Regionalized Lunar South Pole Surface Navigation System Analysis

Bryan W. Welch
Glenn Research Center, Cleveland, Ohio
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Bryan W. Welch
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Bryan W. Welch
National Aeronautics and Space Administration
Glenn Research Center
Cleveland, Ohio 44135

Summary

Apollo missions utilized Earth-based assets for navigation because the landings took place at lunar locations in constant view from the Earth. The new exploration campaign to the lunar south pole region will have limited Earth visibility, but the extent to which a navigation system comprised solely of Earth-based tracking stations will provide adequate navigation solutions in this region is unknown. This report presents a dilution-of-precision (DoP)-based, stationary surface navigation analysis of the performance of multiple lunar satellite constellations, Earth-based deep space network assets, and combinations thereof. Results show that kinematic and integrated solutions cannot be provided by the Earth-based deep space network stations. Also, the stationary surface navigation system needs to be operated either as a two-way navigation system or as a one-way navigation system with local terrain information, while the position solution is integrated over a short duration of time with navigation signals being provided by a lunar satellite constellation.

Introduction

In support of NASA’s Vision for Space Exploration (ref. 1), an extension of the position-fixing capability provided by the global positioning system (GPS) constellation (ref. 2) to the Moon is analyzed. This extension would be afforded by a lunar network of spacecraft orbiting the Moon, Earth-based, deep space network assets (DSN), and combinations thereof. This study provides a dilution-of-precision (DoP)-based stationary surface navigation analysis via radiometric navigation signals from the aforementioned systems for users located on the lunar surface. Although the current study is similar to prior studies on the subject (refs. 3 to 6), several differences are the (1) use of the newly developed DoP technique referred to as “generalized DoP,” (2) combinations of radiometric signals from the lunar and Earth vicinities, and (3) limitations to the regionalization of interest. Generalized DoP provides the ability to assess the navigational performance associated with a receiver that is able to integrate radiometric measurements over time. Such an analysis method allows one to directly compare the navigational capability associated with sparse constellations with that provided by constellations which support full coverage of an appropriate fold. Estimates of a user state derived from multiple radiometric measurements collected over a period of time are herein referred to as being “dynamic,” whereas those provided by full constellations that do not employ integration over time in the receiver are referred to as being “kinematic.” As opposed to standard measures of DoP that are restricted to kinematic position-fixing capabilities, the use of generalized DoP further allows assessment of the constellation to be performed in terms of the latency associated with obtaining a specified level of system performance (refs. 5 and 6).

Several options for the radiometric navigation signal sources are considered in this study and include equally the Earth-based DSN site locations, two inclined elliptical constellations (ref. 7), and combinations thereof. Also included are assessments of a number of augmentations to the system, such as two-way mode of operation, good knowledge of the terrain, and integration of radiometric measurements over periods of time. Comparisons of the system performance under the different system assumptions indicate that system availability performance is significantly improved and latency is reduced by the prescribed augmentations. Results are derived from temporally and spatially averaged system availability numbers associated with prespecified threshold levels of system availability.

Constellations

Three categories of radiometric signal sources are considered: two inclined elliptical lunar-centric constellations, DSN site locations (Canberra, Madrid, and Goldstone), and the combination of the lunar constellations and DSN assets. The notation for the lunar-centric constellations subsequently used, such as elliptical $N/p/f d$ km, is defined as $N$, the number of satellites; $p$, the number of orbital planes; $f$, the binary answer as to whether phasing exists in the mean anomaly between satellites in adjacent planes; and $d$, the semimajor axis (SMA) in kilometers. Table 1 lists the parameters of the two lunar-centric constellations (ref. 7) considered herein. Table 2 lists the latitude and longitude for the three DSN site locations.

Each of the constellations in this study was considered for specific reasons: the two inclined elliptical constellations provided a focus of coverage over the lunar south pole region; the Earth-based DSN assets were useful and available for space navigation; the combinations of the two sets were examined to determine additional benefits of combining the services of both assets in comparison with their individual performance. Figure 1 is an illustration of the elliptical 2/1/0 a6541 constellation in orbit around the Moon. Shown in the background of the illustration are the three Earth-based DSN stations. The image was produced from orbital plots in Satellite Orbit Analysis Program (Wikimedia Foundation, Inc.).
**Analysis**

**Generalized DoP**

The analysis performed is a generalized version of the DoP metric (refs. 4 to 6), of which several forms are subsequently used for analysis. The generalized DoP is derived from the observability gramian, which is obtained by using the navigation user equations of motion and the associated sequence of measurements. The equations of motion and the measurement sequence are given by references 5 and 6. It is shown that the DoP metric takes the following form, derived in references 5 and 6:

\[
\begin{eqnarray}
\text{max} \left\{ \text{eig} \left[ \sum_{t_0}^{t_n} \bar{H}_{0}^{T} W \bar{H}_{0} \right]^{-1} \right\}
\end{eqnarray}
\]

(1)

where

- \(t_n\): \(n^{th}\) time step since time step zero
- \(t_0\): time step zero
- \(\bar{H}_{0}\): matrix transpose of \(\bar{H}_{0}\)
- \(\bar{H}_{0}\): state transitioned partial derivative measurement matrix
- \(W\): measurement weighting matrix

**Variations of Generalized DoP**

To relax the constraint of satellite coverage so as to invert the observability gramian, a number of augmentations to the lunar navigation system are considered in the analysis, as in previous analyses (ref. 7). These augmentations constrain the navigation solution and thereby reduce the number of required satellites in view. The augmentations include clock synchronization and good knowledge of the terrain, and they create four forms of DoP. The selected form of DoP used not only affects the required satellites in view but also affects the state transition and H-matrices used in the calculation. Also, note that throughout the analysis, both range and range-rate
The first form of DoP, geometric dilution of precision (GDoP), is used in the GPS where the solution is obtained for the position of the user in three dimensions and for the time bias, resulting in the requirement of four navigation signals. Since two navigation signals are available from each satellite, only two satellites need be in view to kinematically solve for the user’s position. Without two satellites in view, the solution will have to be integrated over time to be able to invert the solution and solve for the user’s position and time bias. The GDoP metric is used to evaluate a navigation system operating in one-way mode without terrain information. Equations (2) and (3) provide the associated state transition and H-matrices for the GDoP metric:

\[ \Phi_{GDoP}(t_i, t_0) = I + \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \times (t_i - t_0) \]  

where \( \Phi_{GDoP} \) is the state transition matrix for the GDoP metric; \( t_i \) is the \( i \)th time step since time step zero; \( t_0 \) is time step zero; and \( I \) is the identity matrix.

\[
H = \begin{bmatrix}
\frac{\partial p_{r1}}{\partial x_m} & \frac{\partial p_{r1}}{\partial y_m} & \frac{\partial p_{r1}}{\partial z_m} & \frac{\partial p_{r1}}{\partial (ct_{biasu})} \\
\frac{\partial p_{r1}}{\partial r_m} & \frac{\partial p_{r1}}{\partial r_m} & \frac{\partial p_{r1}}{\partial r_m} & \frac{\partial p_{r1}}{\partial (ct_{biasu})} \\
\ldots & \ldots & \ldots & \ldots \\
\frac{\partial r_m}{\partial x_m} & \frac{\partial r_m}{\partial y_m} & \frac{\partial r_m}{\partial z_m} & \frac{\partial r_m}{\partial (ct_{biasu})}
\end{bmatrix}
\]

The second form of DoP, positional dilution of precision (PDoP), provides an estimate of user positioning accuracy for the case in which there is no time bias between orbiter clocks and user clocks, such as in a two-way mode of operation. PDoP results in the requirement of three navigation signals. Thus, the PDoP metric also requires two satellites in view to kinematically solve for the user’s position. The PDoP metric is used to evaluate a navigation system operating in two-way mode without terrain information. Equations (4) and (5) provide the associated state transition and H-matrices for the PDoP metric:

\[ \Phi_{PDoP}(t_i, t_0) = I + \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \times (t_i - t_0) \]  

where \( \Phi_{PDoP} \) is the state transition matrix for the PDoP metric.

\[
H = \begin{bmatrix}
\frac{\partial r_{r1}}{\partial x_m} & \frac{\partial r_{r1}}{\partial y_m} & \frac{\partial r_{r1}}{\partial z_m} \\
\frac{\partial r_{r1}}{\partial r_m} & \frac{\partial r_{r1}}{\partial r_m} & \frac{\partial r_{r1}}{\partial r_m} \\
\ldots & \ldots & \ldots \\
\frac{\partial r_m}{\partial x_m} & \frac{\partial r_m}{\partial y_m} & \frac{\partial r_m}{\partial z_m}
\end{bmatrix}
\]

The third form of DoP, horizontal/time dilution of precision (HTDoP), is applied when a user has knowledge of his altitude above the center of the Moon, but there is still a time bias from the source of the navigation signal. This situation also results in the requirement of three navigation signals, meaning that two satellites must be in view to kinematically solve for the user’s topocentric north and east components along with the time bias. The HTDoP metric is used to evaluate a navigation system operating in one-way mode with terrain information. Equations (6) and (7) provide the associated state transition and H-matrices for the HTDoP metric:

\[ \Phi_{HTDoP}(t_i, t_0) = I + \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \times (t_i - t_0) \]  

where \( \Phi_{HTDoP} \) is the state transition matrix for the HTDoP metric.

\[
H = \begin{bmatrix}
\frac{\partial r_{r1}}{\partial x_m} & \frac{\partial r_{r1}}{\partial y_m} & \frac{\partial r_{r1}}{\partial z_m} \\
\frac{\partial r_{r1}}{\partial r_m} & \frac{\partial r_{r1}}{\partial r_m} & \frac{\partial r_{r1}}{\partial r_m} \\
\ldots & \ldots & \ldots \\
\frac{\partial r_m}{\partial x_m} & \frac{\partial r_m}{\partial y_m} & \frac{\partial r_m}{\partial z_m}
\end{bmatrix}
\]
Finally, the fourth form of DoP is the horizontal dilution of precision (HDoP). It provides an estimate of user positioning accuracy when both time and user altitude are known, only requiring two navigation signals, such as in the case of two-way mode of operation with good knowledge of terrain. This case requires that only one satellite be in view to kinematically solve for the user’s topocentric north and east components. The HDoP metric is used to evaluate a navigation system operating in two-way mode with terrain information. Equations (8) and (9) provide the associated state transition and H-matrices for the HDoP metric.

\[
\Phi_{HDoP}(t_f, t_0) = I + \begin{bmatrix} 1 & t_f - t_0 \\ 0 & 0 \end{bmatrix}
\]

(8)

where \(\Phi_{HDoP}\) is the state transition matrix for the HDoP metric.

\[
H = \begin{bmatrix}
\frac{\partial \tilde{r}_1}{\partial x_1} & \frac{\partial \tilde{r}_1}{\partial y_1} \\
\frac{\partial \tilde{r}_1}{\partial x_2} & \frac{\partial \tilde{r}_1}{\partial y_2} \\
\vdots & \vdots \\
\frac{\partial \tilde{r}_m}{\partial x_m} & \frac{\partial \tilde{r}_m}{\partial y_m} \\
\frac{\partial \tilde{r}_1}{\partial r_1} & \frac{\partial \tilde{r}_1}{\partial r_1} \\
\vdots & \vdots \\
\frac{\partial \tilde{r}_m}{\partial r_m} & \frac{\partial \tilde{r}_m}{\partial r_m} \\
\frac{\partial \tilde{r}_1}{\partial \delta x_m} & \frac{\partial \tilde{r}_1}{\partial \delta y_m} \\
\vdots & \vdots \\
\frac{\partial \tilde{r}_m}{\partial \delta x_m} & \frac{\partial \tilde{r}_m}{\partial \delta y_m}
\end{bmatrix}
\]

(9)

The underlying figure of merit (FOM) used for evaluating the performance associated with a navigation system is system availability (SA). System availability is defined herein as the proportion of time that the navigation system is predicted to provide performance at or below a specified level of DoP. In other words, the navigation system is defined as “available” when the appropriately chosen version of DoP falls below a certain threshold. For this study, as in previous studies, the threshold is set at 10 (refs. 4 to 6). Furthermore, a DoP of 10, coupled with a 1-m user range error (URE) denotes a user state uncertainty of 10 m. Results provided are in terms of system availability for a given latency, whether the solution has zero latency (kinematic) or dynamic solutions of 15 min. or 1 hr. Equation (10) describes how the system availability FOM is calculated, where \(SA\) represents system availability.

\[
SA = 100 \times \frac{\sum_{m=1}^{n_m} \cos(lat_m) \times \sum_{n=1}^{t_f} (DoP_{n,m} <= \text{threshold})}{t_f \times n_{\text{long}} \times \sum_{m=1}^{n_m} \cos(lat_m)}
\]

(10)

where \(t_f\) is the total number of points in the simulation; \(t_0\) is the number of time epochs in the simulation, \(n_{\text{long}}\) is the number of longitude points in the simulation; and \(n_{\text{lat}}\) is the number of latitude points in the simulation.

**Navigation signal**

The navigation signal requirements used in this study are listed in table 3.

<table>
<thead>
<tr>
<th>TABLE 3.—NAVIGATION SIGNAL ASSUMPTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency used for Doppler measurements, GPS L1, GHz ........................................ 1.57545</td>
</tr>
<tr>
<td>User range error, URE, m.......................... 1</td>
</tr>
<tr>
<td>User range rate error, URRE, mm/sec .................. 0.1</td>
</tr>
</tbody>
</table>

**Simulation**

The lunar south pole region is taken as a set of 721 points on the surface, spaced evenly in latitude and longitude. The longitudes for the points go from −175° E. to 180° E. in 5° increments, and the latitudes of the points go from −89° N. to −80° N. in 1° increments. An additional point of interest that is added to the set of data points is located at the exact south pole of −90° N. The analysis is performed over the duration of 1 lunar sidereal month (27.3 Earth days) where DoPs are calculated at an epoch rate of 15 min. The starting epoch for the simulations is Jan 1, 2020, 12:00:00.000 UTC. Visibility to the constellations from the surface points is computed based on a 10° minimum user elevation angle. Figure 2 plots the surface sample points used for the simulation along an orthographic projection of the lunar surface.

**User burden**

Receivers that support a reduced number of satellites will have an increased level of processing or other sensing equipment associated with them. This situation leads to increased user burden in terms of the mass and power the host platform must provide to the navigation receiver. To provide knowledge sufficient to infer user altitude given a horizontal location, a large digital elevation map would have to be available to the user. To provide an error comparable to the 1-m URE assumed for the system, the user is required to store approximately 1 TB of terrain data for global coverage. For the user to have knowledge of terrain within a 30-km radius of a starting point, approximately 100 MB is required for storage.

For a navigation system using two-way radiometric signals as a mode of operation, the two-way radiometric assumption implies that the user would have to be able to transpond the ranging signal that is initialized from the lunar-centric constellation or Earth-based DSN assets. The clock synchronization, which is necessary for one-way radiometric navigation systems, is not a requirement when using two-way radiometric navigation signals for the system’s mode of operation.
Results

Results are reported as the system availability FOM and are presented in tabular form for the lunar south pole regions. The term “no terrain” indicates that there is no detailed cartography of the terrain that would allow determining the altitude of the user. The term “good terrain” indicates that there is such knowledge and that an accurate estimate of user altitude above the lunar datum is available to the navigation receiver. The term “one-way” indicates that the mode of operation for the navigation system is such that signals are transmitted via the lunar-centric constellation or Earth-based DSN assets and are received by the lunar stationary surface user. The term “two-way” indicates that the mode of operation for the navigation system is such that signals are transmitted via the lunar-centric constellation or Earth-based DSN assets and are transponded by the lunar stationary surface user back to the original transmitter, which then sends data back to the lunar stationary surface user for position determination. Tables 4 through 7 provide the system availability results for the four navigation system types previously described.

### Table 4.—System Availability Results for One-Way, No-Terrain Navigation System

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Kinematic</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 min</td>
<td>1 hr</td>
<td></td>
</tr>
<tr>
<td>Elliptical 1/1/0 a6541</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Elliptical 2/1/0 a6541</td>
<td>.00</td>
<td>34.18</td>
</tr>
<tr>
<td>Deep space network (DSN)</td>
<td>.00</td>
<td>.00</td>
</tr>
<tr>
<td>Elliptical 1/1/0 a6541 + DSN</td>
<td>2.83</td>
<td>3.84</td>
</tr>
<tr>
<td>Elliptical 2/1/0 a6541 + DSN</td>
<td>4.91</td>
<td>37.67</td>
</tr>
</tbody>
</table>

### Table 5.—System Availability Results for One-Way, Good-Terrain Navigation System

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Kinematic</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 min</td>
<td>1 hr</td>
<td></td>
</tr>
<tr>
<td>Elliptical 1/1/0 a6541</td>
<td>0.00</td>
<td>71.43</td>
</tr>
<tr>
<td>Elliptical 2/1/0 a6541</td>
<td>46.31</td>
<td>100.00</td>
</tr>
<tr>
<td>Deep space network (DSN)</td>
<td>.00</td>
<td>.00</td>
</tr>
<tr>
<td>Elliptical 1/1/0 a6541 + DSN</td>
<td>3.75</td>
<td>71.62</td>
</tr>
<tr>
<td>Elliptical 2/1/0 a6541 + DSN</td>
<td>49.12</td>
<td>100.00</td>
</tr>
</tbody>
</table>

### Table 6.—System Availability Results for Two-Way, No-Terrain Navigation System

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Kinematic</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 min</td>
<td>1 hr</td>
<td></td>
</tr>
<tr>
<td>Elliptical 1/1/0 a6541</td>
<td>0.00</td>
<td>71.42</td>
</tr>
<tr>
<td>Elliptical 2/1/0 a6541</td>
<td>46.31</td>
<td>100.00</td>
</tr>
<tr>
<td>Deep space network (DSN)</td>
<td>.45</td>
<td>.59</td>
</tr>
<tr>
<td>Elliptical 1/1/0 a6541 + DSN</td>
<td>3.86</td>
<td>71.78</td>
</tr>
<tr>
<td>Elliptical 2/1/0 a6541 + DSN</td>
<td>49.41</td>
<td>100.00</td>
</tr>
</tbody>
</table>

### Table 7.—System Availability Results for Two-Way, Good-Terrain Navigation System

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Kinematic</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 min</td>
<td>1 hr</td>
<td></td>
</tr>
<tr>
<td>Elliptical 1/1/0 a6541</td>
<td>73.31</td>
<td>75.20</td>
</tr>
<tr>
<td>Elliptical 2/1/0 a6541</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>Deep space network (DSN)</td>
<td>4.80</td>
<td>4.94</td>
</tr>
<tr>
<td>Elliptical 1/1/0 a6541 + DSN</td>
<td>74.65</td>
<td>76.49</td>
</tr>
<tr>
<td>Elliptical 2/1/0 a6541 + DSN</td>
<td>100.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>
These results are also summarized in a stoplight chart, which shows the performance of each of the configurations proposed herein in terms of the latency required to achieve 80-percent system availability over the lunar south pole region. The correlation between color and latency in the tables is

1. Green: configuration meets 80-percent system availability kinematically
2. Yellow: configuration meets 80-percent system availability dynamically in 15 min
3. Red: configuration meets 80-percent system availability dynamically in 1 hr
4. Gray: configuration does not meet 80-percent system availability within 1 hr; does not mean that system does not meet 80-percent system availability at all, but does indicate system could take more than 1 hr to do so if it will do so

Table 8 illustrates the performance of the five configurations in the stoplight chart form. This form is useful in comparing the configurations.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>One-way, no terrain</th>
<th>One-way, good terrain</th>
<th>Two-way, no terrain</th>
<th>Two-way, good terrain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elliptical 1/1/0 a6541</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elliptical 2/1/0 a6541</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Deep space network (DSN)</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Elliptical 2/1/0 a6541 + DSN</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The second general trend observed for the lunar south pole region is that the system availability improves for a given constellation as the solution is integrated over time. Also, there are larger improvements when comparing the transition to the 15-min dynamic solution (from the kinematic solution) with the transition to the 1-hr dynamic solution (from the 15-min dynamic solution). It appears that with these sparse constellations, the system availability curve plotted against integrated time levels off to a non-100-percent value. Further analysis would be needed to validate that observation.

The second general trend observed for the lunar south pole region is that the system availability improves for either a one- or two-way navigation system with the inclusion of the local terrain information. This improvement is based on the need for fewer signal sources necessary to solve for a reduced number of states. In a one-way navigation system, four states are solved for by the DoP technique: the local x-, y-, and z-topocentric coordinates and the time bias. Providing local terrain information removes the need to solve for the local topocentric z-coordinate. Similar statements can be made regarding a two-way navigation system.

The third general trend observed for the one- and two-way navigation system modes is that the two-way mode is better than the one-way mode in providing a navigation solution for all the configurations examined. The results in table 8 show improvements in system availability above the 80-percent threshold in the stoplight chart for the two-way system compared with those of the one-way system. This is true for cases with and without local terrain information. Results show that the elliptical 2/1/0 constellation can provide 100-percent system availability with a dynamic solution of 15 min in the two-way mode as compared with a 34-percent system availability in one-way mode with a dynamic solution of 15 min. Only the elliptical 2/1/0 constellation (with and without the DSN augmentation) can provide the 80-percent system availability kinematically when operating in two-way mode with local terrain information.

The fourth general trend observed is that the combinations of the elliptical lunar-centric constellations and the Earth-based DSN ground stations did not significantly improve system availabilities for all the configurations. Data provided in tables 4 through 7 tabulate the system availabilities for all the configurations for the different system schemes. The observation was that the system availabilities obtained for the elliptical 1/1/0 a6541 + DSN and elliptical 2/1/0 a6541 + DSN configurations are not much larger than those for the elliptical 1/1/0 a6541 or elliptical 2/1/0 a6541 configurations, respectively. Therefore, it can be determined that the geometry added via the DSN measurements does not improve upon the geometry from the lunar-centric measurements by a significant factor.

It is important to note that the performance of the DSN configuration should not be expected to be above a system availability of 50-percent. The reason for this is that half the points in the simulation are on the lunar far side, the side that does not face the Earth. Note that due to the wobble of the Moon in orbit, some points near the lunar south pole could face the Earth when the Moon is tilted away from the Earth in the Moon’s northern hemisphere. However, those points would still require a minimum user elevation angle of 10° to be considered visible measurements. Therefore, at most, only the lunar near-side points could have access to the Earth-based DSN ground stations.

**Conclusions**

Generalized dilution of precision (DoP) allows the effects of multiple radiometric measurements to be assessed in the same manner as standard measures of DoP are. In the current case, the effect of integrating multiple radiometric
measurements in time is assessed to allow the performance of sparse constellations around the Moon to be analyzed and compared with kinematic-only solutions. Using this innovation, the basis of comparison can be changed to a domain that is more closely aligned with user requirements, namely, the latency associated with achieving a particular level of precision in the state estimate.

A restriction to the use of kinematic solutions, as is done with an analysis based on static DoP, biases the selection of a constellation to those with more satellites. The use of dynamic solutions permits integrating radiometric signals over a period of time to improve the system availability and thus allows for the consideration of constellations with fewer satellites. The application of generalized DoP for the evaluation of the inherent navigation capability of constellations of lunar-centric satellites, Earth-based, deep space network (DSN) assets, and combinations thereof has eliminated this bias.

Inspection of the results summaries provided in the stoplight charts revealed four trends. First, time integration of the solution improves the system availability metric and lowers the estimated solution error. Second, use of local terrain information can improve system performance. Third, use of a two-way navigation system allows for a 5 percent improvement over a one-way navigation system because of not having to solve for user time bias. Fourth, augmenting the lunar-centric constellations with Earth-based DSN assets provides minimal improvements in system availability performance. Also noted is that the Earth-based DSN configuration exhibits system availabilities of a maximum of 5 percent when operating as a two-way navigation system with local terrain information while integrating the solution for 1 hr.

From this list of possible configurations and using the stated assumptions regarding visibility, the recommended constellation would be the elliptical 2/1/0 a6541. The navigation system should operate in two-way mode, collecting range and range-rate measurements while being augmented via a local terrain map. Under this mode of operation for the elliptical 2/1/0 a6541 satellite constellation, the 80-percent system availability metric can be met kinematically with a predicted system availability of 100 percent. The elliptical 2/1/0 a6541 + DSN constellation also provides this level of system availability but is therefore not necessary to meet a DoP threshold of 10, corresponding to an error on the order of 10 m.

References
Apollo missions utilized Earth-based assets for navigation because the landings took place at lunar locations in constant view from the Earth. The new exploration campaign to the lunar south pole region will have limited Earth visibility, but the extent to which a navigation system comprised solely of Earth-based tracking stations will provide adequate navigation solutions in this region is unknown. This report presents a dilution-of-precision (DoP)-based, stationary surface navigation analysis of the performance of multiple lunar satellite constellations, Earth-based deep space network assets, and combinations thereof. Results show that kinematic and integrated solutions cannot be provided by the Earth-based deep space network stations. Also, the stationary surface navigation system needs to be operated either as a two-way navigation system or as a one-way navigation system with local terrain information, while the position solution is integrated over a short duration of time with navigation signals being provided by a lunar satellite constellation.