Parametric Study of High Frequency Pulse Detonation Tubes

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This paper describes development of high frequency pulse detonation tubes similar to a small pulse detonation engine (PDE). A high-speed valve injects a charge of a mixture of fuel and air at rates of up to 1000 Hz into a constant area tube closed at one end. The reactants detonate in the tube and the products exit as a pulsed jet. High frequency pressure transducers are used to monitor the pressure fluctuations in the device and thrust is measured with a balance. The effects of injection frequency, fuel and air flow rates, tube length, and injection location are considered. Both H₂ and C₂H₄ fuels are considered. Optimum (maximum specific thrust) fuel-air compositions and resonant frequencies are identified. Results are compared to PDE calculations. Design rules are postulated and applications to aerodynamic flow control and propulsion are discussed.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>CJ</td>
<td>Chapman-Jouget</td>
</tr>
<tr>
<td>D</td>
<td>tube diameter</td>
</tr>
<tr>
<td>L</td>
<td>tube length</td>
</tr>
<tr>
<td>Isp</td>
<td>specific thrust (s)</td>
</tr>
<tr>
<td>Ispedialed</td>
<td>calculated specific thrust for a PDE (s)</td>
</tr>
<tr>
<td>PDE</td>
<td>pulse detonation engine</td>
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<tr>
<td>psia</td>
<td>lbf/in² absolute</td>
</tr>
<tr>
<td>p₁, p₂</td>
<td>pressure at closed and open end of tube, respectively (psia)</td>
</tr>
<tr>
<td>SLPM</td>
<td>standard liters per minute</td>
</tr>
<tr>
<td>t</td>
<td>time (s)</td>
</tr>
<tr>
<td>Vg</td>
<td>volume of gas injected into tube in one cycle (at 1atm.)</td>
</tr>
<tr>
<td>Vt</td>
<td>volume of tube between injection point and closed end</td>
</tr>
<tr>
<td>φ</td>
<td>equivalence ratio</td>
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</table>

I. Introduction

A. Background

Actuators of various types play an important role in active boundary layer control. The operating principles for such devices are typically based on mechanical deflection, mass injection, suction, or synthetic jets. Mechanical actuator devices include conventional flap controls, vortex generators, as well as more recent micro-electro-mechanical systems flap devices. Pulsed energy addition actuators, such as combustion-driven jet actuators and sparkjet actuators, have small flow rates of gases in them and gain most of their impulse through addition of thermal energy to the gases. Synthetic jets are devices with a cavity whose internal volume is oscillated and an orifice through which is discharged an oscillating jet. On average, there is no net mass flow contained in a synthetic jet but there is net momentum flow (thrust). Pulsed flow actuators are usually designed to couple with the natural instabilities of the flows they seek to control and thus may have high control authority for relatively little injected flow or energy expended. It is claimed that dimensionless frequencies, defined as \{frequency \times \text{streamwise length of separation}\} / \{\text{free stream velocity}\}, of 1 – 10 are required for boundary-layer separation control (i.e., reduction or removal of separation). For boundary layer control in high-speed applications both high impulse
(thrust) and high frequency (kilocycle level pulsation rates) are required for effective coupling to the flow instabilities.

A new type of pulsed energy addition actuator has been demonstrated\textsuperscript{4,6,7} that is capable of providing both high impulse and high frequency. The actuator is similar to a small pulsejet engine\textsuperscript{8} in that it operates in a resonant mode, taking advantage of wave propagation and reflection within the device to accelerate the combustion, which in turn reinforces the wave motion. Typically, combustion in pulsejets occurs by deflagrations rather than detonations and pressure fluctuations are not very high, perhaps of order 10 psi. Since high impulse is needed for the actuator, detonation combustion, with its associated higher pressure amplitudes, is sought. The actuator is similar to a pulse detonation engine (PDE) in that it employs cyclical detonations in a tube. To date, PDEs have not been demonstrated at frequencies higher than about 100 Hz.\textsuperscript{9} One factor limiting the minimum size and maximum frequency of PDEs is the need to initiate a deflagration to detonation transition (DDT), which typically takes some significant length of tube.

The device under investigation is very simple, consisting of an inlet valve to periodically inject a mixture of fuel and air, a constant area tube open at one end and closed at the other in which the combustion takes place, and an igniter to initiate combustion. Products are discharged into the atmosphere (or flow field to be controlled) through the open end. This study is limited to the development of the “high frequency pulse detonation tube” and not to its application to flow control.

Optimally, the detonation tube operates in the quarter-wave mode of resonance, as illustrated in Figure 1. In this figure, pressure is represented by color (blue is low relative to atmospheric, red is high) and flow direction by arrows. A progression of states of the tube during one cycle is shown, proceeding from top to bottom. Fluid is initially in a process of being discharged from the tube after a detonation (1\textsuperscript{st} image). The reflected expansion wave travels to the left, reflects from the closed end as an expansion wave and the pressure at the closed end is now at its lowest (2\textsuperscript{nd} image). The expansion wave travels right, reflects from the open end of the tube as a compression wave and the velocity of the gas near the exit is now to the left (3\textsuperscript{rd} image). Reactants are now injected into the tube towards the closed end. The compression wave travels to the left, reflects from the closed end as a compression and the pressure there is now high (4\textsuperscript{th} image). Simultaneously, the reactants are ignited by contact with residual reaction products from the previous cycle together with the effects of compression and associated adiabatic heating. After a short ignition delay, the reaction proceeds very fast as a detonation, almost instantaneously raising the pressure of the whole tube. The detonation results in the discharge of product gases from the tube, starting a new cycle.

An important feature of the high frequency pulse detonation tube which distinguishes it from a PDE is that the reactants are not ignited by a spark at each injection cycle, but are ignited (as described above) by contact with the products of reaction from the previous cycle. Additionally, products are not displaced from the device by a non-reacting gas prior to injection of the next charge of reactants. This feature simplifies the fuel and air supply handling issues. Finally, reactants are injected towards the closed end of the tube at relatively high speed, as opposed to the usual injection from the closed end.

The objectives of the present work are to investigate the effects of tube length, location of injection, and reactivity of the fuel-air mixture. Fuels are H\textsubscript{2} and C\textsubscript{2}H\textsubscript{4} reacting with air or an air-O\textsubscript{2} mixture. Tube diameter is held constant (0.75 inches). Performance is assessed qualitatively by examining high frequency pressure measurements and quantitatively by measuring tube thrust. Thrust measurements will be...
compared to computations of thrust in PDEs by the analytical method of Wintenberger et al.,\textsuperscript{10} where the detonation calculations used by this method are by the method of McBride and Gordon.\textsuperscript{11} This method has been validated against experimental data for PDEs, and allows the computation of thrust for a wide range of fuel-O\textsubscript{2}-N\textsubscript{2} mixtures. It represents a baseline with which to compare the high frequency detonation tube results to PDEs and to compare the present results for different reactant mixtures with each other. Design guidelines will be inferred from the results.

II. Experimental Method

A High Frequency Pulsed Detonation Tube

A photograph of the 8 inch detonation tube configuration is shown in Figure 2. The configuration consists of a tube, a high frequency valve driven by a motor, spark plug, gaseous fuel and air supply lines, and instrumentation, all attached to the end of a balance beam. The combustor tube is 0.75 inches in internal diameter and 8 in. long, closed at the bottom and open at the top.

Two sectional views of the high frequency valve are shown in Figure 3: (a) a horizontal section through the axis of the motor shaft and (b) a vertical section through the center of the tube and perpendicular to the motor shaft. The valve consists of a rotating shaft supported by ball bearings, with a hole for the fuel and a slot for the air. The shaft rotates in a close-fitting housing containing a hole and a slot matching the hole and slot in the shaft. When the hole and slot in the shaft align with the hole and slot in the housing (orientation shown in the figures)—which occurs twice per shaft revolution—fuel and air pass through the valve and are combined in the small nozzle, prior to entering the combustor tube. The holes and slots areas are 0.0154 in\textsuperscript{2} and 0.0972 in\textsuperscript{2} respectively. A high-speed (up to about 45,000 rpm) AC/DC motor turns the rotating shaft. The motor speed is controlled by a DC motor controller, which can be set manually or by an external 0-10 V DC control voltage. Pressure taps are provided in the valve for monitoring both inlet fuel and inlet air pressures. It was found that the pressure required to drive the flow through the valve was proportional to the flow rate (in standard liters per minute), where the constant is 0.095-0.14 psia/SLPM for air, 0.2-0.3 psia/SLPM for H\textsubscript{2}, and 0.7-0.8 psia/SLPM for C\textsubscript{2}H\textsubscript{4}.

![Injector valve](image)

**Figure 3** Injector valve: (a) Horizontal section viewed from above, also showing spark plug; (b) vertical section, also showing installation of high frequency pressure gauge.

There is no seal between the housing and the rotating shaft although the clearance between them is small (0.0025 in.). The housing is closed at the fuel flow end but the shaft exits the housing at the air end. To avoid a pressure drop across and possible flow through the ball bearing (which is greased) the housing is vented ahead of the bearing and leakage of air to the atmosphere permitted. This leakage flow was measured and is about 10\% of the total air flow.

Pulsed jets of fuel and air (or an air-O\textsubscript{2} mixture) are provided from the high frequency valve and combined in a small nozzle. The resulting flow enters the tube downwards at an angle of 30\º to the tube axis. The nozzle cross section at the entrance to the tube is 0.335 in. × 0.335 in.

The detonation tube is of modular construction, with tube and fuel/air injector sections that can be screwed into one another. Many different configurations of the detonation tube can be constructed by using interchangeable...
components, and by changing their order of assembly. The results of four configurations illustrated in Figure 4 will be presented: an 8 in. tube, two 12 in. tubes, and a 16 in. tube. The 8 in. and 16 in. tubes have injection at 58.5\% and 54.3\% respectively of the length from the bottom, whereas the 12 in. “top” and 12 in. “mid” configurations have injection at 72.3\% and 39\%.

A spark plug is used to provide an initial ignition source, and to ensure reignition if the combustion is extinguished. The plug is fired continuously at 120 Hz, independent of the injection that occurs at frequencies from 100 Hz to 1500 Hz, and is powered by a commercial ignition transformer. A typical spark plug installation is shown in Figure 3(a). Multiple possible pressure spark plug locations are available, but in the current tests the plugs were located 3.33 in. below injection. In previous work\(^7\) it was found that locating the plug below the injection point produced the most consistent ignition provided that the plug is not located too near the bottom of the tube, in which case reignition will not occur if the combustion is extinguished due to the presence of trapped products.

The valve and tube were fabricated from stainless steel. They are not water-cooled, but short runs (tens of seconds) are possible before the temperatures became high enough to be of concern. After a run, the valve and combustor are cooled by blowing air through the assembly.

### B Instrumentation and method

Flows of fuel, air and O\(_2\) were measured using Hastings mass flow meters of respectively 1000, 2500, and 300 standard liters per minute full scale for H\(_2\), air and O\(_2\) flows respectively. Uncertainty in these measurements was 1\%-2\%. Valve shaft position was measured by means of a helium–neon laser aimed at the rotating shaft, a retroreflective strip on the shaft, and a photodiode to detect the reflected laser light.

Taps were provided in the valve for monitoring both supply H\(_2\) and supply air mean pressures. Pressure taps were also provided in the combustion tube, consisting of 0.076 in. deep by 0.076 in. diameter holes opening into a 10 mm threaded hole. The measurements were made by means of 0-250 lbf/in\(^2\) (psi) PCB-Piezotronics sensors model 113A21 screwed into the threaded hole, as illustrated in Figure 3(b), and a signal conditioning unit. These gauges provide dynamic (i.e., high-frequency pass filtered) pressure measurements. Measurements with closely spaced taps of different diameters (0.076 in. and 0.106 in.) indicated the same pressure except that the amplitude of pressure spikes associated with detonations was about 45\% higher for the larger tap. It is assumed the pressure difference between the tube side of the tap and the transducer depends roughly on the square of the velocity of flow through the tap, i.e., on the inverse square of the tap area. On this basis, extrapolation to taps of large size indicates that actual peak pressure in these spikes is about 60\% greater than measured with the 0.076 in. tap.\(^12\) A larger tap size would reduce this problem, but transducer lifetime is reduced with larger taps due to increased exposure of the probe to high heat flux. Time-resolved pressure data are presented for taps \(p_1\) and \(p_2\), 0.50 in. from the bottom of the tube and 0.25 in. from the top respectively (see Figure 2 and Figure 4).

A force balance was employed to measure thrust.\(^12\) The balance consisted of a beam on a pivot (bearing) whose deflection is opposed by a pair of springs; the detonation tube is mounted on the end of the beam and a counterbalance weight on the other. The distance the beam deflects, which is proportional to thrust, is measured using an inductive non contact displacement sensor. To minimize oscillation in the balance a viscous damper is employed consisting of a piston on the end of a rod which moves in a container of liquid detergent. Typical deflection of the beam at the sensor was 0.5 in. and time response to converge to within 2\% of final value is 0.5 s. Balance uncertainty in the measured force is ±0.25 lbf.

High speed pressure data (3 channels) and data from the valve shaft position photodiode (1 channel) were acquired with a digital oscilloscope. A personal computer (PC) running a Labview program controlled the digital
oscilloscope. After acquisition, data were downloaded to the PC. The PC contained a multipurpose I/O board and preconditioning electronics that allowed simultaneous acquisition of fuel, air, O₂, and leakage flow rates from the mass flow meters, mean pressure data from the strain gauge pressure transducers, and the voltage from the thrust balance potentiometer. A control voltage could also be sent to the motor controller, ramping the motor speed up or down during the course of a run.

During a typical run in which data were to be acquired, the valve motor was started, fuel and air flow rates to the tube were established, and combustion was initiated by the spark. When combustion was established, data acquisition was initiated and simultaneously the valve motor was ramped either from low to high speed or from high to low speed, scanning through a range of injection frequencies. For high-frequency pressure, data were acquired over a period of 5 s, and for the thrust balance data the period was 10 s (longer because of the response time of the balance).

Data from the digital oscilloscope, consisting of 250,000 samples per channel, were broken up into segments of 5000 samples per channel. For each segment, the photodiode data, which were a string of pulses of finite width, were processed to obtain the injection frequency using fast Fourier transforms. An artificial “signal” proportional to the opening area of the valve was derived from the signal and knowledge of the valve geometry.

Test cases consisted of “high” flow cases, a few “medium” flow cases, and “low” flow cases. Nominal gas flow rates for the high flow cases are given in Table 1; the flow rates of oxidant are 1350 standard liters per minute for all of these cases. Medium and low flow rate cases had flows respectively 75% and 50% of the corresponding high flow case. There are cases both with H₂ and C₂H₄ fuels. For the cases shown, where the oxidant is air, the mixtures are stoichiometric. In some cases the oxidant is a mixture of air and O₂, the flow rate of air having been reduced so that the total volumetric flow is the same. For these cases, provided fuel and oxidant are well mixed and complete combustion occurs, the heat release per mole of reactants is the same as for the cases with pure air. Comparison of results for air and with those for air-O₂ mixture provides a direct indication of the effects of fuel-oxidant reactivity. Not indicated in the table are cases with H₂ and air at equivalence ratios (φ) other than 1. In these the air flow rates are as indicated in the table, but the H₂ rate is reduced or increased by the factor φ.

### Table 1 Gas flow rates for the “high” flow rate test cases.

<table>
<thead>
<tr>
<th></th>
<th>air (SLPM)</th>
<th>H₂ (lbm/s)</th>
<th>C₂H₄ (SLPM)</th>
<th>O₂ (lbm/s)</th>
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<tbody>
<tr>
<td>H₂ hi</td>
<td>1350</td>
<td>0.0641</td>
<td>567</td>
<td>0.00187</td>
</tr>
<tr>
<td>H₂ hi (10% O₂)</td>
<td>1215</td>
<td>0.0577</td>
<td>567</td>
<td>0.00187</td>
</tr>
<tr>
<td>C₂H₄ hi</td>
<td>1350</td>
<td>0.0641</td>
<td>94.5</td>
<td>0.00435</td>
</tr>
<tr>
<td>C₂H₄ hi (10% O₂)</td>
<td>1215</td>
<td>0.0577</td>
<td>94.5</td>
<td>0.00435</td>
</tr>
<tr>
<td>C₂H₄ hi (20% O₂)</td>
<td>1080</td>
<td>0.0513</td>
<td>94.5</td>
<td>0.00629</td>
</tr>
</tbody>
</table>

III. Results

The terms “resonant frequency” and “optimum composition” are used in the discussion of these results. In these detonating flows, sharp resonances do not occur as they might in an acoustical tube where response to a periodic excitation jumps sharply at the resonant frequency. These terms are applied to operating conditions (fuel-oxidizer composition and injection frequency) at which the detonation tube provides the highest fuel specific thrust (Isp) relative to an ideal specific thrust computed by the method of Wintenberger et al. The frequencies found in this manner are about 580 Hz for the 8 in. tube, 387 Hz for the 12 in. tube, and 290 Hz for the 16 in. tube, inversely proportional to tube length and independent of reactants (within about ±15%). The frequency of quarter-wave acoustic resonance in the 8 in. tube is about 426 Hz if the tube contains air and it is about 1240 Hz if the tube contains the products of combustion of a stoichiometric mixture of H₂ and air. Processes in the detonation tube are not small perturbations and the composition of gases in the tube varies in the cycle (from reactants to products), so good agreement with the acoustic calculation is not expected; nonetheless, the resonant frequency lies between the two acoustical limits.

A High Frequency Pressure

Typical pressure time traces for a case at optimum composition and resonant frequency are shown in Figure 5(a): the 16 in. tube with H₂-air combustion, φ=0.5, and frequency of 290 Hz. As mentioned previously, the gauges do not measure absolute pressure: measurements have been offset to provide plausible indications of absolute pressure by setting the minimum pressure to zero at the bottom end (p₁) and the nominally flat baseline to one atmosphere at the open end (p₂). The bottom end pressure has an underlying sinusoidal waveform with a large pressure spike near the peak of the sinusoid, which is the detonation. The open end pressure has an underlying flat
waveform with large pressure spike more or less coincident with the spike in \( p_1 \). This confirms our simple conceptual model postulated in the Introduction and illustrated by Figure 1 in which the tube is excited in the quarter wave mode of resonance, and superimposed upon this resonance is a detonation that occurs at the maximum pressure point in the cycle. Peak pressures measured at the closed end are about 240 psia, but, bearing in mind attenuation by the pressure taps, actual pressure peaks are likely to be between 300 psia and 400 psia. Ideal “Chapman-Jouget” (CJ) detonations, computed using the method of McBride and Gordon,\(^{11}\) have a pressure ratio at this composition of about 11.7. This calculation implies that if the pressure after detonation is about 350 psia then the pressure before it is about 30 psia. Indeed, the pressure in the tube measured just prior to detonation varies along its length between about 15 psia and about 50 psia, so these pressures are roughly consistent with CJ detonations. The pressure traces show no evidence of the spark firing, which occurs over a 1/120th sec cycle uncorrelated with the fuel-air injection cycle. In comparisons between simultaneously acquired pressure traces (similar to these) and measurements of the current in the spark it was shown that the spark firing had no effect on the pressure in the tube, provided of course that combustion in the tube was not previously extinguished.\(^{12}\) Although not important to the operation of the tube, it was noted that, due to a reduced breakdown voltage in the combustion products as compared to the reactants, the sparking during the part of the cycle the tube contained products occurred as a series of random sparks rather than just one large spark each cycle as occurred in the presence of reactants.

Figure 5 Pressure traces, 16 in. tube, high flow, H\(_2\) fuel: (a) \( p_1 \) and \( p_2 \), 290 Hz injection, \( \phi = 0.5 \) (optimum); (b) \( p_1 \), 300 Hz injection, \( \phi = 0.5, 0.75, 1.0 \). Also, the valve opening area in arbitrary units (where fully closed is 100).

Also shown in Figure 5(a) is a trace representing the valve opening area, in arbitrary units, where fully closed is 100 and fully open about 115. At this optimum composition and resonant frequency the detonations occur just after the valve closes, which is the point that reactants cease to be injected into the tube. The effects of varying the equivalence ratio, in other words the reactivity of the fuel-air mixture, may be seen in Figure 5(b): cases in order of increasing reactivity are \( \phi = 0.5, 0.75 \) and 1.0, and the frequency is 300 Hz, near resonant. At an equivalence ratio of 1, detonations occur near the peak of the valve opening, i.e., before all the reactants are injected into the tube, and as the equivalence ratio is reduced (i.e., the reactivity is reduced), the detonations occur later.

Pressure traces at the bottom of the 8 in. tube with H\(_2\) fuel are shown in Figure 6. Figure 6 (a) shows the effect of reactivity at 600 Hz, near the resonant frequency: cases are \( \phi = 0.5, 0.75, 1.0 \), and the last case has the same H\(_2\) and oxidizer flow rate as the \( \phi = 1.0 \) case but 10% of the air is replaced by O\(_2\). Detonations if they occur do so just as the valve closes. However, with 10% O\(_2\) (the most reactive case) the pressure rise associated with combustion occurs early, the wave form is erratic and does not peak sharply as with the detonative combustion. There is a tendency for combustion to lag and detonations to fail in the \( \phi = 0.5 \) case. Figure 6(b) shows the effect of injection frequency with \( \phi = 1.0 \): frequencies are 194 Hz, 387 Hz, 580 Hz, and 773 Hz. Pressures are plotted as a function of fraction of a cycle. Detonative combustion occurs at all frequencies, with detonation occurring near the peak of the valve opening (i.e., prior to completion of injection) at 194 Hz, but not until after the valve closes at 773 Hz. At frequencies either side of resonant (580 Hz) the underlying waveform deviates from sinusoidal. At low frequencies, the low portion of the waveform forms a larger fraction of the cycle and contains extra smaller amplitude oscillations.
Pressure traces at the bottom of the 16 in. tube with injection of C₂H₄ fuel and air-10% O₂ or air-20% O₂ near the resonant frequency are shown in Figure 8. In the case of C₂H₄ fuel reacting with air (no O₂), there are no detonations, and combustion, if it occurs at all, is incomplete. With air-10% O₂ detonations are erratic and occur well after the injection valve closes but with 20% O₂ the detonations take place closer to the time the injection valve closes.

An ignition delay time was computed from the pressure traces by finding the time between the peak of the pressure trace in each cycle and the valve reference position, defined somewhat arbitrarily to be 0.1 of a cycle before the point where the valve is fully open. Data were smoothed by averaging over about 0.5 s (1/10th the run time) and rejected where detonations were not observed (i.e., where average peak pressure was less that 60 psia for the 8 in. tube and 80 psia for the 16 in. tube). Results for H₂ fuel at high flow are plotted as a function of frequency normalized by resonant frequency in Figure 7. With both 8 in. and 16 in. tubes increasing fuel-oxidizer reactivity clearly reduces ignition delay, as expected. The ignition delay time is a minimum in the approximate vicinity of the resonant frequency, which suggests a coupling with resonance (i.e., the wave motions in the tube) to reduce ignition delay.

Figure 6 Pressure traces ($p_1$), 8 in. tube, high flow, H₂ fuel: (a) 600 Hz injection, $\phi=0.5$, 0.75, 1, and 10% O₂, (b) various injection frequencies, $\phi=1.0$.

Figure 7. Ignition delay time, 8 in and 16 in. tubes, high flow, H₂ fuel, as a function of normalized injection frequency.

Figure 8. Pressure traces ($p_1$), 16 in. tube, medium flow, C₂H₄ fuel reacting with 10% O₂ and 20% O₂, 300 Hz.
delay time. The ignition delay times at resonance for the 8 in. and 16 in. tube are similar at $\phi=1.0$, but increasingly deviate as the equivalence ratio is reduced, the delay being nearly a factor of two smaller for the 8 in. tube as for the 16 in. tube at $\phi=0.5$ (which is the optimum composition). This is consistent with the previous observations that at the optimum composition detonations occur just after the valve closes and the resonant frequency for the 8 in. tube is twice that for the 16 in. These ignition delay times may be compared with ignition times measured in a PDE by Helfrich et al.\textsuperscript{13} for H\textsubscript{2} air combustion: 0.0026 s at $\phi=0.5$, 0.0014 s at $\phi=0.75$, and 0.0011 s at $\phi=1.0$. The present results represent a factor of 2 to 4 times enhancement relative to these delays.

B Thrust

Thrust measurements are typically normalized by the weight rate of flow of fuel to form specific thrust, $Isp$ (s). This allows comparison between results for low and high flow rates, and various fuel-air compositions. Figure 9 and Figure 10 show a representative selection of results plotted as a function of injection frequency. Figure 9 is for the 8 in. tube with H\textsubscript{2} fuel whereas Figure 10 is for the 16 in. tube with C\textsubscript{2}H\textsubscript{4} fuel. Included in these figures are data for cases in which combustion was not ignited (no spark), labeled “nc”. Vertical lines indicate the resonant frequency (at optimum composition). Each line of data represents a single run as the frequency is scanned. Often runs were repeated, with the frequency scanned from both low to high and from high to low. Results usually overlapped within instrument uncertainty ($\pm$ 5%) but not always—in some cases where detonation combustion was marginal there were significant differences.

![Figure 9](image_url)

**Figure 9.** Specific thrust as a function of injection frequency, 8 in. tube, H\textsubscript{2} fuel: (a) high flow, various equivalence ratios, (b) high and low flow, air and air-10% O\textsubscript{2}. The vertical line is the resonant frequency. NC indicates reactants were not ignited (no combustion).

The effect of equivalence ratio is shown in Figure 9(a). Specific thrust peaks at about $\phi=0.51$ and 580 Hz, the optimum composition and resonant frequency. The peak is quite broad. As the equivalence ratio is decreased $Isp$ decreases rapidly, especially on the low frequency side of the peak, causing the peak to shift to higher frequency. On the other hand, as equivalence ratio is increased $Isp$ decreases more at higher frequency, causing the peak to shift to lower frequency. This behavior may be related to the mass of hot combustion product gases trapped between the injected reactant mixture and the closed end of the tube in each cycle. As the frequency of injection is increased, the tube has less time in a cycle to discharge the products from the previous cycle and the trapped mass is increased. This may aid the formation of detonations and increase the thrust in the less reactive, lower equivalence ratio H\textsubscript{2}-air mixtures. Conversely, it may reduce thrust in the more reactive higher equivalence ratio mixtures by causing premature detonations (before injection is complete). Where the frequency of injection is low, exterior air will enter the tube and convection heat loss to the tube will combine to cool the products, favoring more timely detonation in more reactive mixtures.

The effect of flow rates and reactivity is shown in Figure 9(b). The specific thrust is generally reduced for the low flow rates, more so at lower frequency in a manner similar to the effects of reduced reactivity. The effects of reactivity are directly shown by replacement of 10% of the air with O\textsubscript{2}. Added O\textsubscript{2} (increasing reactivity) reduces specific thrust. This may be compared to Figure 6 (a) which shows that added O\textsubscript{2} causes premature ignition of the reactants—in other words, premature detonation is directly related to decreased $Isp$. 

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Specific thrust is obtained even with no combustion taking place at all (Figure 9). For these no combustion cases specific thrust collapses to a single line at fixed equivalence ratio, but is increased with decreased equivalence ratio. This is not surprising since with no combustion thrust is expected to be proportional to the total flow rate (fuel plus air) but specific thrust is obtained by normalizing with the fuel flow rate. Specific thrust has a broad peak at about 426 Hz, the acoustic frequency of the tube with air.

Results for C$_2$H$_4$ fuel are shown in Figure 10. Specific thrust peaks for the high flow rates with air-20% O$_2$ at about 290 Hz. This is presumed to be the optimum composition and resonant frequency, although higher O$_2$ contents would need to be considered to be certain. Results are consistent with the results for H$_2$-air, where reducing $\phi$ below optimum rapidly reduced the specific thrust at all frequencies, but especially at frequencies below the resonant, shifting the peak to higher frequency. Also consistent, specific thrust is generally lower for the low flow rates.

The thrust at the resonant frequencies for all the H$_2$ fueled cases is plotted as a function of equivalence ratio in Figure 11 and Figure 12. Figure 11 is for the 8 in. and 16 in. tubes and shows the effect of tube length at constant tube diameter and (roughly) constant injection location as a fraction of tube height. Figure 12 is for the 12 in. tubes and shows the effect of injection location. Results are compared to the Wintenberger et al. method calculations. Thrust peaks at about 85% of the calculated value at high flow in the 8 in. and 16 in. tubes (Figure 11). Optimum composition, is around $\phi$=0.55 for the 8 in. tube and a little higher for the 16 in. Thrust drops rapidly to the no combustion value if equivalence ratio is reduced below optimum, whereas above optimum, thrust converges to around 70% of the calculation. For the low flow rates and the 10% O$_2$ cases, thrust is lower, as previously observed. With no combustion, thrust is around 18% of the calculation. Figure (b) gives the thrust (lbf), as opposed to specific thrust (s), for constant (high) flow rate of air (or air-O$_2$). Peak overall thrust (as well as peak specific thrust) is obtained at the optimum composition.

The thrust also peaks at about 85% of the calculated value in the 12 in. tube at high flow, and the optimum $\phi$ is about 0.5 (Figure 12). For the low flow, the thrust is higher relative to the calculation for mid injection as compared...
to top injection, with optimum $\phi$ around 0.6. Perhaps the ratio of the mass of trapped combustion products to the mass of injected reactants determines this behavior, and in the low flow case this ratio is too low with top injection.

The specific thrust for the optimum composition cases, normalized by the specific thrust computed by the method of Wintenberger et al., is plotted as a function of frequency divided by resonant frequency in Figure 13. The data generally show reasonable collapse indicating that these cases are fundamentally similar. The result for the 12 in. tube, mid injection, low flow rate differs in both shape and amplitude. The sharper peak and secondary peak probably occurs simply because the data were not obtained close enough to optimum composition – compare for example $\phi$ = 0.41 and 0.51 in Figure 9(a). In this case, the optimum seems to be closer to 0.6 than 0.51. However, the main point is that $I_{sp}/I_{sp\text{ideal}}$ at optimum is clearly higher for this case.

Table 2 summarizes the measurements of thrust at optimum. Specific thrust values compare favorably with values from the literature for PDEs. For example, Schauer et al. obtained specific thrust in an H$_2$-air PDE over a range of equivalence ratios that was about 0.96 of the ideal for $\phi > 0.8$, and about 0.85 for $\phi = 0.5$. Absolute thrust levels were quite similar ($\leq 6.5$ lbf) operating between 12 Hz and 16 Hz. However, their device was large: their tube was 2 in. diameter and 36 in. long, a volume 32 times that of the present 8 in. tube, and their valves were more complex.

IV. Discussion

The results demonstrate that, for a particular detonation tube length, diameter, fuel type, and air flow rate, an optimum fuel-air equivalence ratio and injection frequency (the optimum condition) exist at which the specific thrust is maximized relative to the ideal specific thrust of a PDE. The optimum or so-called resonant frequency, is approximately constant for a given tube length, independent of fuel type and air flow rate, and inversely proportional to tube length. The resonant frequency is the frequency at which an “underlying” quarter wave oscillation of the flow in the tube is most efficiently excited. The detonations occur each cycle of this oscillation and help to excite the oscillation. At the optimum condition the detonations take place just after the completion of injection of reactants into the tube at the peak pressure of the underlying quarter wave excitation. Shorter ignition delays lead to detonations before injection is complete and if the ignition delay is longer detonations fail to occur, or occur well after injection is complete (past the point at which the underlying quarter wave oscillation of pressure in the tube is a maximum). Operation at other compositions produces local maxima in specific thrust at frequencies other than the resonant due to changes in ignition delay time, or lack of detonation.

Table 2 Summary of results at optimum.

<table>
<thead>
<tr>
<th>res freq Hz</th>
<th>$\phi$</th>
<th>thrust lbf</th>
<th>$I_{sp}$ s</th>
<th>$I_{sp\text{ideal}}$ s</th>
<th>$I_{sp}/I_{sp\text{ideal}}$</th>
<th>$V_{g}/V_{t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_2$ 8&quot; hi</td>
<td>580</td>
<td>0.541</td>
<td>5.67</td>
<td>5562</td>
<td>6425</td>
<td>0.866</td>
</tr>
<tr>
<td>H$_2$ 12&quot; top hi</td>
<td>387</td>
<td>0.602</td>
<td>5.65</td>
<td>5042</td>
<td>6042</td>
<td>0.836</td>
</tr>
<tr>
<td>H$_2$ 12&quot; top lo</td>
<td>387</td>
<td>0.533</td>
<td>2.31</td>
<td>4582</td>
<td>6490</td>
<td>0.706</td>
</tr>
<tr>
<td>H$_2$ 12&quot; mid hi</td>
<td>387</td>
<td>0.512</td>
<td>5.62</td>
<td>5745</td>
<td>6643</td>
<td>0.865</td>
</tr>
<tr>
<td>H$_2$ 12&quot; mid lo</td>
<td>387</td>
<td>0.671</td>
<td>3.02</td>
<td>4860</td>
<td>5635</td>
<td>0.862</td>
</tr>
<tr>
<td>H$_2$ 16&quot; hi</td>
<td>290</td>
<td>0.573</td>
<td>5.46</td>
<td>5102</td>
<td>6211</td>
<td>0.821</td>
</tr>
<tr>
<td>C$_2$H$_4$ 16&quot; hi</td>
<td>290</td>
<td>1.038</td>
<td>6.81</td>
<td>1316</td>
<td>1801</td>
<td>0.835</td>
</tr>
</tbody>
</table>
Although tube diameter has not been varied explicitly in the present experiments, tube diameter is a critical parameter in relation to the reactivity of the fuel-air mixture. For the H$_2$ combustion cases at optimum condition, the diameter is close to the diameter for which a single-head spin mode of detonation propagates in the tube. CJ detonations cannot persist, and transition to detonation cannot take place if the composition is less reactive than the compression that produces this mode of detonation. The onset of a single-head spin occurs where the tube circumference $\pi D$ equals the cell size $\lambda$. For H$_2$ air combustion with $\phi = 0.6$ at 1 atmosphere, $\lambda/\pi$ is in the range 0.69 in$^{16}$ – 0.43 in$^{17}$, close to the tube diameter of 0.75 in. The effect of decreasing $\phi$ is to increase cell size, suppressing detonations and leading to the observed drop in $Isp$. The effect of increasing pressure is to decrease cell size, lowering the equivalence ratio at which detonations are suppressed; this effect may explain the slight reduction in $\phi$ at which detonations are suppressed in going from the low to high flow rates (the underlying quarter wave pressure oscillations are larger for the high flow case). On the other hand, for the C$_2$H$_4$ at the optimum condition the tube diameter is not close to the diameter for which a single-head spin mode of detonation propagates. For stoichiometric C$_2$H$_4$-air combustion at 1 atm. $\lambda/\pi$ is about 0.32 in$^{15}$ and in the present experiment the optimum has 20% added O$_2$, for which the cell sizes are even smaller.

For H$_2$ fuel, there is little effect of tube length on performance at the optimum condition over the range considered ($L/D = 10.7$ – 21.3). The effects of injection location and fuel-air flow rate may be better understood by considering the ratio of the volume of reactants (at ambient pressure) injected at each injection cycle, $V_g$, to the volume of the tube between the point of injection and the closed end, $V_t$, tabulated in Table 2. Specific thrust performance is poor when this ratio falls below roughly 1, and ratios greater than 2 may be advantageous. (Ratios greater than 1 are possible since just prior to detonation the pressure in the tube is between 15 psia at the exit and 45 psia at the closed end.)

The results of this work suggest that injection delay is a critical parameter controlling optimum performance of the device, and that coupling with resonance reduces ignition delay 2 to 4 times compared to delays in PDEs. The reduction was greater for the 8 in. tube than for the 16 in. tube: it is not clear if this difference is some effect of the device, and that coupling with resonance reduces ignition delay 2 to 4 times compared to delays in PDEs. The optimum frequency was the frequency of an underlying quarter wave motion in the tube and the cross-section area, which would reduce the supply pressure requirement below the 39 psia threshold. Additional reduction might be achieved by changing the area-time distribution of the valve so it opens more rapidly and is open for a longer fraction of the cycle. The problem of designing light-weight practical valves for this function is difficult but important.

V. Summary and Conclusions

Experiments have been conducted to investigate high frequency pulsed combustion in a tube closed at one end. Tubes 8 in. and 16 in. long, 0.75 in. in diameter were considered. Mixtures of fuel and air (or air-O$_2$) were injected from near the middle or the open end towards the closed end of these tubes. Fuels considered were H$_2$ and C$_2$H$_4$. Detonation ignition was by contact with products of combustion from the previous cycle coupled with the effects of wave motions in the tube which acted to accelerate ignition. Injection frequency and fuel-air composition were varied to optimize the specific thrust. At the optimum condition, detonations occurred just after completion of injection. For H$_2$-air combustion, the optimum equivalence ratio was in the range 0.5 to 0.6 for both the 8 in. and the 16 in. tube. The optimum frequency was the frequency of an underlying quarter wave motion in the tube and...
was inversely proportional to tube length. Specific thrust values roughly 85% of those for ideal pulse detonation engines (PDEs) were achieved at these optimum conditions. These tubes have much higher thrust per unit tube volume than PDEs, are relatively simple in design, but require the fuel-air mixture to be delivered to the tube at higher dynamic pressure than typical for a PDE. Applications in flow control and propulsion may exist.

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References