_r = R - Y$ is the position tracking-error, and $F_r$ is the "virtual" Cartesian force in the free directions. Thus, in order to implement the force and position controllers in accordance with Equations (122) and (129) in the hybrid control architecture, the joint space control law for the right arm is given by

$$T(t) = J_r(\theta_r)(f_{3r}(t))$$  \hspace{1cm} (130)

where $\theta_r$ is the joint angle vector, $T_r$ is the joint torque vector, and $J_r$ is the Jacobian matrix of the right arm with appropriate reordering of columns of $J_r$ if necessary.

The hybrid controller adaptation laws, Equations (103) through (108) and (123) through (126) are extremely simple, and therefore the control algorithm can be implemented using high sampling rates (rates \( \approx 1 \) kHz); yielding improved performance particularly in force control applications. Since in each sampling period (\( \approx 1 \) m sec) the terms in the robot models (121) and (128) cannot change noticeably, it is reasonable to assume that these terms are "slowly time-varying" compared to the adaptation scheme. Thus the inclusion of the disturbance $f_2$ and $f_3$ in the robot models (121) and (128) does not affect the controller performance.

It is concluded, therefore, that using the position-hybrid control strategy, the left end-effector will track the desired position trajectory despite the interaction forces through the load. The right end-effector will exert the desired force on the load in certain directions while simultaneously tracking the desired position trajectory in the orthogonal directions. It must be noted that in this control strategy, slight fluctuations may be observed on the load force due to very small vibrations of the left arm under position control. The position-hybrid control strategy is now illustrated by the following example.

Let us consider the dual-arm manipulator discussed in Example 2 (with negligible friction) and suppose that the end effectors carry a rigid load in a horizontal plane, as shown in FIG. 7. Suppose that the load 610' as 65 shown, is initially parallel to the x-axis and we wish to make a parallel translation of the load 610' while the right arm, 660, applies a constant desired contact force on the load. This problem is similar to pulling out a drawer using two arms.

The x-component of the force on the load 610' can be written as

$$f_x = k_d(x_2 - x_1) - l_0$$

where \( (x_2 - x_1) \) is the distance between the end-effectors, \( l_0 \) is the length of the load, \( k_d \) is the stiffness of force/torque sensors mounted on the end-effectors to measure the force \( f_x \). The y-component of the force on the load is

$$f_y = \frac{m}{2} \ddot{y}$$

where the arms are assumed to share the load equally; i.e. each arm "sees" the load mass as \( m/2 \). In the simulation, we take \( k_d = 1000.0 \text{ N/m} \) and \( m = 1.0 \text{ Kg} \).

Suppose that the end-effector 605 of the left arm 600 is required to track the desired position trajectories

$$x_{r2}(t) = 0.432 \text{ meter}$$

$$y_{r2}(t) = 0.432[1 + 6e^{x-(t/0.3)} - 8e^{x-(t/0.4)}] \text{ meter}$$

so that the end-effector moves from the initial point \( (0.432, -0.432) \) to the final point \( (0.432, 0.432) \) on the vertical line \( x(t) = 0.432 \). The end-effector 665 of the right arm 660 is required to apply a specified force setpoint \( P_{sf} \) on the load 610' in the x direction and simultaneously track the desired position trajectory \( y_{r2}(t) \) in the y-direction, where

$$P_{sf} = 10 \text{ Newton}$$

$$y_{r2}(t) = 0.432[1 + 6e^{x-(t/0.3)} - 8e^{x-(t/0.4)}] \text{ meter}$$

The adaptive position control law for the left arm is given by

$$T(t) = J_2(\theta_2)(F_{r2}(t))$$

where $\theta_2$ is the joint angle vector, $T_2$ is the joint torque vector, and $J_2$ is the Jacobian matrix of the right arm with appropriate reordering of columns of $J_2$ if necessary.

The hybrid controller adaptation laws, Equations (103) through (108) and (123) through (126) are extremely simple, and therefore the control algorithm can be implemented using high sampling rates (rates \( \approx 1 \) kHz); yielding improved performance particularly in force control applications. Since in each sampling period (\( \approx 1 \) m sec) the terms in the robot models (121) and (128) cannot change noticeably, it is reasonable to assume that these terms are "slowly time-varying" compared to the adaptation scheme. Thus the inclusion of the disturbance $f_2$ and $f_3$ in the robot models (121) and (128) does not affect the controller performance.

It is concluded, therefore, that using the position-hybrid control strategy, the left end-effector will track the desired position trajectory despite the interaction forces through the load. The right end-effector will exert the desired force on the load in certain directions while simultaneously tracking the desired position trajectory in the orthogonal directions. It must be noted that in this control strategy, slight fluctuations may be observed on the load force due to very small vibrations of the left arm under position control. The position-hybrid control strategy is now illustrated by the following example.

Let us consider the dual-arm manipulator discussed in Example 2 (with negligible friction) and suppose that the end effectors carry a rigid load in a horizontal plane, as shown in FIG. 7. Suppose that the load 610' as 65 shown, is initially parallel to the x-axis and we wish to make a parallel translation of the load 610' while the right arm, 660, applies a constant desired contact force on the load. This problem is similar to pulling out a drawer using two arms.

The x-component of the force on the load 610' can be written as

$$f_x = k_d(x_2 - x_1) - l_0$$

where \( (x_2 - x_1) \) is the distance between the end-effectors, \( l_0 \) is the length of the load, \( k_d \) is the stiffness of force/torque sensors mounted on the end-effectors to measure the force \( f_x \). The y-component of the force on the load is

$$f_y = \frac{m}{2} \ddot{y}$$

where the arms are assumed to share the load equally; i.e. each arm "sees" the load mass as \( m/2 \). In the simulation, we take \( k_d = 1000.0 \text{ N/m} \) and \( m = 1.0 \text{ Kg} \).

Suppose that the end-effector 605 of the left arm 600 is required to track the desired position trajectories

$$x_{r2}(t) = 0.432 \text{ meter}$$

$$y_{r2}(t) = 0.432[1 + 6e^{x-(t/0.3)} - 8e^{x-(t/0.4)}] \text{ meter}$$

so that the end-effector moves from the initial point \( (0.432, -0.432) \) to the final point \( (0.432, 0.432) \) on the vertical line \( x(t) = 0.432 \). The end-effector 665 of the right arm 660 is required to apply a specified force setpoint \( P_{sf} \) on the load 610' in the x direction and simultaneously track the desired position trajectory \( y_{r2}(t) \) in the y-direction, where

$$P_{sf} = 10 \text{ Newton}$$

$$y_{r2}(t) = 0.432[1 + 6e^{x-(t/0.3)} - 8e^{x-(t/0.4)}] \text{ meter}$$

The adaptive position control law for the left arm is given by

$$T(t) = J_2(\theta_2)(F_{r2}(t))$$

where $\theta_2$ is the joint angle vector, $T_2$ is the joint torque vector, and $J_2$ is the Jacobian matrix of the right arm with appropriate reordering of columns of $J_2$ if necessary.

The hybrid controller adaptation laws, Equations (103) through (108) and (123) through (126) are extremely simple, and therefore the control algorithm can be implemented using high sampling rates (rates \( \approx 1 \) kHz); yielding improved performance particularly in force control applications. Since in each sampling period (\( \approx 1 \) m sec) the terms in the robot models (121) and (128) cannot change noticeably, it is reasonable to assume that these terms are "slowly time-varying" compared to the adaptation scheme. Thus the inclusion of the disturbance $f_2$ and $f_3$ in the robot models (121) and (128) does not affect the controller performance.

It is concluded, therefore, that using the position-hybrid control strategy, the left end-effector will track the desired position trajectory despite the interaction forces through the load. The right end-effector will exert the desired force on the load in certain directions while simultaneously tracking the desired position trajectory in the orthogonal directions. It must be noted that in this control strategy, slight fluctuations may be observed on the load force due to very small vibrations of the left arm under position control. The position-hybrid control strategy is now illustrated by the following example.

Let us consider the dual-arm manipulator discussed in Example 2 (with negligible friction) and suppose that the end effectors carry a rigid load in a horizontal plane, as shown in FIG. 7. Suppose that the load 610' as 65 shown, is initially parallel to the x-axis and we wish to make a parallel translation of the load 610' while the right arm, 660, applies a constant desired contact force
Note that the initial values of the controller terms are set to zero. The integrals in the control law (131) are computed using a simple trapezoidal rule with $dt=0.5$ msec.

The adaptive hybrid position/force control law for the right arm is given by

$$
\vec{h}(t) = 0.5 \int_{0}^{t} \vec{h}(s) \vec{X}_{d}(s) \, ds
$$

where

$$
\vec{h}(t) = 20000E(t) + 2000E(t)
$$

The integrals in the above adaptation laws are evaluated using the trapezoidal rule with $dt=0.5$ msec.

To evaluate the performance of the position-hybrid control strategy, the nonlinear dynamic models of the arms (Equation 111) and the linear adaptive control laws (Equations 131 through 132) are simulated on a DEC-VAX 11/750 computer with the sampling period of 0.5 msec and the simulation results are shown in FIGS. 10(a) through 10(b). These Figures show that the end-effector coordinates $x_d(t)$ and $y_d(t)$ of the left arm track the desired position trajectories $x_d(t)$ and $y_d(t)$ very closely. FIGS. 10(c) through 10(d) indicate that the right end-effector exerts the desired force of 10 Newtons on the load in the $x$-direction and tracks the desired position trajectory $y_d(t)$ in the $y$-direction. Notice that the oscillations in the force response in FIG. 10(c) are due to the very small variations of the left end-effector $x$-coordinate as shown in FIG. 10(e). These oscillations can be reduced by increasing the gains of the position controller for the left arm. The paths traversed by the end-effectors in the horizontal plane are shown in FIG. 10(e). It is seen that the left end-effector tracks a vertical straight line, whereas the right end-effector moves in initially to produce the desired contact force and then tracks a vertical straight line. Thus, the adaptive position and hybrid controllers for each arm perform well in the dual-arm situation.

SECTION 3- HYBRID-HYBRID

In this section, the hybrid-hybrid control strategy for dual-arm manipulators will be studied in which both arms are in hybrid position/force control. In other words, in a common frame of reference for both arms, the forces exerted by the end-effectors on the load in the constraint directions $\{Z\}$ must be controlled; while simultaneously the end-effectors are required to track desired position trajectories in the free directions $\{Y\}$. Any unwanted forces and torques on the load generated by the relative position and orientation of the end-effectors will act as "disturbances" and the adaptive hybrid controllers ensure that the desired position/force trajectories are tracked despite such disturbances.

Following Section 2, for each manipulator arm the hybrid position/force control law in the joint space can be written as

$$
\bar{h}(t) = 0.5 \int_{0}^{t} \bar{h}(s) \bar{f}(s) \, ds
$$

where

$$
\bar{h}(t) = 2 \int_{0}^{t} \bar{h}(s) \bar{f}(s) \, ds
$$

and

$$
\bar{k}(t) = 2 \int_{0}^{t} \bar{k}(s) \bar{f}(s) \, ds
$$

are the "virtual" Cartesian forces applied to the end-effec-
tor in the constraint directions \( \{Z\} \) and free directions \( \{Y\} \), respectively. The force control law is given by

\[
F_d(t) = P_d(t) + d(t) + K_f(t) \int_0^t \dot{E}_f(t) dt + \int_0^t E_f(t) dt - K_d(t) \ddot{E}_d(t)
\]

(134)

where \( P_d(t) \) is the desired force setpoint, \( E_f(t) = P_d(t) - P_x(t) \) is the force tracking-error and the adaptation laws are:

\[
d(t) = \dot{d}(0) + \delta_1 \int_0^t \dot{q}(s) ds + \delta_2 q(t)
\]

\[
K_f(t) = K_f(0) + \alpha_1 \int_0^t \dot{q}(s) \dot{E}_f^2(s) ds + \alpha_2 q(t) \dot{E}_f^2(t)
\]

\[
K_d(t) = K_d(0) + \beta_1 \int_0^t \dot{q}(s) \dot{E}_d(t) + \beta_2 q(t) \dot{E}_d(t)
\]

where \( \dot{q} = W_p \dot{E}_f(t) + W_p \dot{E}_d(t) - W_p \dot{Z}_d(t) \)

and

\[
E_f(t) = \int_0^t E_f(s) ds \text{ and } \{W_p, W_d, W_d\}
\]

are desired weighting matrices.

The position control law is expressed as

\[
R(t) = R(0) + \bar{R}_d(t) \ddot{E}_d(t) + \bar{R}_d(t) \ddot{E}_d(t) + \bar{R}_d(t) \ddot{E}_d(t) + \bar{R}_d(t) \ddot{E}_d(t)
\]

where \( R(t) \) is the desired position trajectory, \( E_d(t) = R(t) - Y(t) \) is the position tracking-error and the adaptation laws are:

\[
\bar{R}_d(t) = \bar{R}_d(t) + \bar{R}_d(t) \int_0^t \dot{\nu}(s) dt + \bar{R}_d(t) \dot{\nu}(t)
\]

\[
\bar{R}_d(t) = \bar{R}_d(t) + \bar{R}_d(t) \int_0^t \dot{\nu}(s) E_d(t) dt + \bar{R}_d(t) \dot{\nu}(t)
\]

\[
\bar{R}_d(t) = \bar{R}_d(t) + \bar{R}_d(t) \int_0^t \dot{\nu}(s) \dot{E}_d(t) dt + \bar{R}_d(t) \dot{\nu}(t)
\]

\[
\bar{R}_d(t) = \bar{R}_d(t) + \bar{R}_d(t) \int_0^t \dot{\nu}(s) \dot{E}_d(t) dt + \bar{R}_d(t) \dot{\nu}(t)
\]

The adaptation laws for the force controller are:

\[
d(t) = q(t) + \int_0^t q(t) dt
\]

\[
K_f(t) = 100 + 10 \int_0^t \dot{q}(s) q^2(s) dt
\]

\[
K_d(t) = 100 + 4 \int_0^t \dot{q}(s) q(s) dt
\]

\[
K_d(t) = 5000 - 1000 \int_0^t \dot{q}(s) q(s) dt
\]

-continued

\[
\tau(t) = \tilde{W}_p \dot{E}_f(t) + \tilde{W}_p \dot{E}_d(t)
\]

and \( \{\tilde{W}_p, \tilde{W}_d\} \) are desired weighting matrices.

The above controller adaptation laws are extremely simple and therefore the hybrid control algorithm can be implemented using high sampling rates \( (\approx 1 \text{ KHz}) \); yielding improved performance. Under the adaptive hybrid controllers, both end-effectors are expected to exert the desired forces on the load while simultaneously moving on desired trajectories. The hybrid-hybrid control strategy is most available when simultaneous control of both position and force is required. The following example illustrates the hybrid-hybrid control strategy.

Consider the dual-arm manipulator and load, as discussed above in conjunction with FIG. 7, and suppose that both end-effectors are required to exert a constant desired force on the load in the x-direction while moving the load in the y-direction. In this situation, the x and y components of the load force are the same as developed in Section 2.

Suppose further that the desired force and position trajectories for both arms are specified as

\[
P_x(t) = \pm 10 \text{ Newton}
\]

\[
y(t) = 0.432[1 + 6 \exp(-y/0.3) - 8 \exp(-y/0.4)]
\]

so that the end-effectors move from the initial points \([0.432, -0.432], [2.432, -0.432]\) to the final points \([0.432, 0.432], [2.432, 0.432]\) while exerting a force of 10 NT on the load.

The two arms are controlled by identical and independent adaptive hybrid control laws given by:

\[
T(t) = J(t) \begin{pmatrix} F_d(t) \\ F_d(t) \end{pmatrix}
\]

(136)

where

\[
F_d(t) = P_d(t) + d(t) + K_f(t) e_f(t) + K_d(t) e_d(t) - K_d(t)
\]

\[
F_d(t) = \bar{d}(t) + \bar{R}_d(t) e_d(t) + \bar{R}_d(t) e_d(t) + \bar{R}_d(t) e_d(t)
\]

\[
\dot{e}_f(t) = P_{ef}(t) - P_{ef}(t) \text{ force tracking error}
\]

\[
\dot{e}_f(t) = y_{ef}(t) \text{ position tracking error}
\]

\[
\int_0^t e(t) dt = \text{ integral force error}
\]

The adaptation laws for the force controller are:

\[
d(t) = q(t) + \int_0^t q(t) dt
\]

\[
K_f(t) = 5000 - 1000 \int_0^t \dot{q}(s) q(s) dt
\]
The adaptation laws for the position controller are:

\[ \vec{T}(t) = 0.5 \vec{r}(t) + 0.5 \int_0^t \vec{r}(t') \, dt' \]

\[ \vec{R}(t) = 2 \int_0^t \vec{r}(t') \, dt' \]

\[ \vec{C}(t) = 0.5 \int_0^t \vec{r}(t') \, dt' \]

\[ \vec{A}(t) = 0.5 \int_0^t \vec{r}(t') \, dt' \]

where

\[ \vec{r}(t) = 5000 \vec{e}_x(t) + 2000 \vec{e}_y(t) \]

The integrals in the above adaptation laws are evaluated using the trapezoidal rule with \( dt = 0.5 \) msec.

To evaluate the performance of the hybrid-hybrid control strategy, the nonlinear dynamic models of the arms (Equation 111) and the linear adaptive control laws (Equation 136) are simulated on a DEC-VAX 11/750 computer with the sampling period of 0.5 msec and the simulation results are shown in FIGS. 11(a)-(d).

FIG. 11(a) shows that the force exerted by the end-effectors on the load in the x-direction is equal to the desired setpoint of 10 N. FIGS. 11(b) through 11(d) indicate that the coordinates of the end-effectors in the y-direction track the desired position trajectories. The paths traced by the end-effectors in the horizontal plane are shown in FIG. 11(d). It is seen that both end-effectors move in initially to produce the desired force and then track vertical straight lines. Thus the adaptive hybrid controllers perform extremely well for simultaneous force and position control.

Three adaptive control strategies for cooperative dual-arm robots have been described. In these strategies, each robot arm is considered a subsystem of the total system and is controlled independently using an adaptive controller in the low level of the control hierarchy. Each controller ensures that the controlled variables follow desired commands and reject unwanted cross-coupling effects from other subsystems which are treated as "disturbances." The subsystems are coordinated through trajectory generators in the intermediate level, where synchronous desired trajectories for both arms are specified in a common task-related frame of reference. An important feature of the present approach is that the overall control system for \( N \) cooperative arms is reduced to \( N \) decentralized independent single-arm controllers. The control schemes do not require communication and data exchange among individual controllers, which is an appealing feature from both computational and reliability points of view. Furthermore, available techniques for single-arm control can be utilized directly in multiple-arm environments.

The control strategies described herein do not require the knowledge of the load parameters such as mass and stiffness or the robot dynamic parameters such as link masses and inertias, and can therefore cope with uncertainties or variations in the system parameters. Furthermore, the complex dynamic model of the arms are not used in generating the control actions. The control schemes are very simple and extremely fast for on-line implementation with high sampling rates, yielding improved dynamic performance. The control methodology described herein can also be utilized in the coordinated control of \( N \)-arm robots when \( N \) exceeds two.

APPENDIX

In this Appendix, we obtain a simple expression for the force on a rigid load held by two end-effectors in a horizontal plane.

Let us consider a rigid load of length \( l_0 \) held firmly by two end-effectors equipped with force/torque sensors as shown in FIGS. 7 and 8. The sensors are modelled as linear springs with stiffness coefficients \( K_1 \) and \( K_2 \) and natural lengths \( l_1 \) and \( l_2 \). The forces \( F_1 \) and \( F_2 \) exerted by the sensors on the rigid load are given by

\[ F_1 = K_1(l_1 - l_0) \]

\[ F_2 = K_2(l_2 - l_0) \]

where \( l'_1 \) and \( l'_2 \) are instantaneous lengths of the springs. At equilibrium, the forces \( F_1 \) and \( F_2 \) must be equal; otherwise the load will reposition itself under the net force to reach the equilibrium condition. Let us denote the force exerted by the springs on the load by \( F = F_1 = F_2 \); hence

\[ F = F_1 = K_1(l_1 - l_0 + l_0 + l_2 - l_2 + l_2 - l_2) \]

From equation (139), we obtain

\[ F = K_1(l'_1 - l'_1 + l'_0 + l'_2 - l_2 + l_2 - l_2) = K_1(l'_1 - l'_0) \]

where

\[ L' = l'_1 + l'_0 + l'_2 = "natural" \] distance \( AB \) with no force on the load, i.e. \( AB_0 \)

Using \( F = F_2 = K_2(l_2 - l_0) \), equation (140) becomes

\[ F = K_1(l'_1 - L') - K_1 \left[ \frac{F}{K_2} \right] \]

\[ F = \frac{K_1 K_2}{K_1 + K_2} [l'_1 - L'] = K_{eq}[l'_1 - L'] = K_{eq}AB - (AB_0) \]

where \( K_{eq} \) is the equivalent stiffness coefficient of the springs obtained from

\[ \frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} \]

Equation (141) gives a single expression for the force on the load at any instant time.
The above description presents the best mode contemplated in carrying out the invention. The invention is, however, susceptible to modifications and alternate constructions from the embodiments shown in the drawings and described above. Consequently, it is not the intention to limit the invention to the particular embodiments disclosed. On the contrary, the invention is intended to and shall cover all modifications, sizes and alternate constructions falling within the spirit and scope of the invention, as expressed in the appended claims when read in light of the description and drawings.

What is claimed is:

1. In a hybrid controller adapted to issue control signals, each of which control a respective manipulator in a multi-manipulator system wherein each manipulator's position and/or force is applied in controlling a common load being acted upon in coordination by all of the manipulators of the system, the improvement comprising:

   means individually associated with each of said manipulators for independently coordinating its part in controlling said load, and in so doing, introducing into the other manipulators, through the load, a cross-coupled position and/or force term; and

   adaptive control means for generating a variable control signal applied to each of said manipulators, which variable control signal compensates in real-time for the inter-manipulator cross-coupling terms introduced into the other manipulators through said load as it is being acted upon by said multi-manipulator system.

2. The hybrid controller in accordance with claim 1 wherein the manipulators operate in an environment and such manipulators and the environment form a complex, dynamic model specified as having environmental and manipulator parameter values that are unknown, and the improvement is further characterized by:

   said variable control signal generating means include signal handling circuitry which is formulated and operable free from any knowledge of said model environmental or parameter values.

3. The hybrid controller in accordance with claim 2 wherein each manipulator exhibits a force response and includes a driving means that is responsive to a force command signal, and the improvement is further characterized by:

   said adaptive control means including at least one adaptive force controller for controlling each of said manipulator's driving means in the force that it applies to the load; and

   said adaptive force controller includes a realtime force signal generating means for emitting a signal that causes said manipulator to exhibit the desired force response that is indicated by said force command signal.

4. The hybrid controller in accordance with claim 3 wherein each manipulator exhibits a position response and includes a driving means that is responsive to a position command signal, and the improvement is further characterized by:

   at least one adaptive position controller for controlling each of said manipulator's driving means in the position that it applies to said load; and

   said adaptive position controller includes a real-time position signal generating means for emitting a signal that causes said manipulator to exhibit the desired position response that is indicated by said position command signal.

5. The hybrid controller in accordance with claim 4 and wherein the improvement is further characterized by:

   each manipulator's adaptive control means is independently operative from the adaptive control means which control the other manipulators; and

   means for issuing coordinated force and/or position commands to each of said independently operative adaptive control means.

6. The hybrid controller in accordance with claim 5 wherein said adaptive control means is further characterized as comprising:

   an adaptive force control circuit which includes a force signal feedback and feedforward generating means;

   an adaptive position control circuit which includes a position signal feedback and feedforward generating means; and

   means for connecting both said force and position signal generating means to said manipulator driving means in order to control said manipulator's position and/or force exerted on said load in accordance with said position and/or force commands.

7. A hybrid controller in accordance with claim 2 wherein said adaptive control means further comprises:

   means for deriving an auxiliary signal for controlling, at least in part, the position and dynamics of said manipulator; and

   means for generating an auxiliary signal in combination with command signals representing position and dynamics in said position-controlling adaptive feedforward and feedback control loops.

8. A hybrid controller in accordance with claim 1 and including a position and/or force command generator for emitting signals indicative of a desired position and/or force on said load and further wherein each manipulator's actual force/position applied to said load are sensed in order to derive position/force error terms that are thereafter respectively represented as error signals by the hybrid controller, and wherein the improvement is further characterized in that said adaptive control means comprises:

   a signal summing junction for forming said real-time force and/or position control signal(s);

   a feedforward command control loop for applying said desired force/position command signal(s) to said summing junction;

   an adaptive force/position control loop for receiving said desired force command signal; and

   adaptation control means in said adaptive force/position control loop responsive to said command signal(s) and also responsive to actual sensed error signal(s) for modifying the command(s) and error(s) signals and applying the so-modified signal(s) to said summing junction.

9. A hybrid controller in accordance with claim 8 and wherein said signal modifying means in said adaptive control means operates in accordance with a force control law, and said controller further comprises:

   a plurality of variable gain circuits for implementing a force control law characterized as:

$$ F_c(t) = F_d(t) + d(t) + K_d \int_0^t E(j)dt + K_p E(t) - K_v \dot{E}(t) $$
A hybrid controller in accordance with claim 9 and wherein said signal modifying means in said adaptive control means operates in accordance with a position control law, and said controller further comprises:

- A plurality of variable gain circuits for implementing a position control law characterized as:

$$F_d(t) = P \cdot d(t) + K_p \cdot E_p(t)$$

wherein the term $R(t)$ is a desired position trajectory signal term, $[K_p, E_p]$ and $[K_r + B_r + A_r]$ are terms contributed by the feedback and feedforward adaptive control loops.

10. A hybrid controller in accordance with claim 10 and including a position and/or force command generator for emitting signals indicative of a desired position and/or force on said load and further wherein each manipulator's actual force/position applied to said load are sensed in order to derive position/force error terms that are thereafter represented by error signals to be fed back over a feedback control loop to the hybrid controller, and wherein the improvement is further characterized in that said adaptive control means also comprises, in addition:

- A signal summing junction for forming said real time force and/or position control signal(s);
- A feedforward command control loop for applying said desired force/position control signal(s) to said additional summing junction;
- An adaptive control loop for receiving said desired force command signal; and
- Adaptation control means in said adaptive control loop responsive to said force command signal and also responsive to said actual sensed force error signal for modifying the command and error signals and applying the so-modified signal to said summing junction.

11. A hybrid controller in accordance with claim 11 and further comprising:

- A position/force trajectory generator emitting desired force/position trajectory signals including position, velocity and acceleration signals; and
- Means for applying said desired position, velocity and acceleration signals to said adaptive position feedforward and adaptive position feedback loops.

12. A hybrid controller in accordance with claim 12 and further comprising:

- A linear adaptive position control law represented as the output signal from said adaptive position controlling means.

13. A hybrid controller in accordance with claim 13 and wherein said improvement is further characterized by:

- Means for compensating for dynamic cross coupling which results from adaptively controlling both position and force of said load by said manipulators.

14. A hybrid controller in accordance with claim 14 and wherein the manipulator driving means is further characterized as including:

$$F_d(t) = P \cdot d(t) + K_p \cdot E_p(t) + \int_0^t E_i(t) + K_d E_d(t) - K_e \dot{E}_e(t)$$

wherein $P_i(t)$ is said desired force reference signal term, $d(t)$ is an auxiliary force signal term emitted by said adaptation control, and the $K_p, K_d$ and $K_e$ signal terms are varied by the gain control circuits for the PID controller circuits in said feedback force control circuit.
24. A hybrid controller in accordance with claim 1 wherein said positioning of said load is controlled by a control law characterized as:

\[ F(t) = f(t) + K_p E_p(t) + K_v E_v(t) + C(t)R(t) + B(t)\dot{R}(t) + A(t)T(t) \]

wherein the term \( R(t) \) is a desired position trajectory, \( [K_p E_p + K_v E_v] \) and \( [C + B + A] \) are terms contributed by feedback and feedforward adaptive position control loops; and wherein said force control law is characterized as:

\[ P_d(t) = P(t) + K_f d(t) + K_i \int \dot{P}(t) dt \]

wherein \( P_d(t) \) is said desired force reference signal term, \( d(t) \) is an auxiliary force signal term emitted by said adaptation control, and the \( K_f, K_i \) are variable gain terms; \( E \) is an error term; and \( Z \) is a velocity term.

25. A method of cooperative dual-arm robots together manipulating, via end-effectors, a common load by a position-position and position-hybrid control strategy, comprising the steps of:

- controlling the robots by adaptive controllers;
- commanding both arms of one robot to track desired trajectories in Cartesian space despite unknown time-varying interaction forces exerted through the load;
- adaptively controlling, for said robot, one arm's end-effector motions in free directions and applied forces in the constraint directions in Cartesian space, while adaptively controlling the other arm to ensure that its end-effector tracks desired position trajectories in said space.

26. A method of control in accordance with claim 25 wherein said adaptive control includes coordinating force/position control over said arms at a command level.

27. A method of control of cooperative dual-arm robots together manipulating, via end-effectors, a common load by a position-position control strategy, comprising the steps of:

- controlling the robots by adaptive controllers; and, for said position-position control strategy, commanding both arms to track desired trajectories in Cartesian space despite unknown time-varying interaction forces exerted through the load, and further comprising the steps of additionally including a position-hybrid control strategy, wherein the step of adaptive control further includes:

- adaptively controlling one arm's end-effector motions in free directions and applied forces in the constraint directions in Cartesian space, while adaptively controlling the other arm to ensure that its end-effector tracks desired position trajectories in said space.

28. A method of control in accordance with claim 27 and additionally including a hybrid-hybrid control strategy, wherein the step of adaptive control further includes:

- ensuring that both controller's end-effectors track reference position trajectories; while, simultaneously applying desired forces on the common load being acted upon by the dual-arm robots.