ABSTRACT

The effect of hoop stresses in reducing cylindrical roller bearing fatigue life was determined for various classes of inner ring interference fit. Calculations were performed for up to seven interference fit classes for each of ten bearing sizes. Each fit was taken at the tightest, average and loosest values within the fit class for RBEC-5 tolerance, thus requiring 486 separate analyses. The hoop stresses were superimposed on the Hertzian principal stresses created by the applied radial load to calculate roller bearing fatigue life. The method was developed through a series of equations to calculate the life reduction for cylindrical roller bearings based on interference fit. All calculated lives are for zero initial bearing internal clearance. Any reduction in bearing clearance due to interference fit was compensated by increasing the initial (unmounted) clearance. Results are presented as tables and charts of life factors for bearings with light, moderate and heavy loads and interference fits ranging from extremely light to extremely heavy and for bearing accuracy class RBEC 5 (ISO class 5). Interference fits on the inner bearing ring of a cylindrical roller bearing can significantly reduce bearing fatigue life. In general, life factors are smaller (lower life) for bearings running under light load where the unfactored life is highest. The various bearing series within a particular bore size had almost identical interference fit life factors for a particular fit. The tightest fit at the high end of the RBEC-5 tolerance band defined in ANSI/ABMA shaft fit tables produces a life factor of approximately 0.40 for an inner-race maximum Hertz stress of 1200 MPa (175 ksi) and a life factor of 0.60 for an inner-race maximum Hertz stress of 2200 MPa (320 ksi). Interference fits also impact the maximum Hertz stress-life relation.

Keywords: Rolling element bearings; Roller bearings; Interference fit; Bearing life prediction; Stress

INTRODUCTION

Rolling-element bearings often utilize a tight interference fit between the bearing inner ring and shaft or between the outer ring and housing bore to prevent fretting damage at the interface. American National Standards Institute/American Bearing Manufacturers Association (ansi/abma) standards (1-2) as well as catalogs of bearing manufacturers specify suggested fits for various operating conditions that must be based on the most severe operating conditions expected, including highest speeds and highest vibration levels.

A tight fit of the bearing inner ring over the shaft reduces internal bearing clearance. The clearance change can be compensated for by adding initial internal clearance to the bearing. However, the force fit of the inner ring over the shaft also adds a hoop stress on the bearing inner ring. Czyzewski (3) showed that the hoop stresses are generally tensile (designated by a plus (+) sign) and can negatively affect fatigue life.

Coe and Zaretsky (4) analyzed the effect of hoop stresses on rolling-element fatigue. Their work was based on the analysis of Hertzian principle stresses from Jones (5) and the Lundberg-Palmgren bearing life theory (6). Coe and Zaretsky (4) superimposed the hoop stresses on the Hertzian principle stresses whereby the shearing stresses in the stressed volume of the contact between the rolling element (ball or roller) and inner race of the bearing are increased. The increased maximum shearing stress at a depth below the contacting surface due to hoop stress is

$$\tau_{max} = \tau_{max} + \frac{\sigma_h}{2}$$

where \(\tau_{max}\) is the maximum shearing stress, \(\tau_{max}^{h}\) is the maximum shearing stress including the effect of the hoop stress and \(\sigma_h\) is the hoop stress.

Coe and Zaretsky (4) applied Eq. [1] to modify the Lundberg-Palmgren life equation as follows:

$$L_{40} = \left( \frac{\tau_{max}}{\tau_{max}^{h}} \right)^{\frac{c}{e}} \left( \frac{C_D}{P_{eq}} \right)^p$$

where \(c\) is a stress-life exponent, \(e\) is the Weibull slope or modulus, \(C_D\) is the bearing dynamic load capacity, \(P_{eq}\) is the equivalent bearing load and \(p\) is a load–life exponent.

The Coe-Zaretsky analysis assumed for simplicity that all components (inner race, rollers and outer race) were affected equally by the inner-race interference fit. This results in a conservative prediction of bearing life. A more rigorous analysis should apply the life reduction due to hoop stress to only the inner race without modifying the lives of the outer race and rolling-element set.

Subsequent to the Coe-Zaretsky analysis, Zaretsky (7) developed a procedure (Zaretsky’s Rule) for separating the lives of bearing races from the lives of the rolling elements (considered as a set). This procedure allows for calculating the reduction of bearing life from an inner-ring force fit without affecting the rolling-element set and outer race lives; thus providing a more accurate bearing life analysis.

In view of the aforementioned, the objectives of this work were to: (a) independently determine the lives of the inner races, outer races and roller sets for several classes of radially-
loaded, cylindrical roller bearings subject to inner-ring interference fit; (b) calculate the reduction in cylindrical roller bearing fatigue life due to interference fit of the inner ring; and (c) develop life factors applied to the bearing life calculation for force fits according to the ANSI/ABMA standards for shaft fitting practice.

**NOMENCLATURE**

- **b** : semiwidth of Hertzian contact area, m (in)
- **C_D** : Dynamic load capacity, N (lb f)
- **C_0** : Static load capacity, N (lb f)
- **d** : roller diameter, mm (in)
- **d_e** : Bearing pitch diameter, mm (in)
- **D_{IR}** : Inner race diameter, mm (in)
- **D_s** : Common diameter of shaft and inner ring bore, mm (in)
- **E** : Elastic (Young’s) modulus, MPa (psi)
- **e** : Weibull slope
- **L** : Roller life, millions of inner race revolutions
- **L_{10}** : 10-percent life or life at which 90 percent of a population survives, number of inner race revolutions or hr
- **LR** : Life ratio
- **p** : Load-life exponent
- **P** : Radial load on bearing, N (lb f)
- **P_{eq}** : Equivalent bearing load, N (lb f)
- **P_i** : Pressure between shaft and inner race due to force fit, MPa (psi)
- **S or σ** : Stress, MPa (ksi)
- **S'** : Indicates tangential stress including hoop stress superimposed on Hertz stress, MPa (ksi)
- **S_{max,IR}** : Maximum Hertz stress on inner race, MPa (psi)
- **S_{max,OR}** : Maximum Hertz stress on outer race, MPa (psi)
- **u** : Dimensionless depth below surface to maximum shear stress (=z/b).
- **z** : Distance below surface to maximum shear stress due to Hertzian load, m (in)
- **Δ** : Diametral interference, mm (in)
- **ν** : Poisson’s ratio
- **σ_h** : Hoop stress, MPa (psi)
- **τ_{max}** : maximum shear stress, MPa (psi)
- **(τ_{max}_h)** : maximum shear stress modified by hoop stress, MPa (psi)

**Subscripts**

- **adj** : Indicates adjusted life
- **h** : Indicates hoop stress (in tangential or X-direction)
- **IR, OR** : Indicates inner or outer race of bearing
- **n or z** : Indicates normal direction
- **R** : Indicates rollers
- **t or x** : Indicates tangential direction

**ENABLING EQUATIONS**

**Subsurface Shearing Stresses**

A representative cylindrical roller bearing is shown in Fig. 1. The bearing comprises an inner and outer ring and plurality of rollers interspersed between the two rings and positioned by a cage or separator.

Figure 2(a) is a schematic of the contact of a cylindrical roller on a race. Figure 2(b) shows the principle stresses at and beneath the surface. From these principle stresses the shearing stresses can be calculated. There are four shearing stresses that can be applied to bearing life analysis. These are the orthogonal shearing stress, the octahedral shearing stress, the von Mises stress and the maximum shearing stress. For the analysis reported herein, only the maximum shearing stresses are considered.

The maximum shearing stress is one half the maximum difference between principle stresses, thus

\[ \tau_{max} = \frac{\sigma_{z} - \sigma_{x}}{2} \]  \[3\]

Coe and Zaretsky (4) showed that the subsurface shear stress in a cylindrical roller bearing due to Hertzian loading as a function of \( z \), the depth below the surface can be expressed in terms of the maximum Hertz stress \( S_{max} \) and \( u \), the non-dimensional depth below the surface as

\[ \tau = S_{max} \sqrt{1 + u^2} - u - \frac{1}{\sqrt{1 + u^2}} \]  \[4\]
where \( u = \frac{z}{b} \) and \( b \) is the semi-width of the contact area (Fig. 2(a)). By setting the derivative of Eq. [4] with respect to \( u \), equal to zero and solving by iteration, the maximum shearing stress from Hertzian loading is found to occur at \( u = 0.786152 \). Substituting \( u \) into Eq. [4], gives the maximum shear stress due to Hertzian loading:

\[
\tau_{\text{max}} = -0.30028 S_{\text{max}}^{\text{max}} \quad [5]
\]

Coe and Zaretsky (4) considered effects due to the hoop stress from a force fit of the inner ring on the shaft and the effects due to rotation superposed on the Hertz stress. Their analysis showed that the additional effects cause little change in the location of the maximum shear stress on the inner-race surface; the variation in \( u \) due to the added hoop stress is only 0.04 percent.

The principle stresses in the tangential direction and the effect of the added hoop stress are illustrated in Fig. 3. \( S_n \) indicates the normal stress, \( S_t \) indicates the tangential stress due only to Hertzian loading and \( S_t' \) indicates the tangential stress including hoop stress superimposed on \( S_t \). The maximum shear stress is one-half the difference between \( S_n \) and \( S_t' \).

Zaretsky (7) gives a simplified procedure for finding the effect due to hoop stress from inner ring force fits and inner ring rotation, assuming the value \( u=0.78667 \). This simplified procedure, when used to calculate the resulting life of a roller bearing, gives results within one percent of the value found by iterating for the actual location of the maximum shear stress, even with a very heavy press fit.

Zaretsky’s procedure requires the contact pressure, \( P_i \), between the inner race and the shaft. For the case where both components have the same material properties, \( P_i \) is given in Eq. [6] below from Juvinall (8), where \( a \) represents the inside diameter of the shaft, \( b \) represents the diameter of the inner-ring to shaft interface and \( c \) is the outside diameter of the inner ring.

\[
P_i = \frac{E \Delta (b^2 - a^2)(c^2 - b^2)}{2b^3(c^2 - a^2)} \quad [6]
\]

For a bearing race shrunk on a solid shaft, dimensions \( a=0, b=D_S, c=D_R \) and Eq. [6] becomes

\[
P_i = \frac{E \Delta (D_S^2 - D_R^2)}{2D_S D_R^3} \quad [7]
\]

**Strict Series Reliability and Zaretsky’s Rule**

Lundberg and Palmgren (6) first derived the relationship between individual rolling element bearing component life and system life. A bearing is a system of multiple components, each with a different life. As a result, the life of the system is different from the life of an individual component in the system. The fatigue lives of each of the bearing components are combined to calculate the system \( L_{10} \) life using strict-series system reliability (6) and the two-parameter Weibull distribution function (9-11) for the bearing components comprising the system. Lundberg and Palmgren (6) expressed the bearing system fatigue life as follows:
Zaretsky (7) notes that the life of the rolling elements are implicitly included in the inner and outer race lives above. If the life of the rollers, \( L_R \) (taken as a set), is separated from the race lives, Eq. [8] can be rewritten

\[
\frac{1}{L^e} = \frac{1}{L^e_{IR}} + \frac{1}{L^e_{OR}} + \frac{1}{L^e_{IR\rightarrow adj}} + \frac{1}{L^e_{OR\rightarrow adj}} \tag{9}
\]

where \( \text{adj} \) in the subscript indicates adjusted lives for the races that will be greater than the corresponding lives in Eq. [8]. Zaretsky (7) observes that the life of the outer race \( L_{OR} \) is generally greater than the life of the inner race \( L_{IR} \) and the life of the roller set \( L_R \) is equal to or greater than the life of the outer race. In this paper, we assume \( L_R = L_{OR} \), thus Eq. [9] becomes

\[
\frac{1}{L^e} = \frac{1}{L^e_{IR\rightarrow adj}} + \frac{2}{L^e_{OR\rightarrow adj}} \tag{10}
\]

We define the ratio of lives between outer an inner races as

\[
X = \frac{L^e_{OR}}{L^e_{IR}} \tag{11}
\]

If we assume that the life ratio, \( X \) does not change when the roller life is separated from the race lives, Eq. [10] becomes,

\[
\frac{1}{L^e} = \frac{1}{L^e_{IR\rightarrow adj}} + \frac{2}{(XL^e_{IR\rightarrow adj})} \tag{12}
\]

**Bearing Life Factor for Interference Fit**

Coe and Zaretsky (4) show that the life ratio for hoop stress, \( LR_h \), is the ninth power of the ratio of maximum shear stress \( \tau_{\text{max}} \) from Hertz loading alone (from Eq. [5]) to the maximum shearing stress including both Hertzian loading and hoop stress, \( (\tau_{\text{max}})_h \), which can be computed from the simplified procedure of Ref. (7).

\[
LR_h = \left( \frac{L^e_h}{L^e} \right) = \left( \frac{\tau_{\text{max}}}{(\tau_{\text{max}})_h} \right)^9 \tag{13}
\]

Eq. [13] is based on earlier work by Lundberg and Palmgren (6) that uses life exponents for shear-stress that range from 6.9 to 9.3. An exponent of 9 is assumed for the current work. For further discussion of life exponents, see Ref. (12).

Coe and Zaretsky (4) applied the life ratio to the life of the entire bearing, which produces an overly conservative estimate for the life of the bearing. Here, the life ratio will be applied only to the inner ring life. This new value for the inner ring life, \( LR_h L_{IR\rightarrow adj} \) is used in the first term on the right hand side of Eq. [12] to calculate the life of the bearing, \( (L^e_h L) \), including effects of both Hertzian loading and hoop stress.

\[
\frac{1}{(L^e_h L)} = \frac{1}{(LR_h L_{IR\rightarrow adj})} + \frac{2}{(XL_{IR\rightarrow adj})} \tag{14}
\]

Finally, \( (LF)_h \), the life factor for hoop stress, is computed as the ratio of the unfactored life of the bearing, \( (L^e_h L) \) divided by \( L \)

\[
(LF)_h = \frac{(L^e_h L)}{L} \tag{15}
\]

**Determining Life Factor Based On Load and Fit**

For most low speed roller bearing applications (less than one million DN, where DN is the inner-ring speed in rpm multiplied by the bearing bore diameter in mm), the determination of the appropriate life factor based on roller bearing size, radial load and interference fit can be related to the bearing’s static load capacity, \( C_o \), without the need to perform extensive calculations. The bearing static load capacity was first defined by A. Palmgren (13) as “… the allowable permanent deformation of rolling element and bearing ring (race) at a contact as 0.0001 times the diameter of the rolling element….” For roller bearings, this corresponds to a maximum Hertz stress of 4000 MPa (580 ksi). From Hertz theory (5), the relationship between maximum Hertz stress and radial load for a cylindrical roller bearing is

\[
S_{\text{max}} \sim \sqrt{P} \tag{16}
\]

Nearly all bearing manufacturers’ catalogs provide the static load capacity, \( C_o \), for any bearing size. Hence, in order to determine the appropriate stress at the applied radial load on the roller bearing inner race, Eq. [16] can be rewritten as follows:

\[
S_{\text{max}} = k \sqrt{\frac{P}{C_0}} \tag{17}
\]
TABLE 1.—Cylindrical Roller Bearing Properties (from Ref. (14))

<table>
<thead>
<tr>
<th>ABMA Number</th>
<th>Bore mm</th>
<th>OD mm</th>
<th>Number of rollers</th>
<th>Roller diameter, mm</th>
<th>Roller length, mm</th>
<th>Inner race OD, mm (in)</th>
<th>Static Load Capacity, C0 kN (lb.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1906</td>
<td>30</td>
<td>47</td>
<td>18</td>
<td>5</td>
<td>5</td>
<td>33.75 (1.3287)</td>
<td>14.737 (3,313)</td>
</tr>
<tr>
<td>0206</td>
<td>50</td>
<td>72</td>
<td>20</td>
<td>7</td>
<td>7</td>
<td>54.00 (2.126)</td>
<td>32.881 (7,392)</td>
</tr>
<tr>
<td>1910</td>
<td>10</td>
<td>80</td>
<td>14</td>
<td>13</td>
<td>13</td>
<td>56.45 (2.2224)</td>
<td>49.446 (11,116)</td>
</tr>
<tr>
<td>0210</td>
<td>90</td>
<td>110</td>
<td>10</td>
<td>19</td>
<td>19</td>
<td>57.65 (2.2697)</td>
<td>77.226 (17,361)</td>
</tr>
<tr>
<td>0310</td>
<td>110</td>
<td>105</td>
<td>14</td>
<td>9</td>
<td>9</td>
<td>61.95 (2.439)</td>
<td>111.010 (24,956)</td>
</tr>
<tr>
<td>1915</td>
<td>130</td>
<td>130</td>
<td>14</td>
<td>18</td>
<td>18</td>
<td>81.00 (3.189)</td>
<td>68.796 (15,466)</td>
</tr>
<tr>
<td>0215</td>
<td>140</td>
<td>140</td>
<td>14</td>
<td>12</td>
<td>12</td>
<td>84.50 (3.3268)</td>
<td>149.420 (33,591)</td>
</tr>
<tr>
<td>1920</td>
<td>180</td>
<td>180</td>
<td>14</td>
<td>25</td>
<td>25</td>
<td>108.00 (4.252)</td>
<td>123.278 (27,714)</td>
</tr>
<tr>
<td>0220</td>
<td>210</td>
<td>110</td>
<td>14</td>
<td>19</td>
<td>19</td>
<td>115.00 (4.5275)</td>
<td>295.980 (66,539)</td>
</tr>
</tbody>
</table>

Where the conversion constant \( k = 4000 \) for SI units, where \( S_{\text{max}} \) is expressed in MPa, and \( k = 580 \) for English traditional units, where \( S_{\text{max}} \) is expressed in ksi. Table 1 (using data from Ref. (14)) gives the static load capacity for the bearings discussed herein.

The maximum Hertz stress value \( S_{\text{max}} \) as a function of the applied radial load and static load capacity \( C_0 \) is plotted in Fig. 4. The appropriate life factor can be obtained from Tables 2 to 5 for the various interference fits.

As an example of using this procedure, consider a 0210-size bearing with a radial load \( P = 6.95 \text{ kN (1562 lb.)} \). From Table 1, the static load capacity \( C_0 = 77.226 \text{ kN (17,361 lb.)} \). Using either Fig. 4, with \( P/C_0 = 0.09 \) or Eq. [17] with \( \sqrt{P/C_0} = 0.300 \) and the appropriate value for \( k \) yields \( S_{\text{max}} = 1200 \text{ MPa or 175 ksi} \).  

![Graph showing the relationship between Hertz stress and radial load](image)

Figure 4.—Relationship between roller bearing maximum Hertz stress and \( P/C_0 \) (radial load \( P \) divided by static load capacity \( C_0 \)).

TABLE 2.—Life factors for 30 mm bore cylindrical roller bearing with RBEC-5 tolerances (results averaged from ABMA 1906 and 0206 bearings)

<table>
<thead>
<tr>
<th>ABMA Fit Class</th>
<th>Clearance (mm)</th>
<th>IR Hertz stress, MPa (ksi)</th>
<th>LFfit</th>
</tr>
</thead>
<tbody>
<tr>
<td>j5-min</td>
<td>+0.004</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>j5</td>
<td>-0.0035</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>j5-max</td>
<td>-0.011</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.72</td>
</tr>
<tr>
<td>j6-min</td>
<td>+0.004</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>j6</td>
<td>-0.0055</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.93</td>
</tr>
<tr>
<td>j6-max</td>
<td>-0.015</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.59</td>
</tr>
<tr>
<td>k5-min</td>
<td>-0.002</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>k5</td>
<td>-0.0095</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.83</td>
</tr>
<tr>
<td>k5-max</td>
<td>-0.017</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.54</td>
</tr>
<tr>
<td>m5-min</td>
<td>-0.008</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.83</td>
</tr>
<tr>
<td>m5</td>
<td>-0.0155</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.58</td>
</tr>
<tr>
<td>m5-max</td>
<td>-0.023</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.40</td>
</tr>
</tbody>
</table>
**TABLE 3.**—Life factors for 50 mm bore cylindrical roller bearing with RBEC-5 tolerances (results averaged from ABMA 1910, 1010, 0210, 0310 bearings)

<table>
<thead>
<tr>
<th>ABMA Fit Class</th>
<th>Clearance (mm)</th>
<th>IR Hertz stress, MPa (ksi)</th>
<th>LF&lt;sub&gt;fit&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>j5-min</td>
<td>+0.005</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1 1 1</td>
</tr>
<tr>
<td>j5</td>
<td>-0.0045</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.99 0.99 0.99</td>
</tr>
<tr>
<td>j5-max</td>
<td>-0.014</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.74 0.81 0.85</td>
</tr>
<tr>
<td>j6-min</td>
<td>+0.005</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1 1 1</td>
</tr>
<tr>
<td>j6</td>
<td>-0.007</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.92 0.94 0.95</td>
</tr>
<tr>
<td>j6-max</td>
<td>-0.019</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.64 0.73 0.78</td>
</tr>
<tr>
<td>k5-min</td>
<td>-0.002</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1 1 1</td>
</tr>
<tr>
<td>k5</td>
<td>-0.0115</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.80 0.85 0.89</td>
</tr>
<tr>
<td>k5-max</td>
<td>-0.021</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.60 0.70 0.76</td>
</tr>
<tr>
<td>m5-min</td>
<td>-0.009</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.86 0.90 0.92</td>
</tr>
<tr>
<td>m5</td>
<td>-0.0185</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.65 0.74 0.79</td>
</tr>
<tr>
<td>m5-max</td>
<td>-0.028</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.48 0.60 0.67</td>
</tr>
<tr>
<td>m6-min</td>
<td>-0.009</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.86 0.90 0.92</td>
</tr>
<tr>
<td>m6</td>
<td>-0.021</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.60 0.70 0.76</td>
</tr>
<tr>
<td>m6-max</td>
<td>-0.033</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.41 0.53 0.62</td>
</tr>
</tbody>
</table>

**TABLE 4.**—Life factors for 75 mm bore cylindrical roller bearing with RBEC-5 tolerances (results averaged from ABMA 1915 and 0215 bearings)

<table>
<thead>
<tr>
<th>ABMA Fit Class</th>
<th>Clearance (mm)</th>
<th>IR Hertz stress, MPa (ksi)</th>
<th>LF&lt;sub&gt;fit&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>j5-min</td>
<td>+0.007</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1 1 1</td>
</tr>
<tr>
<td>j5</td>
<td>-0.004</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1 1 1</td>
</tr>
<tr>
<td>j5-max</td>
<td>-0.015</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.80 0.86 0.89</td>
</tr>
<tr>
<td>j6-min</td>
<td>+0.007</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1 1 1</td>
</tr>
<tr>
<td>j6</td>
<td>-0.007</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.94 0.96 0.97</td>
</tr>
<tr>
<td>k5-min</td>
<td>-0.002</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1 1 1</td>
</tr>
<tr>
<td>k5</td>
<td>-0.013</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.84 0.88 0.91</td>
</tr>
<tr>
<td>k5-max</td>
<td>-0.024</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.67 0.75 0.80</td>
</tr>
<tr>
<td>m5-min</td>
<td>-0.011</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.87 0.91 0.93</td>
</tr>
<tr>
<td>m5</td>
<td>-0.0122</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.70 0.78 0.82</td>
</tr>
<tr>
<td>m5-max</td>
<td>-0.033</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.55 0.66 0.73</td>
</tr>
<tr>
<td>m6-min</td>
<td>-0.011</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.87 0.91 0.93</td>
</tr>
<tr>
<td>m6</td>
<td>-0.025</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.65 0.74 0.80</td>
</tr>
<tr>
<td>m6-max</td>
<td>-0.039</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.48 0.60 0.68</td>
</tr>
<tr>
<td>n6-min</td>
<td>-0.020</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.73 0.80 0.84</td>
</tr>
<tr>
<td>n6</td>
<td>-0.034</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.54 0.65 0.72</td>
</tr>
<tr>
<td>n6-max</td>
<td>-0.048</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>0.40 0.53 0.61</td>
</tr>
</tbody>
</table>
TABLE 5.—Life factors for 100 mm bore cylindrical roller bearing with RBEC-5 tolerances (results averaged from ABMA 1920 and 0220 bearings)

<table>
<thead>
<tr>
<th>ABMA Fit Class</th>
<th>Clearance (mm)</th>
<th>IR Hertz stress, MPa (ksi)</th>
<th>LFfit</th>
</tr>
</thead>
<tbody>
<tr>
<td>j5-min</td>
<td>+0.009</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
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</tr>
<tr>
<td>j5</td>
<td>-0.0035</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>j5-max</td>
<td>-0.016</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>j6-min</td>
<td>+0.009</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>j6</td>
<td>-0.007</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>j6-max</td>
<td>-0.023</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>k5-min</td>
<td>-0.003</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>k5</td>
<td>-0.0155</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>k5-max</td>
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<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>m5-min</td>
<td>-0.013</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>m5</td>
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<td>1</td>
</tr>
<tr>
<td>m5-max</td>
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<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>m6-min</td>
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<td>1</td>
</tr>
<tr>
<td>m6</td>
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<td>1</td>
</tr>
<tr>
<td>m6-max</td>
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<td>1</td>
</tr>
<tr>
<td>n6-min</td>
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<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>n6</td>
<td>-0.039</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>n6-max</td>
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<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>n6-max</td>
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<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
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<td>P6-min</td>
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<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>p6</td>
<td>-0.053</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
<tr>
<td>p6-max</td>
<td>-0.069</td>
<td>1200 (175) 1700 (250) 2200 (320)</td>
<td>1</td>
</tr>
</tbody>
</table>

RESULTS AND DISCUSSION

We applied the analysis described above to radially-loaded cylindrical roller bearings from four bore sizes, at either two or four dimension series for each bore size. Each bearing was analyzed at three levels of inner ring Hertz stress. The dimension series are shown schematically in Fig. 5. All bearings are made from AISI 52100 steel and have a “square” cross section, with the roller length equal to the diameter. Properties for the bearings analyzed are listed in Table 1.

The calculations were repeated for up to 5 fit classes for each bearing, with each fit taken at tightest, average and loosest values within the fit class for RBEC-5 tolerance. This required 486 separate analyses. A graphical representation of shaft fits is shown schematically in Fig. 6, which was adapted from Ref. (1).

Harris (15) discusses the effect of surface finish on interference fit due to the smoothing of asperities on the surface. He recommends reducing the calculated interference to account for asperity smoothing, depending on the quality of the finish. For very accurately ground surfaces, the recommended reduction is 4 μm (2 μm for each surface, bore ID and shaft OD). In this work, we have reduced the apparent interference by 4 μm (160 μin) to account for surface finish effects.

A commercial bearing analysis code (16) was used to calculate the unfactored $L_{10}$ lives for the inner and outer races operating without force fit. The rollers were modeled with an aerospace crown (17) chosen to minimize the effect of stress concentrations at the ends of the rollers. The crown has a flat length of 61 percent of the actual roller length and a crown radius approximately 100 times the roller length.

All bearings were modeled at the zero internal operating clearance condition. In the case of inner ring force fit, this means the bearings would have an appropriate initial (unmounted) clearance between the rollers and the races.

Each bearing was analyzed for three values of inner-race maximum Hertz stress: 1200, 1700 and 2200 MPa (175, 250 and 320 ksi). The radial load for each bearing was chosen to give the desired stress value. The analysis code estimated the inner and outer race lives using the traditional Lundberg-Palmgren method (6). Therefore, these lives implicitly include the life of the roller set.

Eq. [11] was used to find the ratio of the inner and outer race lives for the bearing and Eq. [12] was used to calculate the adjusted value of the inner race life (with the roller set life separated), assuming that $X$, the ratio of the race lives, does not change when the roller set life is separated from the race lives.

For this study, data for bearing bore sizes were taken from the table for Tolerance Class ABEC-5, RBEC-5 (2). However, this reference does not have information for shaft size limits,
thus values for shaft diameter deviations were taken from the ABMA Shaft Fitting Practice Table (1), which gives tolerancelimits for ABEC-1, RBEC-1 quality level for bearing bore sizes from 3 to 1250 mm (0.1181 to 49.2126 in). The practice of using bore tolerances from an ABEC-5 table and shaft tolerances from an ABEC-1 table is consistent with an example given by Harris (15).

The shaft diameter was subtracted from the bore diameter and then 0.004 mm was added to the difference to account for surface finish. If the resulting fit was positive (indicating clearance) then the interface pressure and thus the hoop stress was assumed to be zero. If the fit was negative, the resulting interference (as a positive number) was used to calculate the interface pressure due to the chosen force fit in Eq. [7].

The simplified procedure described below was used to find the maximum shearing stress, \( \tau_{max} \), including the effect of Hertizan loading and the effect from hoop stress due to the force fit on the inner ring. Eq. [13] was used to calculate the life ratio (hence the revised life) for the inner race and finally, the life of the entire bearing was calculated from Eq [14] using

![Figure 5](image1.png)  
*Figure 5.*—ANSI/ABMA roller bearing dimension series (adapted from Ref. 16).

![Figure 6](image2.png)  
*Figure 6.*—Graphical representation of ANSI/ABMA shaft fits (adapted from Ref. 1). Dimensions illustrate m6 fit, with loosest fit within tolerance band represented by dimension a, tightest fit by dimension d. Fit classes j5 to p6 were considered in this paper.
the reduced life of the inner race and the original lives of the rollers and outer race.

**Analysis of 0210-Size Cylindrical Roller Bearing With m6 Fit**

As an example of the methods presented in this paper, consider a 0210-size cylindrical roller bearing carrying a moderate load of 6.95 kN (1562 lb f) and an average m6 inner-ring press fit. From Eq. [17], the inner race maximum Hertz stress, \( S_{max} \) is 1200 MPa (175 ksi). The analysis code gave lives of the outer and inner races of 14,240 and 2303 million inner race revolutions. From Eq. [8], assuming a Weibull slope of 1.125, the \( L_{10} \) life of the bearing is 2068 million revolutions.

\[
\frac{1}{L_{1.125}} = \frac{1}{2303^{1.125}} + \frac{1}{14240^{1.125}} = \frac{1}{2068^{1.125}} \quad [18]
\]

The ratio \( X \) of outer race life to inner race life, from Eq. [11] is 14,240/2303 = 6.18. The adjusted life of the inner race was found using Eq. [12]

\[
\frac{1}{2068^{1.125}} = \frac{1}{L_{IR-adj}^{1.125}} + \frac{2}{(6.18L_{IR-adj})^{1.125}} \quad [19]
\]

The solution gives \( L_{IR-adj} \) = 2535 million revolutions. (Although not needed for this calculation, the adjusted life of the outer race is 15,675 million revolutions.)

The ABMA Shaft Fitting Practice Table for ABEC-1, RBEC-1 bearings (1) was used to find limiting diameters for a 50-mm shaft with an m6 fit. Shaft fits are illustrated schematically in Fig. 6. This figure shows deviations from the nominal bearing bore and shaft diameter of 50.000 mm. The shaft deviation can range from +0.009 to +0.025 mm (shown as dimensions \( a \) and \( b \) in the figure). The bearing bore deviations were found in the table for Tolerance Class ABEC-5, RBEC-5 (2). The bore deviation can range from 0.000 to -0.008 mm (shown as 0 and dimension c in the figure).

The loosest m6 fit occurs when the largest bore bearing (50-mm) is mounted on the smallest shaft (50.009 mm), producing a fit of 0.009 mm tight (before adjusting for surface finish). This fit is illustrated as dimension \( a \) in Fig. 6. The tightest fit is from the smallest bore (49.992 mm) on the largest shaft (50.025 mm), or 0.033 mm tight, shown as dimension \( d \) in Fig. 6. The average of these extremes is 0.021 mm (0.00083 in) tight. This interference fit was reduced by 0.004 mm to account for asperity smoothing, assuming smooth-ground surfaces. The resulting average m6 fit is 0.017 mm (0.00067 in) tight.

For our example bearing, \( D_S = 50 \) mm (1.9685 in), \( D_{IR} = 57.65 \) mm (2.2697 in), \( d = 13 \) mm (0.5118 in), \( S_{max} = 1200 \) MPa (175 ksi), \( E = 205,878 \) MPa (29.86×10^6 psi), \( v = 0.3 \) and \( \Delta = 0.017 \) mm (0.00067 in). From Eq. [7], the force fit contact pressure, \( P_t = 8.67 \) MPa (125.7 ksi).

Next, the life factor of the inner ring due to the force fit stress was calculated by the following simplified procedure (adapted from Ref. [7]) to calculate the maximum shear stress including the effect of hoop stress.

1. Determine maximum shearing stress \( \tau_{max} \) from [5], where \( \tau_{max} = -(0.3)S_{max} = -360 \) MPa (–52.2 ksi)
2. Determine geometry constant \( B \), where \( B = D_S / D_{IR} = 0.867303 \) (dimensionless)
3. Determine \( m \), where \( m = P_t R^2 / (1 - B^2) = 26.3269 \) MPa (3.818 ksi)
4. Determine \( R' \), where \( R' = D_S / d = 4.4346 \) (dimensionless)
5. Determine \( K_2 \) where \( K_2 = E / (4(1 - v^2))S_{max} = 256.151 \) (dimensionless)
6. Assume value for \( u = 0.78667 \) (dimensionless)
7. Determine \( y \), where \( y = 1 - u / K_2 = 0.996929 \) (dimensionless)
8. Substitute values for \( S_{max} \), \( m \) and \( y \) in Eq. [20] (adapted from Ref. [7]) below to calculate \( (\tau_{max})_h = -386.489 \) Mpa (–56.056 ksi)

\[
(\tau_{max})_h = \tau_{max} - \frac{m}{y^2} \quad [20]
\]

9. Compute the life ratio for the inner ring from Eq. [13]:

\[
LR_h = \left[ -360 / (-386.489) \right]^2 = 0.5278
\]

Using the life ratio from step 9 above in Eq. [14], the life of the bearing including the effect of the force fit was calculated to be 1205 million inner race revolutions:

\[
\frac{1}{(L)^{1.125}_h} = \frac{1}{(0.5278 * 2535)^{1.125}} + \frac{2}{(6.18 * 2535)^{1.125}} = \frac{1}{1205^{1.125}} \quad [21]
\]

The life factor for hoop stress, \( (LF)_h \) is

\[
(\text{LF})_h = \frac{1205}{2068} = 0.58 \quad [21]
\]

Therefore, the average m6 force fit will reduce the life of this bearing by 42 percent.

**Force Fit Life Factors for RBEC-5 Roller Bearings**

The analysis described above was applied to 486 separate configurations, including four bore sizes, up to 4 dimension series (ranging from extremely light to medium), at three values of inner-ring Hertz stress and up to seven inner-ring...
interference fit classes. Each fit class was evaluated at the minimum, maximum and average condition for the RBEC-5 tolerance class.

The shaft interference Table (Ref. (1), and illustrated in Fig. 6) shows fit classes ranging from g6 (loose) to r7 (heavy interference). There is no effect on life for the looser fits that produce no pressure at the bore and no values are given for the heavier fit classes for small bearings. Hence, for 30-mm bearings we calculated life factors for only 4 fit classes (j5 to m5); for 50-mm bearings, 5 classes (j5 to m6); for 75-mm bearings, 6 classes (j5 to n6) and for 100-mm bearings, 7 classes (j5 to p6).

All interference fits were adjusted for the effect of asperity smoothing (assuming accurately ground surfaces) by adding 0.004 mm (160 μm) to the clearance between the shaft and inner ring. If the clearance value was negative (indicating interference), then the resulting pressure was calculated. The results are shown in Tables 2 to 5.

The life factors found in this study range from 1.00 (no effect) where there is no interface pressure to a worst case of 0.36 (64 percent life reduction) for the tightest p6 fit on a 100-mm (220-size) bearing at 1200 MPa (175 ksi) maximum Hertz stress. As should be expected, tighter fits produce smaller life factors (i.e. shorter lives). In general, the life factor is smallest (greatest life reduction) for bearings running under light load where the unfactored life is highest.

Figure 7 and Table 3 show the variation in life factor for 50-mm bore bearings operating under three levels of Hertz stress at the five fit classes considered (j5 to m6). In the three lightest fits shown (j5-k5), the minimum fit will produce no interface pressure, hence the life factor is 1.00. For the heavier fits the life factor is less, ranging to a low value of 0.41 (59 percent life reduction) for the tightest m6 fit on a bearing running at 1200 MPa (175 ksi) maximum Hertz stress.

The various bearing dimension series within a particular bore size had almost identical results for the force fit life factor for a particular fit, despite significant differences in the interface pressure at the bore required to produce that fit. For example, the four 50-mm bearings analyzed (No. 1910, 1010, 0210 and 0310) for the average m6 fit of 0.021 mm interference (before adjusting for finish effect) have force fit pressures of 5.00, 7.55, 8.67 and 12.21 MPa (0.725, 1.10, 1.26, 1.77 ksi), respectively. However, the resulting life factors are nearly identical (0.60, 0.60, 0.58 and 0.60). Therefore, we have combined these results, showing average life factor values for each bore size in Tables 2 to 5.

Interestingly, at a given Hertz stress level, the tightest defined fits produced very similar life factors even on different bore sizes and different fit designations. For example, at the 1200 MPa (175 ksi) maximum Hertz stress level, the tightest (m5) fit on a 30-mm bore bearing has a life factor of 0.40 while the tightest (p6) fit on a 100-mm bearing has a life factor of 0.36. Likewise, for the 2200 MPa (320 ksi) maximum Hertz stress, the m5 fit on a 30-mm bearing has a life factor of 0.61 while the p6 fit on a 100-mm bearing has a life factor of 0.58.
Figure 8 shows the middle of the tolerance band life factor for 30, 50, 75 and 100-mm bore cylindrical roller bearings at the three maximum Hertz stress levels. This plot can be used to estimate the life factor for Hertz stress levels between the values analyzed in this paper. However, for conservative design, life factors should be chosen based on the tight end of the tolerance band, rather than mid-band values.
Effect of Interference Fit on Stress-Life Exponent

Reference (12) states that the theoretical relation between maximum Hertz stress and life in a roller bearing (with line contact) is an inverse eighth power,

\[ L \sim \frac{1}{S_{\text{max}}^n} \]  

where \( L \) represents bearing life, \( S_{\text{max}} \) is the maximum Hertz stress and \( n = 8 \) is the stress-life exponent.

Interference fits can affect the maximum Hertz stress-life relation. Maximum Hertz stress vs. life curves are shown in Fig. 9 for five interference fits on 210-size cylindrical roller bearings. For each interference fit, the Hertz stress-life exponent, \( n \), was calculated. These values are also shown in Fig. 9.

With no press fit, our calculation resulted in a Hertz stress-life exponent \( n = 8.1 \), which is close to the expected value \( n = 8 \). However, with a middle of the tolerance range (m6) force fit, the exponent \( n = 7.7 \). If the results are recalculated based on the tight end of the tolerance range for the m6 interference fit (not shown in Fig. 9), the Hertz stress-life exponent becomes \( n = 7.4 \). Similar results were obtained for other bearing sizes.

This effect can impact the results of accelerated testing on bearings with a heavy press fit performed. If such tests are performed at a high load (thus at high Hertz stress) and then the test results are extrapolated to lower stress levels using the usual stress-life exponent, \( n = 8 \), the predicted value of life may be too high, thus giving a non-conservative design.

It is conjectured that the variation in the Hertz stress-life exponent with interference fit may help explain the load-life exponent \( p = 10/3 \) reported and used by Lundberg and Palmgren (18) for cylindrical roller bearings. Where \( p = 10/3, \ n = 6.66 \). Unfortunately, Lundberg and Palmgren (18) did not report the interference fit that they used in their test bearings.

SUMMARY OF RESULTS

The effect of hoop stresses in reducing roller bearing fatigue life was determined for various classes of inner ring interference fit. Calculations were performed for up to 7 interference fit classes for each bearing series. Each fit was taken at tightest, average and loosest values within the fit class for RBEC-5 tolerance, thus requiring 486 separate analyses.

The hoop stresses were superimposed on the Hertzian principal stresses created by light, moderate and heavy applied radial loads to determine roller bearing fatigue life. Results are presented as life factors for bearings loaded to 1200, 1700 and 2200 MPa (175, 250 and 320 ksi) maximum Hertz stress levels in up to 7 fit classes (extremely light to extremely heavy) and for bearing accuracy class RBEC 5 (ISO class 5). All calculations are for zero initial internal clearance conditions. Any reduction in internal bearing clearance due to the interference fit would be compensated by increasing the initial (unmounted) clearance.

The life factor for interference fit in low-speed roller bearings can be determined through charts or tables from the maximum Hertz stress, which is easily calculated from the applied radial load and the static load capacity. The following results were obtained.

1. Interference fits on the inner bearing ring of a cylindrical roller bearing can significantly reduce bearing fatigue life. A heavy (m6) press fit on a 210-size roller bearing was found to reduce the fatigue life by up to 59, 47 and 38 percent from the standard life at maximum Hertz stresses of 1200, 1700 and 2200 MPa (175, 250 and 320 ksi), respectively.

2. Tighter interference fits produce smaller life factors (i.e. shorter lives). Life factors due to hoop stresses found in this study range from 1.00 (no effect) where there is no interface pressure to as low as 0.35 (65 percent life reduction) for the tightest p6 fit on a 100-mm bore (220-size) bearing with 1200 MPa (175 ksi) maximum Hertz stress.

3. The various bearing series within a particular bore size had almost identical interference fit life factors for a particular fit, despite significant differences in the interface pressure at the bore. Four series (1910-, 110-, 210- and 310-size) 50-mm bore cylindrical roller bearings having an average m6 fit of 0.021 mm interference (before adjusting for finish effect) producing force fit pressures of 5.00, 7.55, 8.67 and 12.21 MPa (0.73, 1.10, 1.26, 1.77 ksi), had resulting life factors of 0.60, 0.60, 0.58 and 0.60, respectively.

4. In general, the life factor is smallest (greatest life reduction) for bearings running under light load where the unfactored life is highest. For any particular bearing size

Figure 9.—Relationship between life and maximum Hertz stress for ABMA 0210 cylindrical roller bearings, showing effect of interference fit on the stress-life exponent, \( n \).
and interference fit, as the maximum Hertz stress on the inner race was increased the effect of the hoop stresses on life was reduced, thus the resulting life factor increased.

5. The tightest fit at the high end of the RBEC-5 tolerance band defined in ANSI/ABMA shaft fit tables produces a life factor of approximately 0.40 for an inner-race maximum Hertz stress of 1200 MPa (175 ksi) and a life factor of 0.60 for an inner-race maximum Hertz stress of 2200 MPa (320 ksi).

6. Interference fits affect the maximum Hertz stress-life relation. With no press fit, a Hertz stress-life exponent \( n = 8.1 \) was found, which is close to the accepted value of \( n = 8 \). With a middle of the tolerance range (m6) force fit, the exponent \( n = 7.7 \) and at the tight end of the range for the m6 interference fit, the Hertz stress-life exponent becomes \( n = 7.4 \).

REFERENCES


