Nucleon-Nucleon Total Cross Section

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Nomenclature

Throughout this paper \( \hbar \equiv c \equiv 1 \).

\( m = 938 \text{ MeV} \) is the nucleon mass, which is used for both protons and neutrons.

\( E_\text{1l} \) is the total energy of the projectile (particle 1) in the lab frame.

\( T_\text{1l} \) is the kinetic energy of the projectile (particle 1) in the lab frame.

\( p_\text{1l} \) is the momentum of the projectile (particle 1) in the lab frame.

\( 10^3 \text{ GeV} = \text{TeV} = 10^{12} \text{ eV} \) (Tera-eV).

\( 10^6 \text{ GeV} = \text{PeV} = 10^{15} \text{ eV} \) (Peta-eV).

\( 10^9 \text{ GeV} = \text{EeV} = 10^{18} \text{ eV} \) (Exa-eV).

\( 10^{12} \text{ GeV} = \text{ZeV} = 10^{21} \text{ eV} \) (Zetta-eV).
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Abstract

The total proton-proton and neutron-proton cross sections currently used in the transport codes HZETRN and BRYNTRN show some disagreement with experiment in the GeV and EeV energy ranges. The GeV range is near the region of maximum cosmic ray intensity. It is useful to correct these cross sections, so that predictions of space radiation environments will be more accurate. Parameterizations of nucleon-nucleon total cross sections are developed which are accurate over the entire energy range of the cosmic ray spectrum.

1 Introduction

A current assignment is to improve the elastic neutron differential cross sections currently used in HZETRN [1] and BRYNTRN [2, 3]. These cross sections and the relevant theory are described in Chapter 4 of Reference [4], where it is shown that nucleon-nucleon total cross sections are fundamental inputs to the formulas for the neutron elastic differential cross sections. It was therefore decided to check how well the nucleon-nucleon total cross sections currently used in HZETRN compare with experiment. It will be seen below that although the global fits are quite good, there are some discrepancies in the region of maximum intensity in the cosmic ray spectrum. The aim of the present paper is to make improvements.

Note that the cross sections discussed in this work have also been parameterized by Kalinovskii et al [5].

The total (elastic plus inelastic) cross sections for proton-proton (pp), neutron-neutron (nn) and neutron-proton (np) scattering have been parametrized in reference [4]. The pp cross section is

\[ \sigma_{pp}^{HZETRN}(T_{1l}) = \left(1 + \frac{5}{T_{1l}}\right) \left\{40 + 109 \cos(0.199\sqrt{T_{1l}}) \exp \left[-0.451(T_{1l} - 25)^{0.258}\right]\right\} \]

(for \(T_{1l} \geq 25\) MeV) (1)

\[ \sigma_{pp}^{HZETRN}(T_{1l}) = \exp \left\{6.51 \left[\exp \left(-\left(\frac{T_{1l}}{134}\right)^{0.7}\right)\right]\right\} \]

(for \(T_{1l} < 25\) MeV) (2)

Note that this second equation is written with a typographical error in reference [4], with the minus sign in the wrong place. Note however that this typographical error is not the cause of the discrepancies noted in the present work. The actual formulas currently used in the transport code HZETRN [1] do not have the typographical error and are written correctly, but nevertheless show some disagreement with experiment as explained below.

The total cross section for neutron-neutron scattering is not available and therefore
the proton-proton cross section is used, namely

\[ \sigma_{nn}(T_{1l}) = \sigma_{pp}(T_{1l}) \]  \hspace{1cm} (3)

This is accurate due to isospin symmetry. The neutron-proton cross section is

\[ \sigma_{np}^{HZETRN}(T_{1l}) = 38 + 12500 \exp \left[ -1.187(T_{1l} - 0.1)^{0.35} \right] \] \hspace{1cm} (for \( T_{1l} \geq 0.1 \) MeV) \hspace{1cm} (4)

\[ \sigma_{np}^{HZETRN}(T_{1l}) = 26000 \exp \left[ -\left( \frac{T_{1l}}{0.282} \right)^{0.3} \right] \] \hspace{1cm} (for \( T_{1l} < 0.1 \) MeV)  \hspace{1cm} (5)

Again, this second equation is written with a typographical error in reference [4], with the minus sign in the wrong place. In the above equations, \( T_{1l} \) is the projectile kinetic energy as measured in the lab (target) frame in units of MeV. The cross sections are in units of mb.

Many parametrizations found in the literature, as well as experimental data [6], are in units of GeV and therefore, when comparing to data and other work it is more convenient to use GeV. The formulas above take on a different form when expressed in GeV. In order to do this note the following. Consider equation (1). The terms \( 5/T_{1l} \) and the arguments of \( \cos \) and \( \exp \) are obviously dimensionless. This means for example, that the 5 should really read 5 MeV, with the same units as \( T_{1l} \). Thus, in GeV this must be written as \( 5000/T_{1l} \). Also the number 0.199 must have units of energy\(^{-1/2}\). The number 25 must have units of energy, and in GeV units it becomes 25,000 GeV. The number 0.451 must have units of energy\(^{-0.258}\), so that it becomes \( 0.451/1000^{0.258} \) GeV\(^{-0.258}\).

The formulas are conveniently re-written as

\[ \sigma_{pp}^{HZETRN}(T_{1l}) = \left( 1 + \frac{a}{T_{1l}} \right) \left\{ 40 + 109 \cos(b \sqrt{T_{1l}}) \exp \left[ -c(T_{1l} - d)^{0.258} \right] \right\} \] \hspace{1cm} (for \( T_{1l} \geq 25 \) MeV) \hspace{1cm} (6)

\[ \sigma_{pp}^{HZETRN}(T_{1l}) = \exp \left\{ 6.51 \left[ \exp \left( -\left( \frac{T_{1l}}{e} \right)^{0.7} \right) \right] \right\} \] \hspace{1cm} (for \( T_{1l} < 25 \) MeV) \hspace{1cm} (7)

\[ \sigma_{np}^{HZETRN}(T_{1l}) = 38 + 12500 \exp \left[ -f(T_{1l} - g)^{0.35} \right] \] \hspace{1cm} (for \( T_{1l} \geq 0.1 \) MeV) \hspace{1cm} (8)

\[ \sigma_{np}^{HZETRN}(T_{1l}) = 26000 \exp \left[ -\left( \frac{T_{1l}}{h} \right)^{0.3} \right] \] \hspace{1cm} (for \( T_{1l} < 0.1 \) MeV) \hspace{1cm} (9)

where the constants \( a, b, c, d, e, f, g \) and \( h \) are given in Table 1 in the MeV unit system. It then becomes a straightforward matter to convert to GeV.
Table 1: Constants for Cross Section Formulas (6) - (9).

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5 MeV</td>
<td>0.199 MeV$^{-1/2}$</td>
<td>0.451 MeV$^{-0.258}$</td>
<td>25 MeV</td>
</tr>
<tr>
<td></td>
<td>e</td>
<td>f</td>
<td>g</td>
<td>h</td>
</tr>
<tr>
<td></td>
<td>134 MeV</td>
<td>1.187 MeV$^{-0.35}$</td>
<td>0.1 MeV</td>
<td>0.282 MeV</td>
</tr>
</tbody>
</table>

2 Comparison of HZETRN formulas with experiment

The HZETRN and BRYNTRN formulas for proton-proton total cross sections are shown as the dashed blue curves in figures 1 and 2 compared to experimental data, shown as the solid triangles, from the Particle Data Group [6]. Figure 1 shows that the apparent overall agreement between theory and experiment is quite good except in the EeV region, where the cosmic ray intensity is very low and therefore of negligible concern for space radiation. The GeV region is of most concern because it is near this region where the cosmic ray spectrum reaches a peak. Therefore, this region is examined in more detail in figure 2. The HZETRN cross section is found to oscillate which is not good behavior. In fact, this oscillation is present at most energies. Furthermore, the HZETRN cross section shows some disagreement with experiment in this energy range. Comparison with neutron-proton data is shown in figures 3 and 4. Again, the global fit looks quite good as shown in figure 3. However, when one looks more carefully at the important GeV region as shown in figure 4, one again notes some discrepancies. The aim of the present work is to correct these discrepancies. Their significance is that transport codes may give slightly incorrect results. The input cross sections should always be as accurate as possible.

3 New parametrizations of nucleon-nucleon total cross sections

The GeV region is often referred to as the intermediate energy region. The high energy region is up to $10^9$ GeV. Recall that $10^3$ GeV = TeV = $10^{12}$ eV and $10^6$ GeV = PeV = $10^{15}$ eV and $10^9$ GeV = EeV = $10^{18}$ eV and $10^{12}$ GeV = ZeV = $10^{21}$ eV. Thus, the region being referred to as the EeV region is of great interest from a pure physics point of view but is not of much interest for space radiation because the cosmic ray flux is very low in
this region. Nevertheless, for the sake of completeness, parametrizations in this region are
also studied because data are available. The dominant process in the very high energy
region is due to Pomeron exchange, such as flavorless gluons [7]. Parameterizations of
this very high energy region has been completed by Collins and Martin [7], Wong [8] and
De et al [9] and many other authors. This work has been condensed into a formula that
provides the best fit between intermediate and high energies by the Particle Data Group
[6]. Their cross section parameterization for both pp and pn reactions is

\[ \sigma^{PDG}(T_{1l}) = Z^{ab} + B \log^2(s/s_0) + Y^{ab}_1(s_1/s)^{\eta_1} - Y^{ab}_2(s_1/s)^{\eta_2} \]  

(10)

with the square of the center-of-momentum energy given by

\[ s = 2m(m + \sqrt{p_{1l}^2 + m^2}) \]  

(11)

where \( m \) is the nucleon mass and

\[ p_{1l} \equiv \sqrt{T_{1l}(T_{1l} + 2m)} \]  

(12)

In equation (10), the constants \( Z^{ab}, B, Y^{ab}_1, Y^{ab}_2 \) are in units of mb and \( s, s_1, s_0 \) are in units of GeV². The values of the constants are listed in Table 2.

Table 2: Constants [6] for Cross Section Formula (10).

<table>
<thead>
<tr>
<th>Beam</th>
<th>Target</th>
<th>( Z ) (mb)</th>
<th>( Y_1 ) (mb)</th>
<th>( Y_2 ) (mb)</th>
<th>( B ) (mb)</th>
<th>( s_1 ) (GeV²)</th>
<th>( \sqrt{s_0} ) (GeV)</th>
<th>( \eta_1 )</th>
<th>( \eta_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>p</td>
<td>35.45</td>
<td>42.53</td>
<td>33.34</td>
<td>0.308</td>
<td>1.0</td>
<td>5.38</td>
<td>0.458</td>
<td>0.545</td>
</tr>
<tr>
<td>p</td>
<td>n</td>
<td>35.80</td>
<td>40.15</td>
<td>30.00</td>
<td>0.308</td>
<td>1.0</td>
<td>5.38</td>
<td>0.458</td>
<td>0.545</td>
</tr>
</tbody>
</table>

3.1 Proton-Proton cross section

For proton-proton (pp) cross sections the HZETRN formula works well for \( p_{lab} < 1.8 \text{ GeV} \) and the Particle Data Group parameterizations work well for \( p_{lab} > 4.7 \text{ GeV} \). Between these energies a better fit is obtained with

\[ \sigma_{pp}^{JWN} = \frac{\gamma}{p_{1l}^{0.16}} \]  

(13)
where $\gamma \equiv 52.5 \text{ mb GeV}^{0.16} = 158.547 \text{ mb MeV}^{0.16}$. The final new parameterization is

$$
\sigma_{pp}^{\text{present}} = \sigma_{pp}^{\text{HZETRN}} \quad \text{for } p_{ll} < 1.8 \text{ GeV} \quad (i.e. \ T_{ll} < 1091.74 \text{ MeV}) \quad (14)
$$

$$
= \sigma_{pp}^{JWN} \quad \text{for } 1.8 \text{ GeV} \leq p_{ll} \leq 4.7 \text{ GeV} \quad (15)
$$

$$
= \sigma_{pp}^{PDG} \quad \text{for } p_{ll} > 4.7 \text{ GeV} \quad (i.e. \ T_{ll} > 3854.69 \text{ MeV}) \quad (16)
$$

Note that these formulas are discontinuous. See figures 1 and 2.

### 3.2 Neutron-Proton cross section

For proton-proton (pp) cross sections the HZETRN formula works well for $p_{lab} < 1.8 \text{ GeV}$ and the Particle Data Group parameterizations work well for $p_{lab} > 4.7 \text{ GeV}$. Between these energies a better fit is obtained with

$$
\sigma_{np}^{JWN}(T_{ll}) = 40 + 10 \cos(\alpha p_{ll} - 0.943) \exp(-\beta p_{ll}^{0.8} + 2) \quad (17)
$$

with $p_{ll}$ given by (12). The constants $\alpha$ and $\beta$ are given in Table 3. The final new parameterization of the present work is

$$
\sigma_{pp}^{\text{present}} = \sigma_{np}^{HZETRN} \quad \text{for } p_{ll} < 0.5 \text{ GeV} \quad (i.e. \ T_{ll} < 124.941 \text{ MeV}) \quad (18)
$$

$$
= \sigma_{np}^{JWN} \quad \text{for } 0.5 \text{ GeV} \leq p_{ll} \leq 2.0 \text{ GeV} \quad (19)
$$

$$
= \sigma_{np}^{PDG} \quad \text{for } p_{ll} > 2.0 \text{ GeV} \quad (i.e. \ T_{ll} > 1271.04 \text{ MeV}) \quad (20)
$$

Note that these formulas are also discontinuous. See figures 3 and 4.

Table 3: Constants for Cross Section Formula (17).

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>0.00369 MeV$^{-1}$</td>
<td>0.00895741 MeV$^{-0.8}$</td>
</tr>
</tbody>
</table>
4 Conclusions

Nucleon-nucleon total (elastic plus inelastic) cross sections are inputs to space radiation transport codes. The nucleon-nucleon total cross sections currently in use in the transport codes HZETRN and BRYNTRN provide an apparently good global fit to the data. However, when one looks more closely at the important GeV energy range the comparison to data is poor. This energy range is near the maximum intensity of the cosmic ray spectrum and is therefore important for space radiation studies. Improved parameterizations have been developed in the present work, which provide a good fit to data over the entire energy range. The new formulas have been incorporated in the HZETRN code.
Figure 1: Total proton-proton cross section versus the projectile momentum in the lab frame, $p_{\text{lab}}$. The blue dashed curve is used in HZETRN [4] and the red solid curve is the present work. Experimental data are from Reference [6].

Figure 2: Detail of Fig. 1.
Figure 3: Total proton-neutron cross section. The blue dashed curve is used in HZETRN [4] and the red solid curve is the present work. Experimental data are from Reference [6].

Figure 4: Detail of Fig. 3.
References


The total proton-proton and neutron-proton cross sections currently used in the transport code HZETRN show significant disagreement with experiment in the GeV and EeV energy ranges. The GeV range is near the region of maximum cosmic ray intensity. It is therefore important to correct these cross sections, so that predictions of space radiation environments will be accurate. Parameterizations of nucleon-nucleon total cross sections are developed which are accurate over the entire energy range of the cosmic ray spectrum.