Eta Meson Production in
Proton-Proton and Nuclear Collisions

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Abstract

Total cross sections for eta meson production in proton - proton collisions are calculated. The eta meson is mainly produced via decay of the excited nucleon resonance at 1535 MeV. A scalar quantum field theory is used to calculate cross sections, which also include resonance decay. Comparison between theory and experiment is problematic near threshold when resonance decay is not included. When the decay is included, the comparison between theory and experiment is much better.

1 Introduction

Protecting astronauts from cosmic radiation is becoming increasingly important with the current plans to establish a permanent human base on the Moon with a follow - on mission to Mars. In developing space radiation transport codes, it is important to include all relevant atomic, nuclear, and particle physics reactions, so that radiation environments can be accurately predicted. The present paper represents a continuing effort to incorporate particle physics reactions into the radiation code HZETRN [1]. Table 1 shows the reaction thresholds of all hadrons produced in proton - proton collisions. The pion is denoted by the symbol, $\pi$, and is often called the $\pi$ meson. It can be seen that $\pi$ production has the lowest threshold and therefore is the most important particle to include. Studies of pion production have been made and have also been included in a transport code called MESTRN [2]. However, it is important to include all possible particles, and Table 1 shows that the particle threshold next highest in energy after pions is due to the eta meson, which is generally denoted by the symbol, $\eta$. The calculation of cross sections for $\eta$ production is the subject of the present paper. The $\eta$ meson is a neutral particle with a mass of 548 MeV, and is the meson next heaviest in mass to the $\pi$ meson. The $\eta$ decays primarily to $\pi$ mesons.

The dominant reaction mechanism for $\eta$ production is via formation and decay of the the nucleon excited resonance at 1535 MeV. This resonance particle is often denoted $N^*(1535)$. The $\eta$ production reaction is

$$pp \rightarrow pN^*(1535) \rightarrow pp\eta,$$  

(1)

where $p$ is the symbol for the proton. Even though other nucleon excited states are produced in $pp$ collisions, such as the resonances at 1440 MeV and 1520 MeV denoted as $N^*(1440)$ and $N^*(1520)$, it is the $N^*(1535)$ which produces copious numbers of $\eta$ mesons due to decay. The $N^*(1440)$ and $N^*(1520)$ decay primarily to $\pi$ particles. Both formation and decay of $N^*(1535)$ is considered in the present work. Compare this to the reactions studied previously in reference [3]

$$pp \rightarrow p\Delta(1232) \rightarrow pp\pi,$$  

(2)
where $\Delta(1232)$ is standard notation for the $\Delta$ particle with a mass of 1232 MeV. This particle is also like an excited state of the nucleon, but now the isospin quantum number is $3/2$ instead of $1/2$. The $\Delta(1232)$ is the lowest mass $\Delta$ resonance. It decays primarily to $\pi$ mesons. Note the resonances are produced via one pion exchange. Also, note that $\eta$ production occurs primarily through excitation of the $N^*(1535)$ resonance. It is not necessary to include production from other resonance channels. Therefore, it is a simple matter to use the theory developed in reference [3] to obtain the $\eta$ production cross section. One simply replaces all $\Delta$ parameters with $N^*$ parameters. Finally, note that the $\Delta(1232)$ does not decay to $\eta$ mesons because its mass is too small.

Table 1: Thresholds for $pp$ reactions [4]. $P$ is the projectile momentum, $T$ is the kinetic energy, and $s$ is the square root of the total center of momentum energy. The units of these quantities are MeV. Particle symbols are as follows. $p$ is the proton, $\pi$ is the pion, $d$ is the deuteron, $n$ is the neutron, $\eta$ is the eta meson, $\Lambda$ is the lambda baryon, $K$ is the kaon, $\Sigma$ is the sigma baryon, $\rho$ is the rho meson, $\omega$ is the omega meson, and $\phi$ is the phi meson. The superscripts $*$ and $t$ are standard notation [5] for higher energy states. Superscripts $+$, $-$, 0 denote the charge state of the particle.

<table>
<thead>
<tr>
<th>Final state</th>
<th>$P$</th>
<th>$T$</th>
<th>$\sqrt{s}$</th>
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<td>$pp\pi^0$</td>
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<td>280</td>
<td>2012</td>
</tr>
<tr>
<td>$d\pi^+$</td>
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<td>288</td>
<td>2015</td>
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<td>$pn\pi^+$</td>
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<td>2017</td>
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<tr>
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<td>600</td>
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<td>$pp\pi^+\pi^-\pi^0$</td>
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<td>2291</td>
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<td>$pp\eta$</td>
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<tr>
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<td>$p\Sigma^+ K^{*0}$</td>
<td>3820</td>
<td>2996</td>
<td>3024</td>
</tr>
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</table>
2 Scalar quantum field theory

A quantum field theory for the interaction and production of scalar particles has previously been developed [3]. This theory has been used for the calculation of differential and total cross sections for both elastic and inelastic scattering and also includes resonance formation and decay. The present paper is concerned only with total cross sections. For the $\text{NN} \rightarrow \text{NN}^*$ reaction, we simply replace the $\Delta$ masses, couplings, and decay widths of reference [3] with the $\text{N}^*(1535)$ values to give

$$\sigma_d = \sigma_e = \frac{g_{N\pi N\pi}^2 g_{\pi NN^*}^2 \sqrt{(s - m_{N^*}^2 - m_N^2)^2 - 4m_{N^*}^2 m_N^2}}{16\pi \sqrt{s(s - 4m_N^2)}} \left[ s^2m^2 + sm^2(m^2 - m_{N^*}^2 - 3m_N^2) + (m_N^3 - m_{N^*}^2 m_N)^2 \right],$$

(3)

where $m$ is the mass particle mass and $s$ is the square root of the center of momentum energy. The interference term is

$$\sigma_i = \frac{g_{N\pi N\pi}^2 g_{\pi NN^*}^2}{4\pi s(s - 4m_N^2)(s + 2m^2 - m_{N^*}^2 - 3m_N^2)} \times \log \left( \frac{s(s + 2m^2 - m_{N^*}^2 - 3m_N^2) + f(s)\sqrt{(s - m_{N^*}^2 - m_N^2)^2 - 4m_{N^*}^2 m_N^2}}{s(s + 2m^2 - m_{N^*}^2 - 3m_N^2) - f(s)\sqrt{(s - m_{N^*}^2 - m_N^2)^2 - 4m_{N^*}^2 m_N^2}} \right).$$

(4)

Here $\sigma_d, \sigma_e, \sigma_i$ are the cross sections coming from the direct, exchange, and interference Feynman diagrams [3]. $g_{N\pi N\pi}$ and $g_{\pi NN^*}$ are the coupling constants. The direct, exchange, and interference terms are added to give the total cross section for production of the $N^*$ resonance. The factor $f(s)$ is defined as

$$f(s) \equiv \sqrt{s(s - 4m_N^2)}.$$

(5)

Using this theory, we calculate the total cross section for $N^*$ production,

$$\text{pp} \rightarrow pN^*(1535).$$

(6)

The $N^*$ is not yet allowed to decay. This is compared to experimental values [6] of the cross section for

$$\text{pp} \rightarrow \text{pp} \eta.$$  

(7)

The comparison is shown in figure 1. It is seen that there is a serious problem near threshold. The theory cuts off at the threshold for $N^*(1535)$ production, yet the experimental cross section continues to smaller energies because the $N^*(1535)$ is decaying.
Figure 1: Comparison of theory and experiment. The experimental data are for the reaction $pp \rightarrow pp\eta$ and the theory is for the reaction $pp \rightarrow pN^*(1535)$, where the $N^*(1535)$ is not allowed to decay. The curve is calculated by summing the direct, exchange and interference terms in equations (3) and (4). Experimental cross sections are taken from reference [6].

3 $N^*$ decay

Consider the general reaction

$$1 + 2 \rightarrow 3 + 4 \rightarrow 3 + 5 + 6,$$

where numbers represent particles. Particle 4 decays into particles 5 and 6. The theory for this process is developed in reference [3] where the total cross section for decay is

$$\sigma(12 \rightarrow 356) = \int d\mu^2 \sigma(12 \rightarrow 34) \rho(\mu^2) \frac{\Gamma(4 \rightarrow 56)}{\Gamma},$$

where

$$\rho(\mu^2) \equiv \frac{m_4\Gamma/\pi}{(\mu^2 - m_4^2)^2 + m_4^2\Gamma^2}.$$

$\Gamma(4 \rightarrow 56)$ is the partial decay width and $\Gamma$ is the total width. Decaying particles such as the $N^*(1535)$, do not have a purely fixed mass. The mass peaks at 1535 MeV, but has a
broad range, which is described by equation (10). Inserting the correct integration limits and changing the integration variable to \( \mu \) gives the resonance decay cross section \[3\],

\[
\sigma(12 \rightarrow 356) = \int_{m_5 + m_6}^{\sqrt{s} - m_3} 2\mu \, d\mu \, \sigma(12 \rightarrow 34) \rho(\mu^2) \frac{\Gamma(4 \rightarrow 56)}{\Gamma}. \tag{11}
\]

The way this is used is by making the cross section for resonance production (particle 4) \( \sigma(12 \rightarrow 34) \) a function of \( \mu \). That is, one simply uses the cross sections from the previous section, but makes the replacement

\[
m_4 \rightarrow \mu. \tag{12}
\]

Finally, note that if the resonance has only one dominant decay channel, then the partial width equals the total width,

\[
\Gamma(4 \rightarrow 56) = \Gamma, \quad \text{(for one dominant decay channel.)} \tag{13}
\]

3.1 Coupling constants

The formula for the decay

\[
4 \rightarrow 5 + 6 \tag{14}
\]

is given by [3]

\[
\Gamma = \frac{1}{\tau} = \frac{g_{456}^2 S}{16\pi m_4^3} \sqrt{m_4^2 + m_5^2 + m_6^2 - 2m_5^2m_6^2 - 2m_5^2m_4^2 - 2m_6^2m_4^2}, \tag{15}
\]

where \( S \) is a product of statistical factors; \( 1/j! \) for each group of identical particles in the final state. For \( N^* \rightarrow N + \eta \), we have \( S = 1 \), \( g_{456} = g_{N^*N\eta} \), \( m_4 = m_{N^*} \), \( m_5 = m_N \), \( m_6 = m_\eta \). The width of the resonance is [5]

\[
\Gamma_{N^*(1535)} = 150 \text{ MeV}, \tag{16}
\]

and therefore \( g_{N^*N\eta} \) could be calculated from equation (15). Instead, the product of coupling constants \( g_{NN\pi}g_{N^*N\pi}g_{N^*N\rho} \) is adjusted to a single number to provide the best fit to the data.

3.2 Comparison between theory and experiment

The above theory, which now includes decay of the \( N^*(1535) \), is compared to experiment in figure 2. The product of coupling constants are adjusted to provide the best fit. It is seen that the inclusion of \( N^*(1535) \) decay clearly fixes the problem near threshold observed in figure 1.
Figure 2: Comparison of theory and experiment for the reaction \( pp \rightarrow p + p + \eta \). The only difference from figure 1 is that now \( N^*(1535) \) decay is included. The curve is calculated from equation (11), with the product of coupling constants adjusted to a single number to provide the best fit.

4 Nuclear reactions

To obtain cross sections for nucleus - nucleus collisions, an approximation is to write

\[
\sigma_{AA \rightarrow \eta X} = (A_p A_T)^{2/3} \sigma_{pp \rightarrow \eta X},
\]

where \( A_p \) and \( A_T \) are the nucleon numbers of the projectile and target respectively, \( \sigma_{AA \rightarrow \eta X} \) is the cross section for \( \eta \) production from nucleus - nucleus collisions, and \( \sigma_{pp \rightarrow \eta X} \) is the cross section for \( \eta \) production from proton - proton collisions discussed above. Note that for \( \pi \) production, small deviations from equation (17) were found in reference [7]. Nevertheless, this provides a first approximation for \( \eta \) production in nuclear collisions.

5 Conclusions

A scalar quantum field theory has been applied to the calculation of total cross sections for \( \eta \) meson production in proton - proton collisions. The \( \eta \) meson decays to \( \pi \) mesons,
which contribute to the electromagnetic cascade, thereby contributing to radiation dose. The tree level Feynman graph was calculated, and the product of coupling constants were adjusted to a single number to provide the best fit to the data. Calculations were made for production of $N^*$ both with and without decay. Comparison between theory and experiment was found to be excellent when decay of the $N^*$ is included. Future work will address this approximation in more detail.

References


Total cross sections for eta meson production in proton-proton collisions are calculated. The eta meson is mainly produced via decay of the excited nucleon resonance at 1535 MeV. A scalar quantum field theory is used to calculate cross sections, which also include resonance decay. Comparison between theory and experiment is problematic near threshold when resonance decay is not included. When the decay is included, the comparison between theory and experiment is much better.