THE DETERMINATION OF TITAN GRAVITY FIELD FROM DOPPLER TRACKING OF THE CASSINI SPACECRAFT

L. Iess¹, J.W. Armstrong², S.W. Asmar², M. Di Benedetto¹, A. Graziani³, R. Mackenzie², P. Racioppa¹, N. Rappaport², P. Tortora¹

(1) Dipartimento di Ingegneria Aerospaziale ed Astronautica, Universita’ di Roma “La Sapienza”, Italy,
(2) Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA, USA,
(3) DIEM, Universita’ di Bologna, Forlì, Italy

Abstract

In its tour of the Saturnian system, the spacecraft Cassini is carrying out measurements of the gravity field of Titan, whose knowledge is crucial for constraining the internal structure of the satellite. In the five flybys devoted to gravity science, the spacecraft is tracked in X (8.4 GHz) and Ka band (32.5 GHz) from the antennas of NASA’s Deep Space Network. The use of a dual frequency downlink is used to mitigate the effects of interplanetary plasma, the largest noise source affecting Doppler measurements. Variations in the wet path delay are effectively compensated by means of advanced water vapor radiometers placed close to the ground antennas. The first three flybys occurred on February 27, 2006, December 28, 2006, and June 29, 2007. Two additional flybys are planned in July 2008 and May 2010. This paper presents the estimation of the mass and quadrupole field of Titan from the first two flybys, carried out by the Cassini Radio Science Team using a short arc orbit determination. The data from the two flybys are first independently fit using a dynamical model of the spacecraft and the bodies of the Saturnian system, and then combined in a multi-arc solution. Under the assumption that the higher degree harmonics are negligible, the estimated values of the gravity parameters from the combined, multi-arc solution are $GM = 8978.1337 \pm 0.0025 \text{ km}^3/\text{s}^2$, $J_2 = (2.7221 \pm 0.0185) \times 10^{-5}$ and $C_{22} = (1.1159 \pm 0.0040) \times 10^{-5}$. The excellent agreement (within $1.7\sigma$) of the results from the two flybys further increases the confidence in the solution and provides an a posteriori validation of the dynamical model.

Introduction

The Cassini spacecraft was captured by Saturn gravity field on July 1, 2004 and is currently carrying out its tour of the Saturn system. The prime mission will end in the summer of 2008, but the additional opportunities for science observations will be possible until 2010, in the extended mission phase. Titan is at the same time one of the primary objectives if Cassini’s scientific investigations and true propulsion system of the mission. The complex Saturn tour has indeed been accomplished by a carefully designed sequence of Titan flybys, each one allowing to vary the orbital elements of the spacecraft to meet the needs of the several instruments hosted onboard. 45 flybys are foreseen in the prime mission, 26 in the extended phase.

In principle such a large number of flybys, occurring with different geometries, could provide an excellent determination of Titan’s gravity field. Unfortunately, due to the lack of a scanning platform for the remote sensing instruments, the orientation of the spacecraft is incompatible with the pointing of the high gain antenna toward the earth, so that tracking data are generally not available near closest approach. Gravity field measurements, requiring continuous tracking from ground stations across the encounter, are possible in only five flybys allocated to radio science. Three of these flybys, T11, T22, and T33, occurred during the prime mission on February 27, 2006, December 28, 2006, and June 29, 2007, while T45 and T68 are scheduled for the extended mission, respectively on 31 July 2008 and 20 May 2010.

The knowledge of Titan gravity field is at the same time a need for precise spacecraft navigation and a crucial scientific objective of the mission. The flybys devoted to gravity science have been allocated to the Cassini Radio Science Team with the main goal of determining the mass of Titan, its quadrupole field coefficients and Love number $k_2$, but they are also used by the Cassini Navigation Team to improve the ephemerides of the Saturn system bodies. From the scientific point of view, the lack of a magnetic field makes gravity data the only available tool to

¹ L.I, M.DB, A.G., P.R and P.T acknowledge support by the Italian Space Agency. J.W.A., S.W.A., R.M.K and N.J.R. performed their work at the Jet Propulsion Laboratory, California Institute of Technology, under contract with NASA. We thank the JPL Radio Science Systems Group, Cassini Navigation Team and NASA’s Deep Space Network for providing tracking data and crucial ancillary information.
probe Titan’s interior. Indeed, the determination of the gravity field provides crucial constraints to models of the interior structure and may also reveal the presence of a subsurface ocean.

This paper presents the results accomplished so far in the analysis of the first two Titan flybys devoted to gravity science, T11 and T22. Section 1 outlines the model of Titan gravity field used in the orbital fit and the interpretation of the results. Sect. 2 presents the data set used in the orbit determination and the corrections applied to the data in order to reduce the effects of propagation media (plasma and troposphere). The results of the orbital fit and the values of the Stokes parameters \( J_2 \) and \( C_{22} \) determined from T11 and T22 are presented in sect. 3, together with a combined solution from the two flybys and some final remarks.

1. Model of Titan gravity field

With a diameter of 2575 km, Titan is the second largest satellite of the solar system. Although its thick atmosphere has prevented a determination of its rotational state by optical imaging, preliminary data from the Cassini radar have confirmed that Titan is tidally locked to Saturn and that the obliquity is small [1]. A remarkable feature of Titan’s orbit is its large eccentricity, about 0.03, a value difficult to explain in the absence of some forcing mechanism. Whatever mechanism is preventing a decay of the eccentricity, its large value bears remarkable implications for Titan’s gravity, the most important of which is a small variability in the quadrupole field. Indeed, the varying distance from Saturn, of order of the product \( ae \) between the semimajor axis and the eccentricity, results in a variation of order \( 3eGM/a^2 \) of the tidal field exerted by Saturn on Titan, modulated at the orbital period (about 16 days). As Titan responds as an elastic body to short term external stresses, the gravity field is expected to change from pericenter to apocenter, by an amount determined by the overall rigidity of the body. A measurement of the variable part of Titan gravity field would be of the highest geophysical interest, as it would allow to infer the elastic properties of the body, strongly dependent on its interior structure.

The static part of the present quadrupole field of Titan is determined by the average tidal field exerted by Saturn and by the centrifugal field associated to the rotation about the polar axis. The effects of these two perturbing fields on Titan are controlled by two parameters, namely

\[
q_t = -3\frac{M_s}{M_t} \left( \frac{R_T}{a} \right)^3 \\
q_r = \frac{\omega^2 R_T^3}{GM_T}
\]

where \( M \) and \( M_s \) are respectively the mass of Titan and Saturn, and \( R_T \), \( a \), \( \omega \) Titan radius, semimajor axis, and rotational angular velocity [2]. The tidal parameter \( q_t \) is the ratio between Saturn tidal field and Titan surface gravity, while the rotational parameter \( q_r \) is the ratio between the centrifugal acceleration at the equator and surface gravity acceleration. The tidal field tends to deform the satellite into a prolate ellipsoid, while the deformation associated to the centrifugal field results in an oblate ellipsoid. As a consequence of the equality between the mean motion and the rotational angular velocity for a satellite tidally locked to the central planet, \( q_t \) and \( q_r \) are no longer independent and \( q_t = -3q_r \). For Titan, \( q_t = -1.19 \times 10^{-6} \).

If one assumes that the satellite is in hydrostatic equilibrium, in the principal axes frame \( J_2 \) and \( C_{22} \) (the only non-null quadrupole coefficients) are a function of a single parameter, the fluid (or secular) Love number \( k_f \) [2,3]:

\[
J_2 = \frac{1}{6} k_f (2q_t - q_r) = \frac{5}{6} k_f q_r \\
C_{22} = -\frac{1}{12} k_f q_r = \frac{1}{4} k_f q_r
\]

\( k_f \) is a function of the rigidity of the satellite; for a liquid body \( k_f = 3/2 \). Adopting this value for the fluid Love number, the putative quadrupole coefficients used as a priori values in the non-linear parameter estimation become

\[
J_2 = 4.93 \times 10^{-5} \\
C_{22} = 1.48 \times 10^{-5} \\
S_{22} = C_{23} = S_{23} = 0
\]

The putative quadrupole field presented here is affected not only by the uncertainty in \( k_f \), but also by the assumption that the reference frame adopted in the estimation of the parameters coincides with the principal axes frame. Furthermore, periodic components of Titan quadrupole field, associated with the short term elastic response to the forcing tidal field, have been neglected. The IAU frame assumes that Titan spin axis is parallel to the normal to the orbital plane, with the prime meridian defined by the eccentricity vector (i.e. by Saturn direction at pericenter). Although Titan obliquity is expected to be small (of the order of 7 arcmin) and Titan should be in a Cassini state, a rough confirmation of this hypotheses is possible by estimating all five components of the degree 2 field. As it will be discussed later, the uncertainties in \( C_{21} \) and \( S_{21} \) are too large for a precise estimate, but the failure to estimate reliably \( C_{21} \) and \( S_{21} \) is an indication that the adopted, body-fixed reference frame is adequate for gravity field determination.
The periodic component of Saturn tidal potential, generated by the eccentricity of Titan orbit, induces additional, periodic terms in the satellite’s quadrupole field. To the lowest order, the tidal perturbing potential is represented by second degree harmonics, and therefore, assuming a linear response from the elastic Titan, the perturbed gravitational potential is still described by degree 2 harmonics. The ratio between the perturbed and external tidal potential is known as the degree 2 Love number, indicated with \( k_2 \). When this additional contribution is added, the model gravity field becomes

\[
J_2 = \frac{5}{6} k_2 q_r + \frac{1}{2} k_2 q_r e \cos(M - \delta) \quad C_{22} = \frac{1}{4} k_2 q_r - \frac{1}{4} k_2 q_r e \cos(M - \delta) \quad S_{22} = -\frac{1}{3} k_2 q_r e \sin(M - \delta)
\]

where \( M \) is the mean anomaly and \( \delta \) is a (positive) tidal phase lag angle, accounting for viscoelastic effects.

The periodic terms generate variations of order \( e \) in the quadrupole coefficients, which could be significantly larger than the measurement accuracies for a broad range of \( k_2 \) values. The prospect of determining \( k_2 \) from measurements of the quadrupole field at different anomalies is of high geophysical interest. Indeed, several models of Titan interior predict the presence of a subsurface ocean under the satellite’s icy crust [4,5]. A liquid layer would dramatically increase the Love number, by an amount dependent on the thickness and viscosity of the ice lid, and the ocean depth. A measurement of \( k_2 \) would not only allow to discriminate the presence or absence of an ocean, but also put strong constraints on Titan interior structure, making it one of the most important measurements of the Cassini mission.

2. Doppler data and propagation noise

The Cassini orbit determination is carried out by using a combination of radio-metric and optical observables. For gravity field determination, however, by far the most valuable data type is obtained from range rate (Doppler) measurements, where a highly stable microwave carrier is sent from a ground station to the spacecraft and coherently retransmitted to earth by means of a transponder. Using X-band (7.2-8.4 GHz) in both the uplink and the downlink, frequency stabilities as low as \( 1 \times 10^{-14} \) (or 0.003 mm/s) over 1000 seconds integration times can be attained under favourable conditions [6], the main limitation being due to propagation effects in space plasmas.

The effect of charged particles in the solar corona, interplanetary space and the Earth ionosphere were almost fully removed during Cassini cruise radio science experiments [7,8], thanks to the availability of a Ka-band (32.5-34 GHz), two-way radio link enabled by an onboard frequency translator. This instrument was part of a scientific subsystem named RFIS (Radio Frequency Instrument Subsystem), comprising a Ka-band TWTA, a Ka-band exciter (providing a Ka-band downlink coherent with the X-band uplink used for telecommand and navigation), an ultrastable oscillator and a S-band transponder. In mid-2003, however, this crucial instrument suffered an unrecoverable malfunction and was therefore unavailable during the Saturn tour. Thus, in gravity science experiments the effects of the solar plasma could only be mitigated by using X/Ka (X-band uplink, Ka-band downlink) Doppler data rather than X/X data, when the former were available. As plasma noise is inversely proportional to the square of the carrier frequency, a higher downlink frequency renders Doppler observables less affected by plasma noise. The expected reduction is about a factor \( \sqrt{2} \), essentially as a consequence of the strong suppression of downlink plasma noise (by a factor \( (32.5/8.4)^2 = 15 \)). Besides the overall noise reduction, Titan flybys data analysis has shown that the use of Ka-band downlink data reduces also harmful signatures affecting the orbit determination process, leading to adjustments of the central values of the estimated quantities in addition to some reduction in the estimation errors. In the effort to maximize the phase stability, 2-way data were preferred to 3-way data, if simultaneously acquired.

Another important contribution to the overall noise budget comes from propagation through the earth troposphere. Path delay variations from water vapour, particularly large at low elevation angles, significantly affects Doppler measurements. In interplanetary spacecraft navigation, tropospheric effects are partially removed at the Deep Space Station (DSN) complexes using data and software tools named Tracking System Analytical Calibrations (TSAC) [9]. This system, based upon a combination of weather data and multidirectional, dual frequency GPS measurements at each station complex, allows a good removal of the dry component of the tropospheric delay, but fails to detect short time scale variations (60-1000 s) associated to changes in the columnar water vapour content. A new, more accurate tropospheric calibration system was developed for Cassini radio science cruise experiments [10]. Two Advanced Media Calibration (AMC) instruments, consisting of water vapor radiometers, digital pressure sensors and microwave temperature profilers assembled in a single unit, were originally installed close to Deep Space Station (DSS) 25, the ground antenna on NASA’s Deep Space Network (DSN). After Cassini’s successful Saturn orbit insertion in 2004, one of the AMC units was moved to Spain, at the DSN Madrid complex, and installed close to DSS 55, thus extending the capabilities of precise calibration of the Earth troposphere to a second ground station.
During T11 and T22 both AMC systems were used whenever the spacecraft was in visibility at the tracking station. In T11, one of the four tracking passes used in the analysis was carried out from the Goldstone antenna DSS 14, located at about 10 km from DSS 25 and the AMC unit, forcing the use of TSAC calibrations. In T22, one pass was carried out from DSS 34 (Canberra, Australia), not equipped with an AMC system. In the remaining four passes, scheduled over the Goldstone and Madrid complexes (two at DSS 25 and two at DSS 55) several problems were experienced. For the DSS 55 passes, the wet path delay was not available until the spacecraft reached about 50 deg elevation, due to blockage of the water vapour radiometer field of view from the antenna main dish. For both passes over DSS 25, the dry path delay data were not valid due to a malfunction at the meteo station. In addition, another unknown failure affected the dry path delay data for one of the two passes over DSS 55. In summary, almost 50% of the AMC data acquired during T22 could not be used in the final processing, although reliable AMC wet data were available at Titan closest approach. In spite of the incomplete acquisition, the use of the AMC-reconstructed dry and wet path delays in the orbit determination process proved crucial in achieving a full consistency between T11 and T22 gravity results.

Short arc gravity field determination

The determination of the Titan gravity field in the T11 and T22 flybys was carried out using a short data arc spanning roughly ±24 hours from closest approach. The observable quantities were Doppler data acquired at NASA’s DSN antennas, supplemented by a combination of TSAC and AMC tropospheric calibration. A total of 1147 and 1292 Doppler data, compressed at 60 s integration time, were used respectively in the orbital fit of T11 and T22. Ka-band data are a significant fraction of the whole data set, namely 28% in T11 and 33% in T22. With the exception of a short (2 hours) pass from the Canberra complex, the tracking was supported from the Goldstone and Madrid DSN stations.

The two flybys were analyzed separately, with each arc providing two largely independent orbital solution. The solutions for T11 and T22 were later combined in a single multi-arc solution, providing estimates of local and global parameters. Due to the different flyby geometries (see Table 1), and especially the different flyby altitudes, the dynamical model and the set of estimated parameters were different for T11 and T22. The most remarkable difference was the 1297 km altitude of the spacecraft closest approach in T22, a level at which the denser than expected Titan atmosphere causes a significant aerodynamic drag. With densities between $10^{-12}$ to $10^{-11} \text{ kg/m}^3$ at 1300 km, peak decelerations of about 10−3 m/s² are expected (see fig. 1). This evaluation was obtained by assuming a density of $7 \times 10^{-12} \text{ kg/m}^3$, a cross sectional area of 23 m² and a drag coefficient equal to 10. (Such a fictitiously large value of the $C_D$ is required to ensure consistency between the observed acceleration and the density determined by the onboard instruments, and may point to scaling errors in the density profile.)

Although the drag and quadrupole field acceleration are of the same order of magnitude, their signature is sufficiently different to allow a good discrimination of the two effects by the orbital filter. In the dynamical model Titan’s atmosphere was modeled as a two-layer, exponential atmosphere, with base altitudes of 1300 and 1500 km. This assumption is supported by in situ measurements carried out by the onboard Ion and Neutral Mass Spectrometer (INMS), which indicates that an exponential profile with varying scale height accurately models Titan atmospheric density up to about 1700 km [11]. In each of the two layers (1300-1500 km and 1500-2000 km) the density variation is determined by two different scale heights and by the density at the base of the first layer. These three parameters were solved for in the analysis of T22. The value of the drag coefficient (not separable from the density) was arbitrarily assumed equal to 10, while the cross sectional area was computed from a model of the spacecraft elements and the known attitude with respect to the ram direction. As the density estimated by INMS at a given altitude varies significantly in each flyby, the base density was given very loose a priori constraints. This conservative approach was further motivated by the discrepancy (up to a factor of four) in the estimated density between INMS and other methods (radio occultations and control torques exerted by the reaction wheel assembly to counteract the aerodynamic torque). On the other hand, all methods agree in the value of the scale height, consistently throughout all flybys. The scale heights in the two layers have been strongly constrained to the rms value of a set of ten scale heights, each obtained by fitting the ten density profile measured by INMS in different flybys. A priori values of 86 km and 101 km were used in the orbital fit for the scale heights in the lower and upper layer, with a priori uncertainties of 2 km and 16 km, respectively.

The dynamical model included the gravitational accelerations from all solar system bodies, obtained from JPL planetary ephemerides DE410. The gravitational parameters and the state vectors of Saturn and its satellites, as well the first three zonal coefficients of Saturn, were adopted from the ephemerides provided by the Cassini Navigation Team (NAV) by fitting a large number of radio and optical data. The initial Cassini state vector was obtained from a reconstructed trajectory provided by NAV. The Cassini and Titan initial state vectors were constrained a priori using typical uncertainties attained during the Saturn tour. The integration of the trajectory was carried out in a Saturn-system barycentric frame. The acceleration due to solar radiation pressure and anisotropic thermal emission from the three onboard radio-isotope thermoelectric generators (RTG) were also included in the model. The latter (about $5 \times 10^{-9} \text{ m/s}^2$ and constant in the spacecraft frame) is by far the most important.
Crucial for the analysis of gravity data is also the rotation model of Titan. Cassini NAV currently uses a model slightly different from the one proposed by the IAU. The differences between the two are too small to affect the gravity results, but the NAV model was adopted for internal consistency. Both assume a polar axis close to the normal to the orbital plane and a prime meridian almost oriented to Saturn at pericenter and apocenter, as expected from a tidal lock and Titan’s occupancy of a Cassini state with small obliquity (a few arcmin). As the thick cloud layers prevent a direct identification of surface features by means of optical remote sensing, improvements of current rotation models are only possible from SAR observations.

In T11, where the atmospheric drag is negligible, 18 parameters were initially estimated, namely the state vector of Cassini and Titan at a reference epoch (24 hours before Titan closest approach), and the $GM$ and the five quadrupole coefficients of Titan, with loose a priori constraints for $J_2$, $C_{22}$ and $S_{22}$. However, the coefficients $C_{21}$ and $S_{11}$, associated with a tilt of the polar axis from the principal axis, could not be estimated, as their formal uncertainties were larger than the nearly zero central values. The lack of sensitivity to the pole location is also indicated by the almost identical results obtained from a solution where only $J_3$, $C_{22}$ and $S_{22}$ were estimated (with $C_{22}=S_{22} = 0$). The results of the gravity field determination using X- and Ka-band data with AMC calibrations are shown in Table 2. All data were weighted to the rms value of each tracking pass. The implicit assumption of white noise is rather well confirmed by the actual power spectra of the Doppler residuals. The estimate of $C_{22}$ and $S_{22}$ were remarkably good, as expected for an almost equatorial flyby. The Doppler residuals from the T11 orbital fit are shown in fig. 2.

The small but non-zero $S_{22}$ (determined with a 10% accuracy) can be combined with the much larger $C_{22}$ (determined to 0.4%) to provide an estimate of the angle between the principal axis of Titan and the least moment of inertia (Titan’s long axis). This offset is about -0.86 degrees. It is remarkable that in T11, occurring at a mean anomaly of 173 degrees, the theoretical model predicts a positive value of $S_{22}$ (see eq. 4), unless the phase lag angle $\delta$ is negative, an assumption inconsistent with the current understanding of the tidal deformations of Titan. Although this result is puzzling, a negative $S_{22}$ may be due, for example, to the adopted rotation model (leading to a non-zero longitude of Saturn when Titan is at pericenter), to anomalous displacements of the surface ice lid, or to non-hydrostatic features of Titan interior.

The data from the second Titan flyby (T22) were analyzed with a similar filter setup. As explained above, Titan atmospheric parameters had to be estimated to account for the drag near closest approach. In general, the large inclination led to a poorer estimation of some parameter. For $S_{22}$, where a small value was expected, the estimate turned out to be impossible. Since the adopted value of $S_{22}$ may produce a bias in $C_{22}$, the approach followed in the orbit determination was to confine the estimation to $J_3$ and $C_{22}$ and treat $S_{22}$ as a consider parameter, adopting the value obtained from orbital fit of T11. (This simplified approach neglects the variations of $S_{22}$ predicted by Titan tidal model, eq. 4; for the mean anomalies of T11 and T22 and assuming $\delta = 0$, $k_2 = 0.45$, as predicted by some models with a subsurface ocean, a change in $S_{22}$ by about $2 \times 10^{-7}$ is expected.) In addition, the gravity parameters were constrained with an a priori uncertainty of ten times the formal uncertainty obtained from T11 (loosening the constraints by a further factor of 10 leads to nearly identical results). The estimated values and associated errors of the quadrupole coefficients for this case are shown in Table 2. The Doppler residuals are plotted in fig. 3.

The remarkable agreement and consistency between the two flybys surely strengthen the confidence in the results and the adopted dynamical model. A clearer quantitative indication of such agreement is apparent from fig. 4, which shows the 1σ probability ellipses in the $J_2$-$C_{22}$ plane. The discrepancy in $C_{22}$ for the two flybys is about 1.7σ. At least three remarks are at hand. First, Titan quadrupole coefficients deviate appreciably from the expectations for a body in hydrostatic equilibrium. In particular, the ratio $J_2/C_{22}$ is significantly lower than the hydrostatic value of 10/3 (see eq. 2). Second, the agreement in the presence of quite different flyby geometries indicates that the higher degree and order gravity field, not considered in our dynamical model, must be small. A sensitivity analysis aiming to determine upper bounds to degree 3 and 4 harmonics is planned in a future work. Third, the determination of the $k_2$ Love number, based upon variations of the quadrupole coefficients at pericenter and apocenter, can be accomplished as planned.

Given the small variation of $J_2$ and $C_{22}$ predicted by the tidal model, one may combine the results from the two flybys in a single solution for Titan gravity field at apocenter. In this T11-T22 combined solution local and global parameters are separated and a new information matrix is inverted. The combined estimate leads to the following values for Titan’s gravitational parameter and quadrupole coefficients:

\[
GM = 8978.1337 \pm 0.0025
\]

\[
J_2 = (272.21 \pm 1.85) \times 10^{-7}
\]

\[
C_{22} = (111.59 \pm 0.40) \times 10^{-7}
\]

The significant reduction in the $J_2$ uncertainty is largely due to the strong $J_2$-$C_{22}$ correlation (-0.4 in T11 and -0.95 in T22).
Initial results, obtained when only X-band data and the less accurate TSAC calibrations were used, were significantly different, with a poorer agreement between T11 and T22. For comparison, the probability ellipses for $J_2$ and $C_{22}$ obtained from this data set (X-band only with TSAC) are plotted in fig. 4 (thin lines) together with the ones previously obtained using X- and Ka-band data with AMC (thick lines). Strikingly, data more affected by propagation effects produce shifts in the central values rather than significant increases in the Doppler noise (and therefore in the estimation errors). Even if the availability of Ka-band and AMC data was far from complete, our results show that precision tracking experiments may significantly benefit from the use of such systems. Further benefits are expected if a full Ka-band up, Ka-down radio link is available.

References


Table 1. The Cassini’s Titan gravity flybys.

<table>
<thead>
<tr>
<th></th>
<th>T11</th>
<th>T22</th>
<th>T33</th>
<th>T45</th>
<th>T68</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude (km)</td>
<td>1812</td>
<td>1297</td>
<td>1960</td>
<td>1591</td>
<td>1400</td>
</tr>
<tr>
<td>Inclination (°)</td>
<td>180</td>
<td>68</td>
<td>8</td>
<td>123</td>
<td>131</td>
</tr>
<tr>
<td>Mean anomaly (°)</td>
<td>173</td>
<td>197</td>
<td>15</td>
<td>324</td>
<td>81</td>
</tr>
<tr>
<td>Sun-earth-probe angle (°)</td>
<td>147</td>
<td>132</td>
<td>45</td>
<td>29</td>
<td>119</td>
</tr>
<tr>
<td>Noise (mm/s)</td>
<td>0.024</td>
<td>0.023</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Number of points</td>
<td>1152</td>
<td>1292</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

Table 2. Titan $GM$ and quadrupole coefficients from T11 and T22 flybys.

<table>
<thead>
<tr>
<th></th>
<th>A priori value</th>
<th>A priori uncertainty</th>
<th>Estimated 1σ uncertainty</th>
<th>Estimated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T11) $GM$</td>
<td>8.97813542E+03</td>
<td>8.000E-01</td>
<td>3.538E-03</td>
<td>8.97813601E+03</td>
</tr>
<tr>
<td>(T22) $GM$</td>
<td>8.97813668E+03</td>
<td>3.583E-02</td>
<td>3.982E-03</td>
<td>8.97812994E+03</td>
</tr>
<tr>
<td>(T11) $J_2$</td>
<td>4.93414561E-05</td>
<td>5.000E-05</td>
<td>3.319E-06</td>
<td>2.81291854E-05</td>
</tr>
<tr>
<td>(T22) $J_2$</td>
<td>2.87219604E-05</td>
<td>3.319E-05</td>
<td>6.450E-07</td>
<td>2.85795991E-05</td>
</tr>
<tr>
<td>(T11) $C_{22}$</td>
<td>1.47936735E-05</td>
<td>5.000E-05</td>
<td>4.639E-07</td>
<td>1.11742992E-05</td>
</tr>
<tr>
<td>(T22) $C_{22}$</td>
<td>1.11381341E-05</td>
<td>4.639E-07</td>
<td>2.227E-07</td>
<td>1.07914396E-05</td>
</tr>
<tr>
<td>(T11) $S_{22}$</td>
<td>0.0</td>
<td>5.000E-06</td>
<td>4.446E-08</td>
<td>-3.39150623E-07</td>
</tr>
</tbody>
</table>

Fig. 1. The accelerations due to $J_2$ (blue curve), $C_{22}$ (green curve), and atmospheric drag (red curve) for the T11 flyby (1297 km closest approach altitude). The upper and lower plots are respectively the magnitude of the acceleration vector and its projection along the earth-spacecraft direction, in km/s$^2$. 

7
Fig. 2. Two-way range rate residuals (in mm/s) for the T11 flyby. Closest approach to Titan (marked by the arrow) occurred on 27 February 2006 08:26:24 SCET (09:33:25 ERT). The text labels indicate the transmitting and receiving stations of NASA’s Deep Space Network, and the uplink and downlink bands. The rms value of the residuals, at 60 s integration time, is 0.022 mm/s.

Fig. 3. Two-way range rate residuals (in mm/s) for the T22 flyby. Closest approach to Titan (marked by the arrow) occurred on 28 December 2006 10:06:26.67 SCET (11:17:21 ERT). The rms value of the residuals, at 60 s integration time, is 0.023 mm/s, down from 0.034 mm/s if AMC calibrations were not applied.
Fig. 4. 1σ probability ellipses of the quadrupole coefficients $J_2$ and $C_{22}$, for the short arc solutions of the T11 and T22 Titan flybys, using different tropospheric calibrations (AMC and TSAC) and Doppler observables (X-band only and X/Ka-band). TSAC is the standard calibration system used by Cassini Navigation Team, AMC is the radio science system using water vapor radiometers. Axes are not at the same scale.