Anticipating Cycle 24 Minimum and Its Consequences

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November 2007
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## TABLE OF CONTENTS

1. INTRODUCTION .......................................................................................................................... 1

2. RESULTS AND DISCUSSION ....................................................................................................... 2
   2.1. Overview of Cycle 23’s Key Characteristics ................................................................. 2
   2.2. Cyclic Trends ..................................................................................................................... 7
   2.3. Correlative Results ........................................................................................................... 21

3. SUMMARY .................................................................................................................................... 37

REFERENCES ................................................................................................................................... 38
LIST OF FIGURES

1. Key characteristics of cycle 23 and initial markers for cycle 24 ......................................... 3
2. Superposed epoch analyses of $R$, LAT and HLS for the 30 mo preceding the epoch of sunspot minimum using 12-mo moving averages ........................................... 5
3. Superposed epoch analysis of NSD for the 30 mo preceding the epoch of sunspot minimum using observed monthly counts ....................................................... 6
4. Cyclic variation of NSD$_{f\rightarrow Rm}$ for cycles 11–23 .......................................................... 8
5. Cyclic variation of NSD$_{Rm\rightarrow l}$ for cycles 11–23 ........................................................... 9
6. Cyclic variation of NSD for cycles 11–23 ........................................................................ 10
7. Cyclic variation of $t_{f\rightarrow l}$ for cycles 11–23 ................................................................ 11
8. Cyclic variation of $t_{Rm\rightarrow l}$ for cycles 11–23 ................................................................. 12
9. Cyclic variation of $t_{l\rightarrow Rm}$ for cycles 11–23 ............................................................... 13
10. Cyclic variation of NSD$_1$ for cycles 11–23 .................................................................... 14
11. Cyclic variation of NSD$_{10}$ for cycles 11–23 ............................................................... 15
12. Cyclic variation of NSD$_{20}$ for cycles 11–23 .................................................................. 16
13. Cyclic variation of $t_{LATm\rightarrow Rm}$ and $t_{HLSm\rightarrow Rm}$ for cycles 11–23 ......................... 17
14. Cyclic variation of $t_1$, $t_{10}$, and $t_{20}$ for cycles 11–23 ............................................... 18
15. Cyclic variation of $Rm$ and $RM$ for cycles 11–23 ......................................................... 19
17. Scatter plots of $Rm(n+1)$ and $RM(n+1)$ versus NSD$_1(n)$ ............................................. 22
18. Scatter plots of $Rm(n+1)$ and RM($n+1$) versus NSD$_{10}(n)$ ........................................... 23
19. Scatter plots of Rm($n+1$) and RM($n+1$) versus NSD$_{20}(n)$ ........................................... 24
LIST OF FIGURES (Continued)

20. Scatter plot of PER versus NSD \(_1\) ................................................................. 25
21. Scatter plot of PER versus NSD \(_{10}\) ................................................................. 26
22. Scatter plot of PER versus NSD \(_{20}\) ................................................................. 27
23. Scatter plot of \(t_1(n+1)\) versus NSD \(_1(n)\) ......................................................... 28
24. Scatter plot of \(t_{10}(n+1)\) versus NSD \(_{10}(n)\) .................................................... 29
25. Scatter plot of \(t_{20}(n+1)\) versus NSD \(_{20}(n)\) ..................................................... 30
26. Scatter plots of \(Rm\) and \(RM\) versus \(t_1\) ............................................................. 31
27. Scatter plots of \(Rm\) and \(RM\) versus \(t_{10}\) ........................................................... 32
28. Scatter plots of \(Rm\) and \(RM\) versus \(t_{20}\). The large dot at (5,3.4) means that two cycles had the same values (cycles 17 and 19) ......................................................... 33
29. Scatter plots of \(RM\) and DES versus ASC .......................................................... 34
30. Scatter plot of \(RM(n+1)\) versus PER\((n)\) ............................................................ 35
# LIST OF ACRONYMS AND ABBREVIATIONS

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASC</td>
<td>ascent duration (the elapsed time in months from cycle minimum to cycle maximum using smoothed monthly mean sunspot number)</td>
</tr>
<tr>
<td>cl</td>
<td>confidence level</td>
</tr>
<tr>
<td>DES</td>
<td>descent duration (the elapsed time in months from cycle maximum to subsequent cycle minimum using smoothed monthly mean sunspot number)</td>
</tr>
<tr>
<td>E(FSD)</td>
<td>epoch of first spotless day</td>
</tr>
<tr>
<td>E(HLSm)</td>
<td>epoch of minimum value of HLS</td>
</tr>
<tr>
<td>E(HLSM)</td>
<td>epoch of maximum value of HLS</td>
</tr>
<tr>
<td>E(LATm)</td>
<td>epoch of minimum value of LAT</td>
</tr>
<tr>
<td>E(LATM)</td>
<td>epoch of maximum value of LAT</td>
</tr>
<tr>
<td>E(LSD)</td>
<td>epoch of last spotless day</td>
</tr>
<tr>
<td>E(NSD_{10f})</td>
<td>epoch of first occurrence of 10 or more spotless days</td>
</tr>
<tr>
<td>E(NSD_{20f})</td>
<td>epoch of first occurrence of 20 or more spotless days</td>
</tr>
<tr>
<td>E(Rm)</td>
<td>epoch of sunspot minimum (cycle minimum)</td>
</tr>
<tr>
<td>E(RM)</td>
<td>epoch of sunspot maximum (cycle maximum)</td>
</tr>
<tr>
<td>HLS</td>
<td>highest latitude spot</td>
</tr>
<tr>
<td>LAT</td>
<td>weighted (by area) mean latitude of spots</td>
</tr>
<tr>
<td>LO</td>
<td>lower observed</td>
</tr>
<tr>
<td>n</td>
<td>cycle number</td>
</tr>
<tr>
<td>NSD</td>
<td>number of spotless days</td>
</tr>
<tr>
<td>NSD_1</td>
<td>elapsed time in months between first occurrences of spotless days for 2 consecutive sunspot cycles</td>
</tr>
<tr>
<td>NSD_{10}</td>
<td>elapsed time in months between first occurrences of 10 or more spotless days for 2 consecutive sunspot cycles</td>
</tr>
<tr>
<td>NSD_{20}</td>
<td>elapsed time in months between first occurrences of 20 or more spotless days for 2 consecutive sunspot cycles</td>
</tr>
</tbody>
</table>
LIST OF ACRONYMS AND ABBREVIATIONS (Continued)

NSD$_{f\rightarrow Rm}$ number of spotless days between first occurrence of a spotless day and the epoch of sunspot minimum

NSD$_{Rm\rightarrow L}$ number of spotless days between the epoch of sunspot minimum and the last occurrence of a spotless day

P probability, a departure from independence (chance) (based on Fisher’s exact test for 2×2 contingency tables)

PER period (= ASC + DES)

PI prediction interval

$r$ coefficient of correlation

$r^2$ coefficient of determination

$R$ smoothed monthly mean sunspot number (the 12-mo moving average of monthly mean sunspot number)

$Rm$ minimum value of $R$ (minimum amplitude)

$RM$ maximum value of $R$ (maximum amplitude)

$sd$ standard deviation

$se$ standard error of estimate

$t_1$ elapsed time in months from first occurrence of a spotless day and the epoch of sunspot minimum

$t_{10}$ elapsed time in months from first occurrence of 10 or more spotless days and the epoch of sunspot minimum

$t_{20}$ elapsed time in months from first occurrence of 20 or more spotless days and the epoch of sunspot minimum

$t_{f\rightarrow L}$ elapsed time in months from first occurrence of a spotless day and the last occurrence of a spotless day

$t_{Rm\rightarrow L}$ elapsed time in months from the epoch of sunspot minimum and the last occurrence of a spotless day

$t_{L\rightarrow RM}$ elapsed time in months from last occurrence of a spotless day and the epoch of sunspot maximum

$t_{LATm\rightarrow Rm}$ elapsed time in months from the minimum value of LAT to the epoch of sunspot minimum
LIST OF ACRONYMS AND ABBREVIATIONS (Continued)

$t_{HLS\rightarrow Rm}$  elapsed time in months from the minimum value of HLS to the epoch of sunspot minimum

UO  upper observed

$x$  independent variable

$y$  regression equation

$y_{LP}$  regression equation for long-period cycles

$y_{SP}$  regression equation for short-period cycles

$\Sigma_{NSD}$  sum of spotless days from first occurrence to last occurrence.
ANTICIPATING CYCLE 24 MINIMUM AND ITS CONSEQUENCES

1. INTRODUCTION

Especially important for the prediction of a sunspot cycle is knowing the date of occurrence and size of its sunspot minimum.\textsuperscript{1-11} Conventionally, sunspot cycles are described using the 12-mo moving average of monthly mean sunspot number ($R$), also called smoothed monthly mean sunspot number, as reported by the Solar Influences Data Analysis Center of the Royal Observatory of Belgium at <http://sidc.oma.be/index.php3>. The minimum value of $R$ is called sunspot minimum or minimum amplitude ($R_m$), and its occurrence date represents the epoch of sunspot minimum ($E(Rm)$). Similarly, the largest value of $R$ is called sunspot maximum or maximum amplitude ($R_M$), and its occurrence date represents the epoch of sunspot maximum ($E(RM)$). The elapsed time between $E(Rm)$ and $E(RM)$ is called the ascent duration (ASC), and the elapsed time between two consecutive sunspot cycle minimums is called the cycle duration or period (PER). Strictly speaking, sunspot minimum and maximum are better described as extended periods of about 1–3 yr in length, rather than specific instances in time, where sunspot activity is predominantly low or high, respectively.

In addition to sunspot number, three other parameters useful for the determination of sunspot minimum include the number of spotless days (NSD), the 12-mo moving averages of the monthly weighted mean latitude (weighed by spot area) of the sunspots (LAT), and the monthly mean highest latitude spot (HLS). Near sunspot minimum, NSD increases dramatically reaching a peak near $E(Rm)$.\textsuperscript{12-13} Also, in the vicinity of $E(Rm)$, there is a transition from old cycle spots of lower latitude to new cycle spots of higher latitude, typically $\geq25$ deg, thereby causing both LAT and HLS to increase from lower values to higher values, usually several months in advance of cycle minimum (see <http://solarscience.msfc.nasa.gov/greenwch.shtml>).

In this Technical Publication, examination of the behaviors of the number of spotless days (through May 2007) and the 12-mo moving averages (through November 2006) of $R$, LAT, and HLS is accomplished to ascertain the nearness of conventional sunspot minimum for cycle 24, the next sunspot cycle, and consequences of its occurrence.
2. RESULTS AND DISCUSSION

2.1. Overview of Cycle 23’s Key Characteristics

Figure 1 displays $R$ (the lower panel), the NSD (middle panel), and the LAT and HLS (upper panel) for the interval January 1994 through November 2006. A number of important descriptors for the continuing cycle 23 and the initial stages of cycle 24 are identified. Examples are as follows:

- $E(Rm)_{23}$ denotes the conventional occurrence date of sunspot minimum for cycle 23, which was May 1996.

- $E(RM)_{23}$ denotes the conventional occurrence date of sunspot maximum for cycle 23, which was April 2000.

- $Rm(23)$ is the minimum amplitude of cycle 23, which measured 8.0.

- $RM(23)$ is the maximum amplitude of cycle 23, which measured 120.8.

- ASC(23) is the ascent duration in months for cycle 23, which measured 47 mo.

- PER(23) is the minimum-to-minimum duration in months, which presently is unknown, yet will measure 126 or more mo.

- $R$ measured 12.7 in November 2006.

- $E(FSD)_{23}$ is the first occurrence date of a spotless day during the decline of cycle 22, which occurred in April 1994.

- $E(NSD_{10f})_{23}$ is the first occurrence of 10 or more spotless days during the decline of cycle 22, which occurred in April 1995.

- $E(NSD_{20f})_{23}$ is the first occurrence of 20 or more spotless days during the decline of cycle 22, which occurred in September 1996.

- $E(LSD)_{23}$ is the last occurrence date of a spotless day following sunspot minimum for cycle 23, which occurred in January 1998.

- $E(FSD)_{24}$ is the first occurrence date of a spotless day during the decline of cycle 23, which occurred in January 2004.

- $E(NSD_{10f})_{24}$ is the first occurrence date of 10 or more spotless days during the decline of cycle 23, which occurred in February 2006.
Figure 1. Key characteristics of cycle 23 and initial markers for cycle 24.
• $E(\text{NSD}_{20f})_{24}$ is the first occurrence date of 20 or more spotless days during the decline of cycle 23, which occurred in April 2007.

• $\Sigma\text{NSD}$ is the sum of spotless days from first occurrence to last occurrence, which numbered 303 for cycle 23 and 121, so far (through May 2007), for cycle 24.

• $E(\text{LAT}m)_{23}$ is the occurrence date for the minimum value of the 12-mo moving average of LAT for cycle 23 during the decline of cycle 22, which occurred in December 1995 and measured 7.8 deg.

• $E(\text{HLS}m)_{23}$ is the occurrence date for the minimum value of the 12-mo moving average of HLS for cycle 23 during the decline of cycle 22, which occurred in October 1995 and measured 16.1 deg.

• $E(\text{LAT}M)_{23}$ is the occurrence date for the maximum value of the 12-mo moving average of LAT during the rise of cycle 23, which occurred in February 1998 and measured 23.1 deg.

• $E(\text{HLS}M)_{23}$ is the occurrence date for the maximum value of the 12-mo moving average of HLS during the rise of cycle 23, which occurred in December 1999 and measured 38.4 deg.

Also given are the November 2006 values of LAT and HLS, which are 7.9 and 14.6 deg, respectively, and $\text{NSD}_{1}$, $\text{NSD}_{10}$, and $\text{NSD}_{20}$, which represent the elapsed times in months between consecutive occurrences of FSD, $\text{NSD}_{10f}$ and $\text{NSD}_{20f}$ respectively, and measure 117, 130, and 127 mo, respectively.

Inspection of figure 1 clearly shows that cycle 23 was double-peaked, with the first peak being the largest. Also noticeable are the characteristic dips and steep rises prior to $E(\text{Rm})$ of LAT and HLS. Additionally, the current value of $R$ (November 2006) is near the upper observed value of $Rm$ for the most recent cycles 16–23, identified in the lower right-hand corner of figure 1, and, in fact, is nearly equal in value to that observed about 7 mo prior to $E(\text{Rm})_{23}$. Similarly, the November 2006 value of LAT is of equivalent value to that observed about 4–6 mo prior to $E(\text{Rm})_{23}$, although the November 2006 value of HLS is below the lowest value observed in cycle 23 (14.6 deg as compared to 16.1 deg).

Figure 2 (left panel) shows the mean value of the 12-mo moving averages of $R$ and the envelope of observed values, where UO is the upper observed value and LO is the lower observed value (true for all subpanels) based on the most recent cycles 16–23 for elapsed time in months from 30 mo prior to $E(\text{Rm})$ to $E(\text{Rm})$. The November 2006 value for $R$ equals 12.7 and suggests that $E(\text{Rm})_{24}$ should be expected within the next 1–18 mo from November 2006, based on the envelope, or about 8 mo from November 2006, based on the mean of the most recent cycles 16–23, which, if true, suggests that $E(\text{Rm})_{24}$ should occur about July 2007 and that cycle 23 will be classified as a fast-riser long-period cycle (like cycles 11 and 13).

Figure 2 (lower right panel) shows the mean value of the 12-mo moving average of LAT and its envelope, which suggests that once a significant upturn takes place (due to the presence of high-latitude new cycle spots), $E(\text{Rm})_{24}$ should follow within 6–11 mo. Similarly, figure 2 (upper right panel) shows the mean value of the 12-mo moving average of HLS and its envelope, which suggests that once
Figure 2. Superposed epoch analyses of $R$, LAT and HLS for the 30 mo preceding the epoch of sunspot minimum using 12-mo moving averages.

As yet, LAT and HLS have shown no significant upturns, so $E(Rm)_{24}$ very likely will fall some time in 2007 or later. It should be noted that the November 2006 values for LAT and HLS are below the means of cycles 16–23, and, when compared to the LO envelopes, it appears likely that $E(Rm)_{24}$ may follow November 2006 by about a year or more.

Figure 3 displays the behavior of NSD relative to $E(Rm)$. Recall that the first occurrence of 10 or more spotless days during the decline of cycle 23 occurred in February 2006. Compared to the UO envelope, NSD is found to first equal 10 mo or about 27 mo before $E(Rm)$. Likewise, recall that the first occurrence of 20 or more spotless days during the decline of cycle 23 occurred in April 2007. Compared to the UO envelope, NSD is found to first equal 20 or more about 8 mo before $E(Rm)$. Together, these findings suggest that $E(Rm)_{24}$ probably should not be expected before December 2007.
Figure 3. Superposed epoch analysis of NSD for the 30 mo preceding the epoch of sunspot minimum using observed monthly counts.

Although not shown, the 12-mo moving average of the number of groups measures 1.39 in November 2006. For the most recent cycles 16–23, this number averages about 0.79 at $E(Rm)$, having a range of 0.36 (cycle 19) to 1.13 (cycle 21). Compared to the mean of the most recent cycles 16–23, the November value of the 12-mo moving average of the number of groups is equivalent to that seen about 10 mo prior to $E(Rm)$, suggesting cycle minimum for cycle 24 about September 2007.
2.2 Cyclic Trends

Figure 4 displays the cyclic variation of the number of spotless days between $E(FSD)$ and $E(Rm)$ ($NSD_{f→Rm}$), for cycles 11–23. A strong downward trend is noticeable, especially between cycles 12 and 18, with a flattening occurring between cycles 18 and 23 (alternatively, there may exist an inherent behavioral difference between cycles 12–15 and cycles 11 and 16–23). Cycles 16–23 (also true for cycle 11) have all had $NSD_{f→Rm}$ measuring 316 or less (range 114–316). The inferred trend suggests that cycle 24 will have fewer than 383 spotless days between its $E(FSD)$ and $E(Rm)$, with 95-percent probability. Through May 2007, there have been 121 spotless days, which already is more than was seen for cycles 18 and 20 and nearly equal to that seen in cycle 23 (134 spotless days).

In figure 4, the parametric median value (the thin horizontal line), the inferred regression ($\gamma$), the linear correlation coefficient ($r$), the coefficient of determination ($r^2$) (a measure of the amount of variance explained by the inferred regression), the standard error of estimate ($se$), the confidence level ($cl$) (a measure of the statistical importance of the inferred regression, with $cl \geq 95$-percent indicating a statistically significant result and $\geq 90$-percent indicating a marginally significant result), and the 90-percent prediction interval (PI) (based on the given sample size) are given. This same format is used in all subsequent parametric cyclic variations and in the scatter plots section.

Figure 5 depicts the cyclic variation of the number of spotless days between $E(Rm)$ and $E(LSD)$ ($NSD_{Rm→L}$), for cycles 11–23. Again, a strong downward trend is noticeable. The inferred trend suggests that cycle 24 will have fewer than 225 spotless days between its $E(Rm)$ and $E(LSD)$, with 95-percent probability.

Figure 6 shows the cyclic variation of NSD (from $E(FSD)$ to $E(LSD)$) for cycles 11–23. Again, a strong downward trend is apparent, which suggests that cycle 24 will have fewer than 560 spotless days, in total, with 95-percent probability.

Figure 7 displays the cyclic variation of the elapsed time in months from $E(FSD)$ to $E(LSD)$ ($t_{f→L}$), for cycles 11–23. Plainly, with the passage of time, the elapsed time between first and last occurrences of spotless days has decreased by more than 50 mo, from an average of 107 mo for cycles 11–15 to 53 mo for cycles 16–23. The inferred trend suggests that $t_{f→L}$ for cycle 24 will be shorter than 62 mo, with 95-percent probability. Through May 2007, the elapsed time since $E(FSD)$ is 40 mo, suggesting that fewer than 22 mo remain before $E(LSD)$ should occur (fewer than 18 mo, when compared to the median).

Figure 8 shows the cyclic variation of the elapsed time in months from $E(Rm)$ to $E(LSD)$ ($t_{Rm→L}$), for cycles 11–23. A strong downward trend is apparent, which suggests that once $E(Rm)$ occurs for cycle 24, $E(LSD)$ will follow within 28 mo, with 95-percent probability.

Figure 9 displays the cyclic variation of the elapsed time in months from $E(LSD)$ to $E(RM)$ ($t_{L→RM}$), for cycles 11–23. A strong upward trend is apparent, which suggests that once $E(LSD)$ occurs for cycle 24, $E(RM)$ will follow within 42 mo, with 95-percent probability. (Note: It could follow in as few as 20 mo.)
Note: Thru May 2007, ΣNSD = 121

\[ y = 1,038.621 - 40.951x \]
\[ r = -0.675, r^2 = 0.456 \]
\[ se = 181.956, cl > 98\% \]

⇒NSD_{f→Rm} (24) = 56 ± 327 (90% PI)

Figure 4. Cyclic variation of NSD_{f→Rm} for cycles 11–23.
\[ y = 401.819 - 12.170x \]
\[ r = -0.612, r^2 = 0.375 \]
\[ se = 63.882, cl > 95\% \]

\( \Rightarrow \text{NSD}_{Rm \rightarrow l}(24) = 110 \pm 115 \) (90\% PI)

**Figure 5.** Cyclic variation of \( \text{NSD}_{Rm \rightarrow l} \) for cycles 11–23.

Figure 10 depicts the cyclic variation of the elapsed time in months between consecutive occurrences of \( E(\text{FSD}) \) (denoted here as \( \text{NSD}_1 \)), for cycles 11–23. Because the inferred trend is not statistically important, use of the mean and standard deviation (sd) is preferred. Hence, \( \text{NSD}_1(24) \) should be about 131 mo
Figure 6. Cyclic variation of NSD for cycles 11–23.

(±23 mo), inferring that \( E(\text{FSD})_{25} \) should not be expected until about 2015 (±2 yr). However, it seems noteworthy that six of the past eight cycles have had NSD\(_1\) equal to 131 or fewer mo (range 109–131 mo), suggestive that \( E(\text{FSD})_{25} \) might more likely be expected about 2013–2015.

Figure 11 shows the cyclic variation of the elapsed time in months between consecutive occurrences of \( E(\text{NSD}_{10}) \) (denoted here as NSD\(_{10}\)), for cycles 11–23. Again, the inferred trend is not statistically important. Based on the mean and \( sd \), NSD\(_{10}(24)\) should be about 130 mo (±22 mo), inferring that \( E(\text{NSD}_{10})_{25} \) will not occur until about 2017 (±2 yr). As before, it seems noteworthy that six of the past eight cycles have had NSD\(_{10}\) equal to 130 or fewer mo (range 113–130 mo), suggestive that \( E(\text{NSD}_{10})_{25} \) might more likely be expected about 2015–2017.

Figure 12 displays the cyclic variation of the elapsed time in months between consecutive occurrences of \( E(\text{NSD}_{20}) \) (denoted here as NSD\(_{20}\)) for cycles 11–23. As with NSD\(_1\) and NSD\(_{10}\), the inferred trend is not statistically important. Based on the mean and \( sd \), NSD\(_{20}(24)\) should be about 128 mo (±22 mo),
inferring that $E(\text{NSD}_{20f})_{25}$ will not occur until about 2019 ($\pm 2$ yr). Also, since six of the past eight cycles have had $\text{NSD}_{20}$ equal to 128 or fewer mo (seven of eight have had $\text{NSD}_{20}$ equal to 129 or fewer mo), $E(\text{NSD}_{20f})_{25}$ seems more likely to be expected about 2017–2019.

Figure 13 (lower panel) shows the cyclic variation of the actual elapsed times in months between $E(\text{LAT}m)$ and $E(Rm)$ ($t_{\text{LAT}m\rightarrow Rm}$) for cycles 12–23, and figure 13 (upper panel) shows the cyclic variation of the actual elapsed time in months between $E(\text{HLS}m)$ and $E(Rm)$ ($t_{\text{HLS}m\rightarrow Rm}$) for cycles 12–23, in contrast to that shown in figure 2 based on superposed epoch analyses. No significant trend is found for
\[
y = 66.797 - 2.368x \\
r = -0.702, r^2 = 0.492 \\
se = 9.778, cl > 99.\%
\]

\[\Rightarrow t_{Rm\rightarrow L}(24) = 10 \pm 18 \text{ months (90\% PI)}\]

Figure 8. Cyclic variation of \(t_{Rm\rightarrow L}\) for cycles 11–23.

either parameter. Based on the means and \(sds\), the elapsed time between \(E(LATm)\) and \(E(Rm)\) for cycle 24 should be about 15 mo (±17 mo), and the elapsed time between \(E(HLSm)\) and \(E(Rm)\) for cycle 24 should also be about 15 mo (±12 mo). For the former, four of the past eight cycles had an elapsed time of only 2–6 mo, while, for the latter, five of the past eight cycles had an elapsed time of 16–23 mo (three of the past eight cycles had an elapsed time of 7–10 mo). Thus, the determination of \(E(LATm)\) and \(E(HLSm)\) does not in itself allow for the reliable prediction of \(E(Rm)\). Rather, the sharpness of the increases seems to more strongly suggest the closeness of \(E(Rm)\).
Figure 9. Cyclic variation of $t_{L\rightarrow RM}$ for cycles 11–23.

Figure 14 depicts the cyclic variation for cycles 11–23 of the elapsed times in months between $E(FSD)$ and $E(Rm)$ ($t_1$) (bottom panel), between $E(NSD_{10})$ and $E(Rm)$ ($t_{10}$) (middle panel), and between...
$y = 157.648 - 1.549x$

$r = -0.468, r^2 = 0.219$

$se = 11.861, c > 90%$

$\Rightarrow NSD_1(24) = 120 \pm 21$ months (90% PI)

mean = 131 months

sd = 13 months

$\Rightarrow NSD_1(24) = 131 \pm 23$ months

---

**Figure 10.** Cyclic variation of NSD$_1$ for cycles 11–23.

$E(\text{NSD}_{20f})$ and $E(\text{Rm})$ ($t_{20}$) (top panel). For all parameters, a strong downward trend is observed, although the one for $t_1$ could alternatively be interpreted as an inherent change between cycles 11–15 and 16–23. The value of $t_1$ for cycle 24 should be less than 41 mo (with 95-percent probability). Through November 2006, it has already been 34 mo since $E(\text{FSD})$, indicating that fewer than 7 mo (before July 2007) should remain before $E(\text{Rm})$ unless, of course, cycle 24 is more like cycles 11–15 than cycles 16–23.
The value of $t_{10}$ should be less than 26 mo (with 95-percent probability). Through November 2006, it has already been 9 mo since $E(\text{NSD}_{10})$, indicating that fewer than 17 mo (before April 2008) should remain before $E(Rm)$.

The value of $t_{20}$ should be less than 15 mo (with 95-percent probability). Through November 2006, it has a value of −5 mo (the first occurrence of 20 or more spotless days was April 2007), indicating that fewer than 14 mo (before July 2008) should remain before $E(Rm)$. However, because eight of the past eight cycles had $t_{20}$ equal to −2 to 4 mo, $E(Rm)$ might more likely be expected sometime between February 2007 and August 2007.

Figure 15 displays the cyclic variation of $Rm$ (lower panel) and $RM$ (upper panel) for cycles 11–23. For $Rm$, the trend is upward and is statistically very important; while for $RM$, the trend is upward but only of marginal statistical significance. $Rm$ for cycle 24 should measure about $10.8 \pm 4.7$, inferring that there is a 95-percent probability that $Rm$ will be larger than 6.1 (or smaller than 15.5). Recall that $R$ equals 12.7 in November 2006, so $R$ already is within the 90-percent prediction interval expected for cycle 24’s $Rm$. 

Figure 11. Cyclic variation of $\text{NSD}_{10}$ for cycles 11–23.
On the basis of the inferred marginally significant regression, $RM$ for cycle 24 is expected to measure about $160.3 \pm 64.3$, inferring that it will be larger than about 96.0. Instead, using the mean ($=121.3$) and $sd$ ($=40.6$) for cycles 11–23, $RM$ for cycle 24 should be larger than 48.4. It should be noted that six of the past eight cycles have had $RM$ equal to or larger than 119.2 (the median) and seven of eight have had $RM$ equal to or larger than 110.6.

Figure 12. Cyclic variation of NSD$_{20}$ for cycles 11–23.
Figure 13. Cyclic variation of $t_{LATm\rightarrow Rm}$ and $t_{HLSm\rightarrow Rm}$ for cycles 11–23.
Figure 14. Cyclic variation of $t_1$, $t_{10}$, and $t_{20}$ for cycles 11–23.
Figure 15. Cyclic variation of $Rm$ and $RM$ for cycles 11–23.

Figure 16 shows the cyclic variation of ASC (lower panel) and PER (upper panel) for cycles 11–23. For ASC, no preferred trend is discerned. Hence, ASC for cycle 24 should measure about $46 \pm 13$ mo (the 90-percent prediction interval), although seven of the past eight cycles have had ASC
Figure 16. Cyclic variation of ASC and PER for cycles 11–23. Notice the period “gap” between 127–134 mo.
equal to 49 or fewer mo (range 34–49 mo). For PER, while no preferred linear trend is discerned, on the basis of 12-mo moving averages, cycle periods appear to be distributed into a long-period cycle group (PER ≥135 mo) and a short-period cycle group (PER ≤126 mo) with seven of the past eight fully known cycles being cycles of shorter periods. Because cycle 23 already has persisted for at least 126 mo (through November 2006) and no upturn in LAT or HLS has been seen (fig. 1), it appears certain that PER for cycle 23 either will fall within the 8-mo period gap, a first for a sunspot cycle, or within the long-period group, indicating $E(Rm)$ on or after August 2007.

### 2.3 Correlative Results

Figure 17 displays the scatter plots of $Rm$ (lower panel) and $RM$ (upper panel) for the following cycle versus NSD$_1$ for the current cycle. Of the two plots, only the latter is statistically significant. The value of NSD$_1$ for cycle 23 is shown as the downward pointed arrow (=117 mo). Such a value suggests that $RM(24)$ will equal about $150\pm 64$ (the 90-percent prediction interval), or, more likely, ≥114.9, based on the 2×2-contingency table (2:4:1:5).

The parametric medians (the thin horizontal and vertical lines), the probability of obtaining the observed result or one more suggestive of a departure from independence (chance) (P) based on Fisher’s exact test for 2×2-contingency tables, the inferred linear regression, and the value from cycle 23 (shown as the downward pointing arrow), are also shown in figure 17. This same format follows for all scatter plots.

Figure 18 similarly depicts the scatter plots of $Rm$ (lower panel) and $RM$ (upper panel) for the following cycle versus NSD$_{10}$ for the current cycle. While neither plot is statistically significant based on linear correlation analysis, the latter one is statistically significant based on Fisher’s exact test of the 2×2-contingency table (1:5:1:5), inferring the probability of obtaining the observed result, or one more suggestive of P, to be equal to 4 percent. Cycle 23’s NSD$_{10}$ (=130 mo) suggests that $RM(24)$ will lie in the upper-left quadrant (≥114.9).

Figure 19 shows the scatter plots of $Rm$ (lower panel) and $RM$ (upper panel) for the following cycle versus NSD$_{20}$ for the current cycle. Unfortunately, NSD$_{20}$ does not seem to be a reliable predictor for $Rm$ or $RM$ of the following cycle.

Figures 20–22 display the scatter plots of PER versus NSD$_1$, NSD$_{10}$, and NSD$_{20}$, respectively, for the same cycle. Inspection reveals that none of the plots is statistically significant. Hence, the elapsed time between first occurrences of spotless days of two consecutive cycles (or of 10 or more, or 20 or more) does not provide a reliable prediction for the cycle duration of the ongoing cycle. (For NSD$_1$, it is nearly marginally significant, suggesting that cycle 23 might have been expected to be a cycle of shorter period, rather than one of longer period.)

Figure 23 depicts the scatter plot of $t_1$ for the following cycle versus NSD$_1$ for the current cycle. Similarly, figures 24 and 25 depict the scatter plots of $t_{10}$ and $t_{20}$ for the following cycle versus NSD$_{10}$ and NSD$_{20}$ for the current cycle, respectively. Inspection reveals that none of the plots is statistically significant. Hence, the elapsed time between first occurrences (or of 10 or more, or 20 or more) of spotless days of two consecutive cycles for the current cycle does not provide a reliable prediction for the elapsed time
between first occurrence (or the occurrence of 10 or more or 20 or more) of a spotless day and $E(Rm)$ of the following cycle.

Figure 26 shows the scatter plots of $Rm$ (lower panel) and $RM$ (upper panel) versus $t_1$, with the former being highly statistically significant and the latter being marginally significant. So, once $E(Rm)$ occurs, $t_1$ and $Rm$ are set, and $RM$ can be inferred. The current value of $R$ (through November 2006) is 12.7 and $t_1$ measures at least 34 mo. As $t_1$ increases in length, $R$ decreases in value such that, for $t_1 > 40$ mo, $Rm < 5.2$ would be expected; and, for $t_1 \leq 40$ mo, $Rm \geq 5.2$ would be expected. An elapsed time of 40 mo implies $E(Rm)$ in May 2007. Since high-latitude new cycle spots have yet to appear, this
seems to suggest that $E(Rm)$ will occur after May 2007 and the value of $R$ will significantly decrease between November 2006 and May 2007 (or cycle 24’s $Rm$ will be an outlier). For $RM$, if $t_1 \leq 40$ mo, it is expected to be $\geq 119.2$, while a longer value for $t_1$ suggests a smaller $RM$ (an exception is cycle 11, which had $RM=140.5$).
Figure 19. Scatter plots of Rm(n+1) and RM(n+1) versus NSD$_{20}$(n).

Figure 27 depicts the scatter plots of Rm (lower panel) and RM (upper panel) versus $t_{10}$. Both plots are found to be statistically significant. So, once $E(Rm)$ occurs, $t_{10}$ and $Rm$ are set, and $RM$ can be inferred. As noted above, in November 2006, $R=12.7$ and $t_{10}$ measures at least 9 mo. As $t_{10}$ increases in length, $R$ decreases in value such that, for $t_{10}>20$ mo, $Rm<5.2$ would be expected; and, for $t_{10}\leq20$ mo,
$R_m \geq 5.2$ would be expected. An elapsed time of 20 mo implies $E(R_m)$ in October 2007. For $RM$, if $t_{10} \leq 20$ mo, it is expected to be $\geq 119.2$, while a longer value for $t_{10}$ suggests a smaller $RM$.

Figure 28 displays the scatter plots of $R_m$ (lower panel) and $RM$ (upper panel) versus $t_{20}$. Both plots are found to be statistically significant. So, once $E(R_m)$ occurs, $t_{20}$ and $R_m$ are set, and $RM$ can be inferred. In November 2006, $R = 12.7$ and $t_{20}$ measures at least $-5$ mo. As $t_{20}$ increases in length, $R$ decreases in value such that, for $t_{20} > 5$ mo, $R_m < 5.2$ would be expected and, for $t_{20} \leq 5$ mo, $R_m \geq 5.2$ would be expected. An elapsed time of $5$ mo implies $E(R_m)$ in September 2007. For $RM$, if $t_{20} \leq 5$ mo, it is expected to be $\geq 119.2$, while a longer value for $t_{20}$ suggests a smaller $RM$. (For $t_{20} \leq 8$ mo, corresponding to 9 of the 13 cycles, $R_m$ has averaged 7.5, with an $sd$ of 3.4, and $RM$ has averaged 138.4, with an $sd$ of 35.9; $t_{20} = 8$ mo implies $E(R_m)$ in December 2007.)

Figure 29 shows the scatter plots of $RM$ (lower panel) and the descent duration (the elapsed time in months from $E(RM)$ to the next cycle’s $E(R_m)$), denoted here as DES (upper panel) versus ASC. The former plot is better known as the Waldmeier effect, which reveals faster rising (shorter ASC) cycles to be cycles of larger amplitude (larger $RM$) and slower rising (longer ASC) cycles to be cycles of smaller amplitude (smaller $RM$). On the basis of Fisher’s exact test for 2×2-contingency tables, there is only a 7.7-percent
probability of obtaining the observed result, or one more suggestive of a departure from independence. Thus, cycles with ASC ≤ 47 mo tend to be larger than $RM = 119.2$ (true for seven of eight cycles, the exception being cycle 13), while those of ASC > 47 mo (true for five of five cycles) have always had $RM < 119.2$. The given inferred regression ignores the two cycle extremes (cycles 14 and 19). Thus, a cycle of $RM = 160$ should have an ascent duration of about 37 mo ($± 7$ mo, the 90-percent prediction interval), while a cycle of about 80 should have an ascent duration of about 54 mo ($± 7$ mo).

Concerning DES versus ASC, while there is no inferred statistically significant regression between them, ignoring period, an interesting feature is apparent. Namely, shorter period cycles have always fallen below the PER = 127-mo line, with DES, given by $y_{SP}$, and longer period cycles have always fallen above the PER = 134-mo line, with DES given by $y_{LP}$. Because ASC(23) = 47 mo, one deduces that DES(23) either will be equal to $75 ± 7$ mo if it is a shorter period cycle or $93 ± 3$ mo if it is a longer period cycle (90-percent prediction intervals). Hence, cycle 23 will have a period of either $122 ± 7$ mo

Figure 21. Scatter plot of PER versus NSD$_{10}$.
if it is a shorter period cycle or 140±3 mo if it is a longer period cycle. Thus, one expects $E(Rm)$ for cycle 24 before March 2007 if it is a shorter period cycle or after September 2007 if it is a longer period cycle. Because no high-latitude new cycle spots have, as yet, been seen (through May 2007), this seems to indicate that cycle 23 more likely is a cycle of longer period and $E(Rm)$ for cycle 24 will occur after September 2007.

Figure 30 depicts the scatter plot of $RM$ for the following cycle versus PER for the current cycle. Although the inferred linear regression is not statistically important, the resulting contingency table does suggest a significant association between the two parameters. For example, five of six cycles (the exception being cycle 20) having PER equal to 126 mo or longer have been followed by cycles having $RM < 114.9$, while five of six cycles (the exception being cycle 15) having PER shorter than 126 mo have been followed by cycles having $RM \geq 114.9$. Cycle 23 has already persisted for 126 mo (through November 2006). Hence, statistically speaking, it is very likely that it will be a cycle of longer period, so one probably expects that it will be followed by a cycle of smaller maximum amplitude. This finding is in stark contrast to that previously reported for the predicted size of cycle 24 based on a specific dynamo model and geomagnetic precursors.14–16
Figure 23. Scatter plot of $t_1(n+1)$ versus NSD$_1(n)$. 
Figure 24. Scatter plot of $t_{10}(n+1)$ versus $\text{NSD}_{10}(n)$. 

$P = 50.0\%$
Figure 25. Scatter plot of $t_{20}(n+1)$ versus NSD_{20}(n).
Figure 26. Scatter plots of $Rm$ and $RM$ versus $t_1$. 

\[
y = 173.927 - 1.099x \\
r = -0.522, r^2 = 0.273 \\
se = 36.125, cl > 90\%
\]

\[
y = 12.319 - 0.131x \\
r = -0.692, r^2 = 0.479 \\
se = 2.752, cl > 99\%
\]

Note: Thru November 2006, $t_1$ (24) $\geq$ 34 months
Figure 27. Scatter plots of $R_m$ and $RM$ versus $t_{10}$. 

$\begin{align*}
\text{Note: Thru November 2006, } t_{10} (24) &\geq 9 \text{ months} \\
\{ y = 157.855 - 1.648x, \quad r = -0.614, r^2 = 0.377, \quad se = 33.419, \quad cl > 95\% \} \\
\{ y = 9.884 - 0.167x, \quad r = -0.655, r^2 = 0.429, \quad se = 2.876, \quad cl > 98\% \}
\end{align*}$
Figure 28. Scatter plots of $Rm$ and $RM$ versus $t_{20}$. The large dot at (5,3.4) means that two cycles had the same values (cycles 17 and 19).
Figure 29. Scatter plots of $RM$ and DES versus ASC.
\{ y = 333.547 \pm 1.655x \\
r = -0.375, r^2 = 0.141 \\
se = 40.712, cl < 90\%
\}

Figure 30. Scatter plot of RM(n+1) versus PER(n).
3. SUMMARY

This study has shown that cycle 23, the current ongoing sunspot cycle, in all probability is a cycle of longer period (PER ≥ 135 mo). This implies that cycle 24’s conventional onset will occur on or after August 2007, probably about January 2008, give or take a few months. Through November 2006, cycle 23 has now persisted for 126 mo. For cycles 11–22, all cycles either have had PER ≤ 126 mo (cycles 15–19 and 21 and 22) or ≥ 135 mo (cycles 11–14 and 20). Also, because (through May 2007) there has not, as yet, been a single occurrence of a high-latitude (≥ 25 deg) new cycle spot nor has a sharp upturn in LAT or HLS been seen, these being characteristics that have always indicated the imminent onset for the following new sunspot cycle, cycle 23 undoubtedly will continue to persist for several additional months beyond November 2006. A consequence of this then is that cycle 24 might not be as large as previously predicted; based on cycles 11–22, only one of five cycles having PER ≥ 135 mo has been followed by a cycle of larger than average maximum amplitude, the exception being cycle 20, which was followed by the second largest cycle (cycle 21) of the modern era, having $RM = 164.5$. 
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Anticipating Cycle 24 and Its Consequences

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On the basis of the 12-mo moving average of monthly mean sunspot number (R) through November 2006, cycle 23 has persisted for 126 mo, having had a minimum of 8.0 in May 1996, a peak of 120.8 in April 2000, and an ascent duration of 47 mo. In November 2006, the 12-mo moving average of monthly mean sunspot number was 12.7, a value just outside the upper observed envelope of sunspot minimum values for the most recent cycles 16–23 (range 3.4–12.3), but within the 90-percent prediction interval (7.8±6.7). The first spotless day during the decline of cycle 23 occurred in January 2004, and the first occurrence of 10 or more and 20 or more spotless days was February 2006 and April 2007, respectively, inferring that sunspot minimum for cycle 24 is imminent. Through May 2007, 121 spotless days have accumulated. In terms of the weighted mean latitude (weighed by spot area) (LAT) and the highest observed latitude spot (HLS) in November 2006, 12-mo moving averages of these parameters measured 7.9 and 14.6 deg, respectively, these values being the lowest values yet observed during the decline of cycle 23 and being below corresponding mean values found for cycles 16–23. As yet, no high-latitude new-cycle spots have been seen nor has there been an upturn in LAT and HLS, these conditions having always preceded new cycle minimum by several months for past cycles. Together, these findings suggest that cycle 24’s minimum amplitude still lies well beyond November 2006. This implies that cycle 23’s period either will lie in the period “gap” (127–134 mo), a first for a sunspot cycle, or it will be longer than 134 mo, thus making cycle 23 a long-period cycle (like cycle 20) and indicating that cycle 24’s minimum will occur after July 2007. Should cycle 23 prove to be a cycle of longer period, a consequence might be that the maximum amplitude for cycle 24 may be smaller than previously predicted.
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