Lightning Charge Retrievals:
Dimensional Reduction, LDAR Constraints, and
a First Comparison w/LIS Satellite Data.

William Koshak, MSFC
E. Philip Krider, UA
Natalie Murray, UA
Dennis Boccippio, MSFC
SDMetrics Entry - Presentations

Title: Lightning Charge Retrievals: Dimensional Reduction, LDAR Constraints, and a First Comparison w/LIS Satellite Data

Presenters: W. J. Koshak, E. P. Krider, N. Murray, D. J. Boccippio

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Organization: VP61
What is an Inverse Problem?

Unknown $f_i$

Law

Measurements $\tilde{g}$
First Inversion Scientist?

- Hunger ...
- Tracks ...
- Invert ...
- Decide to Follow ...
- Find ...
- Kill ...
- Eat!
1st International Interactive Workshop on Inverse Methods
1976

General Inversion:

\[ \tilde{g}_t + \tilde{\epsilon} \]

\[ \tilde{a}(\tilde{f}_t) + \mu \]

\[ \tilde{f} \]

\[ \tilde{f}_t \]

\[ \mu(f_t) \]

Nonlinear Inversion Method on

External Constraints

Numerical Stabilization
Linear Inversion:

\[ \vec{g}_t + \vec{\varepsilon} \]

\[ \vec{f}_t \]

- External Constraints
- Linear Inversion Method on
- Numerical Stabilization

\[ A \quad K \]
Regression:

\[ \vec{g}_t + \vec{\epsilon} \]

\[ \vec{f}_t \]

\[ A \]

\[ K \]

\[ \text{Regression Method on} \]

\[ \text{External Constraints} \]

\[ \text{Numerical Stabilization} \]
Examples

Temperature Sounding
(IR Spectrometer)

Lightning Charge
(Electric Field Change)

Earth Interior Density
(Sound Wave Propagation)

Cardiac Parameters
(Electric Potential)

... Many More!
We are ALL Inverters
\[ \Delta E_i = \int_{\mathcal{V}} K_i(r) \Delta \rho(r) d\mathcal{V} \]
KSC Field Mill Network

Diagram of the KSC Field Mill Network.
Chi-Squared Goodness-of-Fit

\[ \chi^2(a) = \frac{1}{m-n} \sum_{i=1}^{m} \frac{(M_i(a) - \Delta E_i)^2}{\sigma_i^2} \]

\[ a = (a_1, \ldots, a_n) \]
Ockham’s Razor

"Pluralitas non est ponenda sine neccesitate"

Circa 1288-1348

JOC/EFR © February 2005
http://www-history.mcs.st-andrews.ac.uk/Mathematicians/Ockham.html
**NEEDS** (neccesitate)
- Air
- Water
- Food
- Minimal Shelter/Clothing
- Medical Care
- Love

**WANTS** (Pluralitas)
- A/C
- SUV
- Jet Ski
- Designer Clothing
- Boat/Yacht
- Video Camera
- Gourmet Foods
- Swimming Pools
- Sporting Goods
- Stereos
- Plastic Surgery
- Bungee Jumping
- Vacations
- Opulent Home(s)
- etc.
Electrostatic Field Changes Produced by Florida Lightning

ELIZABETH A. JACOBSON AND E. PHILIP KRIDER

Institute of Atmospheric Physics, The University of Arizona, Tucson 85721
(Manuscript received 15 June 1975, in revised form 21 September 1975)

ABSTRACT

The electrostatic behavior of thunderstorms triggered by local heating and sea-breeze convergence, a low pressure disturbance, and a weak blast passage has been studied at the NASA Kennedy Space Center, Florida. A nonlinear least-squares estimation procedure has been developed to describe changes in the mean electrostatic field produced by lightning in terms of point charge models for cloud charge destabilization. The results of this analysis indicate that discharges to ground usually neutralize cloud charge in the range from -10 to -60 C. The measured charge altitudes for Florida are generally higher than for other geographical locations, 6 to 9.5 km, but the corresponding ambient air temperatures, -20 to -34C, are lower. A large fraction of the discharges to ground have field changes which are small in comparison in velocity within 3 km of the discharge. An analysis of three cases suggests that ground discharges often neutralize a small positive charge, 0.3 to 0.4 C at altitudes 3 to 5 km, in addition to the large negative charge higher in the cloud.

1. Introduction

The NASA Kennedy Space Center (KSC) has constructed and is currently operating a large network of ground-based electric field mills, in order to identify clouds which might be an electrical hazard to space vehicles prior to and during launch. These instruments have been installed to minimize the chance of lightning disturbances such as those which occurred during the launch of Apollo 12 (Krider et al., 1974). The KSC network represents a unique facility for the study of lightning and thunderstorms not only because of its size and the quality of its data acquisition system, but also because it is located in Florida, a region of high thunderstorm frequency which has received little previous study. In this paper, we present and analyze typical electric fields produced by thunderstorms and lightning at KSC. Data were obtained during two storms in 1973 using a single field mill and for a number of storms in 1974 using 21 instruments.

2. Instrumentation

A map showing the locations of the field mill sites at KSC is given in Fig. 1. Each field mill contains a vertically oriented stator section which is alternatively covered and uncovered by a grounded rotor turning at 1800 rpm. The differential voltage between the covered and uncovered stator is filtered to remove high-frequency components, amplified, and rectified using a synchronous sample and hold circuit. The resulting signal is passed through a low-pass filter with a 0.1 s time constant to remove ac components and provides both the amplitude and polarity of the external electric field. The 0.1 s time constant is too slow to time-resolve individual return strokes in discharges to ground, but is more than adequate to resolve an entire flash. The analog output of each field mill is transmitted to a central receiving station, where it is

Fig. 1. The locations of the electric field measuring sites at the NASA Kennedy Space Center, Florida. WS is the site of the KSC weather office, and WS-8 the location of the X-band radar.
1-Charge Model (4 parameters)

\[ M_{li}(\mathbf{a}_1) = \frac{-Qz}{2\pi \varepsilon_0 \left[ (X_i - x)^2 + (Y_i - y)^2 + z^2 \right]^{3/2}}, \]

\[ \mathbf{a}_1 = (x, y, z, Q) \]
An Analysis of the Charge Structure of Lightning Discharges to Ground

Paul R. Kaspari, Mark Brook, and Ron A. McCrory

New Mexico Institute of Mining and Technology, Socorro, New Mexico 87801

1. INTRODUCTION

Wilson (1916) first suggested that simultaneous measurements in a sufficiently large number of points of the electromagnetic field change caused by lightning could be used to construct a measure of the discharge, and he proceeded to specify the equations relating the field change at the ground to the height and distance of simple one- and two-point charge models. Although it was not feasible for him to make such measurements, he appreciated their significance in this and a later study (Wilson, 1919) by increasing the electric field change as a function of distance from various lightning discharges. From this, he was able to obtain estimates of the average electric moment destroyed by a lightning flash and of the height and magnitudes of the discharge involved.

The first multiparameter lightning study was conducted in New Mexico by Woodman et al. (1962), who made simultaneous measurements of the electric field change caused by lightning at eight stations over an area 5 km in diameter. The data were used to compute details of charge structures for individual strokes of discharges to ground and for complex interstroke discharges. Both the negative and positive changes were found to be within a limited range of ambient static field, temperature, between 5° and 25°C, and were distributed horizontally within the area. Evidence of anhedral electric structure was identified which followed the motion of the storm and was associated with one sheet below cloud base.

Krehbiel, Brook, and McCrory 1979
4-Station Solution (Krehbiel et al., 1979)

\[ |\Delta E_i| = \frac{|Q|_z}{2\pi \varepsilon_0 [(X_i - x)^2 + (Y_i - y)^2 + z^2]^{3/2}} \]

Algebra & Differencing Stations GIVES:

\[
\begin{bmatrix}
2(X_1 - X_2) & 2(Y_1 - Y_2) & U_{12} \\
2(X_1 - X_3) & 2(Y_1 - Y_3) & U_{13} \\
2(X_1 - X_4) & 2(Y_1 - Y_4) & U_{14}
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
\eta
\end{bmatrix} =
\begin{bmatrix}
X_1^2 + Y_1^2 - X_2^2 - Y_2^2 \\
X_1^2 + Y_1^2 - X_3^2 - Y_3^2 \\
X_1^2 + Y_1^2 - X_4^2 - Y_4^2
\end{bmatrix},
\]

\(U_{ij}\)'s depend on the \(\Delta E_i\)s

\[ |Q|_z = \frac{1}{2} \eta^{3/2} \]
Overdetermined Fixed Matrix (OFM) Approach

\[
|\Delta E_i| = \frac{|Q|^z}{2\pi\varepsilon_o \left[ (X_i - x)^2 + (Y_i - y)^2 + z^2 \right]^{3/2}}.
\]

Raise to -2/3 power, Algebra & NO Differencing of Stations GIVES:

\[
\begin{bmatrix}
  d_1 \\
  \vdots \\
  d_m
\end{bmatrix} = \begin{bmatrix}
  (X_1^2 + Y_1^2) & -2X_1 & -2Y_1 & 1 \\
  \vdots & \vdots & \vdots & \vdots \\
  (X_m^2 + Y_m^2) & -2X_m & -2Y_m & 1
\end{bmatrix} \begin{bmatrix}
  w \\
  wx \\
  wy \\
  wr^2
\end{bmatrix} \equiv T_s,
\]

\[
d_i \equiv |\Delta E_i|^{-2/3}, \quad w \equiv \left( \frac{2\pi\varepsilon_o}{|Q|^z} \right)^{2/3}, \quad r^2 \equiv x^2 + y^2 + z^2.
\]

Extraction:

\[
\begin{align*}
x &= s_2/s_1 \\
y &= s_3/s_1 \\
z &= (s_4s_4 - s_2^2 - s_3^2)/s_1 \\
Q &= (2\pi\varepsilon_o \Delta E_i) \left[ (s_4^2s_4 - s_2s_2^2 - s_3s_3^2)^{1/2} |\Delta E_i| \right], \text{ where } s = (T^T T)^{-1} T^T d
\end{align*}
\]
# 4-Station vs. OFM

<table>
<thead>
<tr>
<th>Topic</th>
<th>4-Station</th>
<th>OFM</th>
</tr>
</thead>
<tbody>
<tr>
<td># stations</td>
<td>4</td>
<td>m</td>
</tr>
<tr>
<td>Withstand errors?</td>
<td>No</td>
<td>Yes (least squares)</td>
</tr>
<tr>
<td>Fixed Matrix?</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Understand eigenvalues?</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Computationally efficient?</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Bipolar Retrievals?</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Replaces Chi-Square Analyses?</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>
360,000 monopoles retrieved

96,179 ruptured

456,179 analyzed
360,000 monopoles retrieved
198 ruptured
360,198 analyzed
2-Charge Model (8 Parameters)

\[
M_{2t}(a_2) = \frac{-1}{2\pi\varepsilon_0} \left[ \frac{Qz}{[(X_i - x)^2 + (Y_i - y)^2 + z^2]^{3/2}} + \frac{Q'z'}{[(X_i - x')^2 + (Y_i - y')^2 + z'^2]^{3/2}} \right],
\]

\[
a_2 = (x, y, z, Q, x', y', z', Q')
\]
DR Model (2 Charges but 4 Parameters!)

\[ D_i(a_1) = M_{li}(a_1) + Y_i(a_1), \]

\[ Y_i(a_1) = M_{li}(a_1) = \frac{-Q'(a_1)z'(a_1)}{2\pi \varepsilon_0 \left[ (X_i - x'(a_1))^2 + (Y_i - y'(a_1))^2 + (z'(a_1))^2 \right]^{3/2}}, \]

\[ a_1 = (x, y, z, Q) \]
DR Chi-Squared

\[ \chi^2(a_1) = \frac{1}{m-4} \sum_{i=1}^{m} \frac{(D_i(a_1) - \Delta E_i)^2}{\sigma_i^2} \]

\[ a_1 = (x, y, z, Q) \]
Scan Slab that Intersects Negative Charge Region

\[ \mathbf{a}_1 = (x, y, z, Q) \]

\[ \mathbf{a}'_1 = (x', y', z', Q') \]

\[ \Delta E_i - M_{ii}(a_i) = \text{Residual Field Change} \]
Typical DR Retrieval
(Lines Up With LDAR)
LDAR-Constrained Charge Solutions

\[ \begin{bmatrix} Q \\ Q' \end{bmatrix} = (K^T K)^{-1} K^T g. \]
May 14, 1999 (Day 134)

Q=56.168, Q'=8.209
June 11, 1999 (Day 162)
Q = 40.072, Q' = -36.123
June 29, 1999 (Day 180)

Q = 33.633, \( Q' = 3.760 \)

Time: 19:02:34.393 - 19:02:36.393

GBFM Flashtime (GMT) = 19:02:57.100
September 21, 1998 (Day 264)

$Q = 9.317, \ Q' = -3.337$

```
+-----------------+-----------------+
|                  |                  |
|                  |                  |
|                  |                  |
+-----------------+-----------------+
```
September 21, 1998 (Day 264)

$Q = 2.462$, $Q' = -2.412$
September 21, 1998 (Day 264)

Q=39.719, Q'=10.397

GBFM Flashtime (GMT) = 20:40:47.100
Strike to Launch Pad 39B (STS-115)

(August 25, 2006 @ 17:49:17.78 GMT)
Thank You!