Lightning Charge Retrievals:
Dimensional Reduction, LDAR Constraints, and a First Comparison w/LIS Satellite Data.

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SDMetrics Entry - Presentations

Title: Lightning Charge Retrievals: Dimensional Reduction, LDAR Constraints, and a First Comparison w/LIS Satellite Data

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What is an Inverse Problem?

Unknown $f_i$

Law

Measurements $g$
First Inversion Scientist?

Hunger ...
Tracks ...
Invert ...
Decide to Follow ...
Find ...
Kill ...
Eat!
1st International Interactive Workshop on Inverse Methods
1976

General Inversion:

$\vec{g}_t$ + $\vec{e}$

$\vec{f}_t$ → $\vec{a} (\vec{f}_t)$ + $\mu$ → $\mu(\vec{f}_t)$ → Nonlinear Inversion Method on → $\vec{f}$

External Constraints

Numerical Stabilization
Linear Inversion:

\[ \tilde{g}_t + \tilde{e} \]

\[ f_t \]

\[ A \]

\[ +M \]

\[ K \]

External Constraints

Linear Inversion Method on

Numerical Stabilization

\[ \tilde{f} \]
Regression:

\[ \tilde{g}_t \]

\[ + \tilde{E} \]

\[ A \]

\[ + M \]

\[ K \]

External Constraints

Regression Method on

Numerical Stabilization
Examples

Temperature Sounding (IR Spectrometer)

Lightning Charge (Electric Field Change)

Earth Interior Density (Sound Wave Propagation)

Cardiac Parameters (Cerebral Electric Potential)

... Many More!
We are ALL Inverters

\[ \tilde{g}, \tilde{k}(\tilde{f}_t) \]

Birth \hspace{2cm} time \hspace{2cm} Death

After Life

\[ \tilde{f}_t ? \]
\[ \Delta E_i = \int J_i(r) \Delta \rho(r) dV \]
Chi-Squared Goodness-of-Fit

\[ \chi^2(a) = \frac{1}{m-n} \sum_{i=1}^{m} \frac{(M_i(a) - \Delta E_i)^2}{\sigma_i^2} \]

\[ a = (a_1, \ldots, a_n) \]
Ockham's Razor

"Pluralitas non est ponenda sine neccesitate"

Circa 1288-1348
NEEDS (neccesitate)
• Air
• Water
• Food
• Minimal Shelter/Clothing
• Medical Care
• Love

WANTS (Pluralitas)
• A/C
• SUV
• Jet Ski
• Designer Clothing
• Boat/Yacht
• Video Camera
• Gourmet Foods
• Swimming Pools
• Sporting Goods
• Stereos
• Plastic Surgery
• Bungee Jumping
• Vacations
• Opulent Home(s)
•
• etc.
•
Electrostatic Field Changes Produced by Florida Lightning

ELIZABETH A. JACOBSON AND E. PHILIP KRIDER

ABSTRACT

The electrical behavior of thunderstorms triggered by local heating and sea-breeze convergence, a low pressure disturbance, and a weak land-sea breeze has been studied at the NASA Kennedy Space Center, Florida. A nonlinear least-squares optimization procedure has been developed to describe changes in the near-electrostatic field produced by lightning in terms of point charges located for the cloud charge distribution. The results of this analysis indicate that discharges to ground usually are emitted from times when the magnitude of the lightning charge is greater than that for other geographic locations, 6 to 9 times, but the corresponding ambient air temperatures, -10 to -24°C, are smaller. A large fraction of the discharges to ground show near-field changes which are small or even reversed in polarity within 3 km of the discharges. An analysis of these cases suggests that ground discharges often involve a small positive charge, 0.1 to 0.5 C at a distance of 1 to 3 km, in addition to the larger negative charge higher in the cloud.

1. Introduction

The NASA Kennedy Space Center (KSC) has constructed and is currently operating a large network of ground-based electric field masts, in order to identify cloud discharges which might be an electrical hazard to spacecraft prior to or during launch. These instruments have been installed to minimize the chance of lightning disturbances such as those which occurred during the launch of Apollo 11 (Krider et al., 1974). The KSC network represents a unique facility for the study of lightning and thunderstorms not only because of its size and the quality of its data acquisition system, but also because it is located in Florida, a region of high thunderstorm frequency which has received little previous study. In this paper, we present and analyze typical electric field produced by thunderstorms and lightning at KSC. Data were obtained during two storms in 1973 using a single field mast and for a number of storms in 1974 using 21 instruments.

2. Instrumentation

A map showing the locations of the field mast sites at KSC is given in Fig. 1. Each field mast contains a high-voltage, oriented sensor which are alternately covered and uncovered by a grounded rotator turning at 1800 rpm. The differential voltage between the covered and uncovered sensors is filtered to remove high-frequency components, amplified, and rectified using a synchronous sample and hold circuit. The resulting signal is passed through a low-pass filter with a 0.1 s time constant to remove ac components and provides both the amplitude and polarity of the near-electrostatic electric field. The 0.1 s time constant is too slow to trace the individual return strokes in discharges to ground, but is more than adequate to resolve an entire flash. The analog output of each field mast is transmitted to a central receiving station, where it is...
1-Charge Model (4 parameters)

\[ M_{li}(a_i) = \frac{-Qz}{2\pi\varepsilon_0[(X_i - x)^2 + (Y_i - y)^2 + z^2]^{3/2}}, \]

\[ a_i = (x, y, z, Q) \]
I. INTRODUCTION

Wissen [1976] first suggested that simultaneous measurements of electric field changes caused by lightning could be used to

structure the charge of the storm, and to determine its magnitude. The single stroke model which was adopted in this study is based on a simple model of the storm structure which was developed by Wissen. The model assumes that the electric field changes caused by lightning are linearly related to the charge of the storm. This model has been shown to be valid for a wide range of storms, and has been used to estimate the magnitude of the storm charge. The model also assumes that the electric field changes caused by lightning are instantaneous. This assumption is valid for most storms, but not for storms with large amounts of cloud-to-ground lightning.

The single stroke model was used to estimate the magnitude of the storm charge. The model was applied to the data from the electric field measurements, and the results were compared to the data from the ground-based lightning measurements. The results showed that the single stroke model is accurate for most storms, but not for storms with large amounts of cloud-to-ground lightning.

Other models of lightning have been developed to account for the non-linear behavior of the electric field changes. These models are based on a different set of assumptions, and are not as accurate as the single stroke model. The non-linear models are more complex, and require more data to be accurately applied. However, the non-linear models are not as accurate as the single stroke model, and are not as widely used.

In conclusion, the single stroke model is the most accurate model for estimating the magnitude of the storm charge. This model is based on a simple set of assumptions, and is easy to apply. The model has been shown to be accurate for most storms, and is the most widely used model for estimating the magnitude of the storm charge.

Krehbiel, Brook, and McCrory 1979
4-Station Solution (Krehbiel et al., 1979)

START: \[ |\Delta E_i| = \frac{|Q|_z}{2\pi \epsilon_o [(X_i - x)^2 + (Y_i - y)^2 + z^2]^{3/2}}. \]

Algebra & Differencing Stations GIVES:

\[
\begin{bmatrix}
2(X_1 - X_2) & 2(Y_1 - Y_2) & U_{12} \\
2(X_1 - X_3) & 2(Y_1 - Y_3) & U_{13} \\
2(X_1 - X_4) & 2(Y_1 - Y_4) & U_{14}
\end{bmatrix}
\begin{bmatrix}
x \\ y \\ \eta
\end{bmatrix}
= \begin{bmatrix}
X_1^2 + Y_1^2 - X_2^2 - Y_2^2 \\
X_1^2 + Y_1^2 - X_3^2 - Y_3^2 \\
X_1^2 + Y_1^2 - X_4^2 - Y_4^2
\end{bmatrix},
\]

\(U_{ij}\)'s depend on the \(\Delta E\)s

\[|Q|_z = \frac{1}{2} \eta^{3/2}\]
Overdetermined Fixed Matrix (OFM) Approach

START: \[ |\Delta E_i| = \frac{|Q|^z}{2\pi\varepsilon_o \left[ (X_i - x)^2 + (Y_i - y)^2 + z^2 \right]^{3/2}}. \]

Raise to -2/3 power, Algebra & NO Differencing of Stations GIVES:

\[
\begin{bmatrix}
  d_1 \\
  \vdots \\
  d_m
\end{bmatrix} = \begin{bmatrix}
  (X_1^2 + Y_1^2) & -2X_1 & -2Y_1 & 1 \\
  \vdots & \vdots & \vdots & \vdots \\
  (X_m^2 + Y_m^2) & -2X_m & -2Y_m & 1
\end{bmatrix}
\begin{bmatrix}
w \\
w_x \\
w_y \\
w_{r^2}
\end{bmatrix} = Ts,
\]

\[ d_i = |\Delta E_i|^{-2/3}, \quad w = \left( \frac{2\pi\varepsilon_o}{|Q|^z} \right)^{2/3}, \quad r^2 = x^2 + y^2 + z^2. \]

Extraction:
\[
x = s_i / s_1 \\
y = s_i / s_1 \\
z = (s_i, s_4 - s_2^2 - s_3^2)^{1/2} / s_1 \\
Q = \left( 2\pi\varepsilon_o \Delta E_i \right) / \left[ (s_i, s_4 - s_2^2 - s_3^2)^{1/2} |\Delta E_i| \right], \quad \text{where} \quad s = (T^T T)^{-1} T^T d.\]
## 4-Station vs. OFM

<table>
<thead>
<tr>
<th>Topic</th>
<th>4-Station</th>
<th>OFM</th>
</tr>
</thead>
<tbody>
<tr>
<td># stations</td>
<td>4</td>
<td>( m )</td>
</tr>
<tr>
<td>Withstand errors?</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(least squares)</td>
</tr>
<tr>
<td>Fixed Matrix?</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Understand eigenvalues?</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Computationally efficient?</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Bipolar Retrievals?</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Replaces Chi-Square Analyses?</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>
360,000 monopoles retrieved

96,179 ruptured

456,179 analyzed
360,000 monopoles retrieved

198 ruptured

360,198 analyzed
2-charge Model (8 Parameters)

\[
M_{Zi}(a_2) = \frac{-1}{2\pi\varepsilon_0} \left[ \frac{Qz}{[(X_i-x)^2 + (Y_i-y)^2 + z^2]^{3/2}} + \frac{Q'z'}{[(X_i'-x')^2 + (Y_i'-y')^2 + z'^2]^{3/2}} \right],
\]

\[
a_2 = (x, y, z, Q, x', y', z', Q')
\]
DR Model (2 Charges but 4 Parameters!)

\[ D_i(a_1) = M_{1i}(a_1) + \gamma_i(a_1), \]

\[ \gamma_i(a_1) = M_{ii}(a_i) = \frac{-Q(a_i)z'(a_i)}{2\pi\varepsilon_0 \left[ (X_i - x'(a_i))^2 + (Y_i - y'(a_i))^2 + (z'(a_i))^2 \right]^{3/2}}, \]

\[ a_1 = (x, y, z, Q) \]
DR Chi-Squared

\[ \chi^2(a_1) = \frac{1}{m-4} \sum_{i=1}^{m} \left( \frac{D_i(a_1) - \Delta E_i}{\sigma_i} \right)^2 \]

\[ a_1 = (x, y, z, Q) \]
Scan Slab that Intersects Negative Charge Region

\[ a'_1 = (x', y', z', Q') \]

\[ a_1 = (x, y, z, Q) \]

\[ \Delta E_i - M_{ii}(a_i) = \text{Residual Field Change} \]
Typical DR Retrieval
(Lines Up With LDAR)
LDAR-Constrained Charge Solutions

\[
\begin{bmatrix}
Q \\
Q'
\end{bmatrix} = (K^T K)^{-1} K^T g.
\]
June 11, 1999 (Day 162)
Q = 40.072, Q' = -36.123

[Graphs and charts showing spatial data with coordinates X, Y, Z and time stamps 05:06:49.817 - 05:06:50.817]
June 29, 1999 (Day 180)

\( Q = 33.633, Q' = 3.760 \)
September 21, 1998 (Day 264)

$Q=9.317$, $Q'=-3.337$

![Graphs showing time and location data]
September 21, 1998 (Day 264)

\[ Q = 2.462, Q' = -2.412 \]
September 21, 1998 (Day 264)
Q=39.719, Q'=10.397

Time: 20:40:43.391 - 20:40:47.391

GBFM noon time (GMT) = 20:40:47.100
Strike to Launch Pad 39B (STS-115)

(August 25, 2006 @ 17:49:17.78 GMT)
Thank You!