Scaling of Performance in Liquid Propellant Rocket Engine Combustors

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What is Scaling?

- “The ability to develop new combustion devices with predictable performance on the basis of test experience with old devices.”

- Can be used to develop combustion devices of any thrust size from any thrust size
  - Applied mostly to increase thrust

- Objective is to use scaling as a development tool
  - Move injector design from an “art” to a “science”
Why is Scaling Important?

• Provides guidance and validation to the combustor design and development
  - Develop full-size designs that are closer to success more quickly
  - Validate key requirements earlier in the development process

• May allow use of smaller and lower flow rate hardware during development
  - Reduce costs for manufacturing development hardware
  - Reduce iterations of full-size hardware
  - Reduce development testing costs
    • (-) Smaller, lower flow rate test facilities
    • (-) Less propellant consumption, fewer test personnel
    • (+ ?) Higher pressure test facilities
  - Increase reliability with more thorough evaluation of margins

Is There a Scaling “Holy Grail”?

• Is there a development scaling methodology for combustion devices that offers:
  1. Reduced size and lower flow rate than original
  2. Lower pressure than original
  3. Easily and inexpensively producible
  4. Complete validation for performance, combustion stability, heat transfer, and ignition
Exact Combustion Similarity

- All processes occur in identical fashion, even though they occur with different scales
  - Flow paths
  - Flame patterns
  - Locations and time histories of specie generation
  - Locations and time histories of heat release
  - Contours of temperature, pressure, and velocity
- Focus on steady internal aerothermochemistry
- Note that unsteady flows are not expected to have the same scaling rules

Similarity Parameters from Mass, Momentum, and Energy Equations for Exact Combustion Similarity

Reynolds No. = \( Re = \frac{\rho v L}{\mu} \)

Schmidt No. = \( Sc = \frac{\mu}{\rho D} \)

Prandtl No. = \( Pr = \frac{c_p \mu}{k} \)

Mach No. = \( M = \left( \frac{\rho v^2}{\gamma p} \right)^{1/2} \)

Froude No. = \( Fr = \frac{v^2}{g_x L} \)

\( \Phi = \frac{1/2v^2}{(c_p / \gamma)T} \)

Specific Heat Ratio = \( \gamma = \frac{c_p}{c_v} \)

First Damköhler Group = \( Da,i = \frac{L}{\nu \tau_i} \)

Third Damköhler Group = \( Da,iii = \frac{q' L}{\nu_c \tau_i} \)

Defined by Penner, 1955
Constant properties will result in competition between $Re$ and $Da, i$.

Reynolds No. $Re = \frac{\rho v L}{\mu}$

Schmidt No. $Sc = \frac{\mu}{\rho D}$

Prandtl No. $Pr = \frac{c_p \mu}{k}$

First Damköhler Group $Da, i = \frac{L}{v \tau_i}$

Third Damköhler Group $Da, iii = \frac{q' L}{v c_p T \tau_i} = Da, i * \frac{q'}{c_p T}$

Scaling Between Large & Small $\rightarrow$ Penner-Tsien Rule & Constant Pressure

- Properties $= \text{Constant (}\mu, \rho, D, c_p, k)$
  - $Sc = \frac{\mu}{\rho D} = \text{Constant}$
  - $Pr = \frac{c_p \mu}{k} = \text{Constant}$

- $Re = \frac{\rho v L}{\mu} = \text{Constant}$
  - $v L \mid_{\text{subscale}} = v L \mid_{\text{fullscale}} \rightarrow \left(\frac{v_S}{v_F}\right)\left(\frac{L_S}{L_F}\right) = 1$

- $Da, i = \text{Constant}$
  - $\frac{L}{v \tau_i} \mid_{\text{subscale}} = \frac{L}{v \tau_i} \mid_{\text{fullscale}} \rightarrow \left(\frac{\tau_{i,S}}{\tau_{i,F}}\right) = \left(\frac{L_S}{L_F}\right)^2$
Penner's Conclusions

- Penner concluded control of chemical conversion rate is obtained by artificial modification of droplet size
  - Variation of surface tension by, e.g., surface active agents
- Successful scaling probably accomplished only for bipropellants with greatly different volatilities
- Engine development involves testing small scale injectors with high injection velocities and fine sprays for the less volatile propellant
  - Injector dimensions scale same as chamber dimensions

Scaling Between Large & Small $\rightarrow$
Crocco & Pressure Dependence $\tau \sim p^m$

- $Re = \frac{\rho v L}{\mu} = \text{Constant}$, and $\rho \sim p$
  $$pvL_{\text{subscale}} = pvL_{\text{fullscale}} \rightarrow \left( \frac{p_s}{p_f} \right) \left( \frac{v_s}{v_f} \right) \left( \frac{L_s}{L_f} \right) = 1$$
- $Da, i = \text{Constant}$
  $$\frac{L}{v\tau_i}_{\text{subscale}} = \frac{L}{v\tau_i}_{\text{fullscale}} \rightarrow \left( \frac{\tau_{i,s}}{\tau_{i,F}} \right) = \left( \frac{L_s}{L_f} \right)^{2m/(m+1)}$$
- Note that $\left( \frac{v_s}{v_f} \right) = \left( \frac{L_s}{L_f} \right)^{(1-m)/(1+m)}$ and $\left( \frac{d_s}{d_f} \right) = \left( \frac{L_s}{L_f} \right)^{m/(m+1)}$
Crocco’s Conclusions

- Control of chemical conversion rate is obtained by control of pressure
- Engine development involves testing small scale injectors with high pressures
  - Injector dimensions are *not* scaled the same as chamber dimensions

Conclusions From Early Scaling Studies

- Similarity of some of the parameters resulted in difficult design situations
  - Penner-Tsien: small injectors with increased pressure drops in chambers with distorted contraction ratios, uncertain requirements for $\tau$
  - Crocco: small injectors at higher chamber pressures with distorted injector dimensions, uncertain requirements for $\tau$
Scaling with Constant Element Dimensions – Typical Chamber Configurations Used Today

Constant Pressure

\[ L_b = \text{constant} \]

\[ L' = \text{constant} \]

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M Differences in Short and Long Barrel Chambers

Distance from Injector Face

Short Lb
Long Lb
Short Lb Mach No.
Long Lb Mach No.
Hewitt $d/V$ for Scaling

- Injector characteristic $d/V$ is fixed to chamber diameter:
  \[
  \left( \frac{d_S}{d_F} \right) \frac{v_F}{v_S} = \left( \frac{D_{c,S}}{D_{c,F}} \right)
  \]

Scaling the Combustion Chamber with Geometric Photoscaling

- Photoscaled chamber too short
- Full-length chamber too long

Element size is reduced

Constant Pressure

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Re-evaluate the Required Similarity Groups

Example: Primary Atomization

Significant Reduction of Scales Does Not Change Primary Atomization Regimes
Historical Examples

- **M-1**
  - 6670 kN thrust (1.5 Mlbf)
  - 100:1 ratio between fullscale and subscale thrust

- **Space Shuttle Orbital Maneuvering System**
  - 26.7 kN (6 Klbf)
  - 6:1 and 10:1 ratios between fullscale and subscale thrust

- **NASA Lewis Research Center Thrust/Element**
  - 50:1 ratio between thrust/element in constant chamber diameter

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M-1 Thrust Chamber

- Thrust = 6670 kN (1,500 Klbf)
- LO$_2$/LH$_2$ Propellants
- Pc ~ 6.9 MPa (1000 psia)
- Upper stage concept considered for Apollo and other missions
- Terminated in advanced component development
M-1 Main Injector

M-1 Fullscale Combustor

106.7 cm
Comparison of M-1 Fullscale and Subscale Thrust Chambers

Comparison of M-1 Subscale to Fullscale Performance

Characteristic Velocity Efficiency, %

Mixture Ratio

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M-1 Performance Comparisons

- Measured total $\eta_{C*} \sim 96.0\%$ (or total loss $\Delta \eta_{C*} \sim 4.0\%$)
- Core efficiency based on subscale chamber
  - Core $\eta_{C*} \sim 99.3\%$, or $\Delta \eta_{C*} \sim 0.7\%$
  - No barrier cooling
  - Small maldistribution losses in small hardware
  - Face coolant distribution same in subscale and fullscale
- Intentional Maldistributions
  - $\Delta \eta_{\text{core}} \sim 1.8\%$ due to redistributing fuel for wall and baffle surface cooling
  - $\Delta \eta_{\text{core}} \sim 0.6\%-0.8\%$ due to altering single element oxidizer flow for baffle surface cooling
- $M$ variations between subscale and fullscale $\Delta \eta_{C*} \sim 0.3\%$
- Total accounted $\Delta \eta_{C*} \sim 3.4\%-3.6\%$ out of 4.0\%
  - Not yet considered unintentional maldistributions which can be quite large for very large diameter injectors

NASA LeRC Thrust/Element Studies

- Thrust $= 67$ kN (15 Klb)
- $\text{LO}_2/\text{LH}_2$ propellants
- $P_c \sim 2.1$ MPa (300 psia)
- Part of extensive injector research program in the U.S. during the 1960s
Coarse Elements are Vaporization-limited

**Thrust per Element $T/E,$**

<table>
<thead>
<tr>
<th>$T/E$</th>
<th>L' (in cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>30.5 (12&quot;)</td>
</tr>
<tr>
<td>50</td>
<td>55.9 (22&quot;)</td>
</tr>
<tr>
<td>100</td>
<td>73.7 (29&quot;)</td>
</tr>
</tbody>
</table>

**M-1 Coarse Element**

- Open Symbol $L' = 30.5$ cm (12")
- Solid Symbol $L' = 55.9$ cm (22")

- 1267 (5635)
Summary –
Scaling with Constant Element Size

- Element dimensions kept approximately constant while chamber dimensions (diameter and length scales) are changed
  - Keeps element combustion characteristics similar
  - Violates combustor scaling rules where $L \sim \tau$
    - Injector retains $\sim$ constant $\tau$’s
  - Performance can scale well
    - Maintain constant $M$ in chamber, or ensure reaction completed in barrel
    - Maintain performance subelements
  - Heat transfer scaling has issues
    - Outer row wall spacings not the same
    - Injector $Re$ are similar
  - Combustion stability not scaled well
    - Elements subjected to higher frequency chamber resonances

Summary –
Scaling with Geometric Photoscaling

- Element dimensions change proportionally with chamber diameter
  - Some relationships between chamber and element retained
  - Violates scaling rules where element $Re = constant$
  - Performance scaling uncertain
  - Heat transfer scaling uncertain
    - Outer row element spacings similar
    - Injector $Re$ not the same
  - Combustion stability can scale well
    - Hewitt d/V characteristic is maintained
• Continue to “mine” the historical data base to help define the scaling relationships
  – History provides a wealth of scaling information – thousands of thousands of tests with thousands of combustors!
  – Don’t let this expensive progress go to waste
• Establish scaling relationships for all important individual processes in LPRE
  – Research activities in injection, primary atomization, secondary atomization, vaporization, mixing, reaction
  – Include scaling studies in your physics-based activities
• Use combustion Computational Fluid Dynamics (CFD) analyses to perform scaling “numerical experiments”
Objectives

- Re-introduce to you the concept of scaling
- Describe the scaling research conducted in the 1950s and early 1960s, and present some of their conclusions
- Narrow the focus to scaling for performance of combustion devices for liquid propellant rocket engines
- Present some results of subscale to fullscale performance from historical programs

Scaling H-1 to F-1

\[ \eta_{C*} = 93.8\% \]

\[ \eta_{C*} = 97.3\% \]

53.1 cm \rightarrow 99.6 cm
Scaling H-1 to F-1 – Why didn’t it work?

- $\eta C^* = 97.3\% \quad \longrightarrow \quad \eta C^* = 93.8\%$
- $Dch = 53.1\, \text{cm} \quad \longrightarrow \quad Dch = 99.6\, \text{cm}$
- $L' = 79.2\, \text{cm} \quad \longrightarrow \quad L' = 101.6\, \text{cm}$
- $L^* = 121.9\, \text{cm} \quad \longrightarrow \quad L^* = 121.9\, \text{cm}$
- $P_c = 4.85\, \text{MPa} \quad \longrightarrow \quad P_c = 7.76\, \text{MPa}$
- $F_{sl} = 912\, \text{kN} \quad \longrightarrow \quad F_{sl} = 6770\, \text{kN}$
- 365 ox, 612 fuel \quad \longrightarrow \quad 714 ox, 702 fuel
- $F/E = 2.5\, \text{kN} \quad \longrightarrow \quad F/E = 9.5\, \text{kN}$

H-1 Baffle Compartment is Smaller than F-1 Baffle Compartment

$20.6'' \longrightarrow 39.2''$

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Seven Similarity Parameters for Non-Reacting Flow Processes

Reynolds No. = \( Re = \frac{\rho v L}{\mu} \)

Schmidt No. = \( Sc = \frac{\mu}{\rho D} \)

Prandtl No. = \( Pr = \frac{\frac{c_p \mu}{k}} \)

Mach No. = \( M = \left( \frac{\rho v^2}{\gamma p} \right)^{1/2} \)

Froude No. = \( Fr = \frac{v^2}{g_L} \)

\[ \Phi = \frac{1/2 v^2}{(c_p / \gamma) T} \]

Specific Heat Ratio = \( \gamma = \frac{c_p}{c_v} \)

First Damköhler Group = \( Da, i = \frac{L}{v \tau_i} \)

Third Damköhler Group = \( Da, iii = \frac{q' L}{v c_p T \tau_i} \)

Two Similarity Parameters for Reacting Flow Processes

Reynolds No. = \( Re = \frac{\rho v L}{\mu} \)

\[ \Phi = \frac{1/2 v^2}{(c_p / \gamma) T} \]

Specific Heat Ratio = \( \gamma = \frac{c_p}{c_v} \)

First Damköhler Group = \( Da, i = \frac{L}{v \tau_i} \)

Third Damköhler Group = \( Da, iii = \frac{q' L}{v c_p T \tau_i} \)
Reduced Set from Penner

\[ \text{Reynolds No.} = Re = \frac{\rho v L}{\mu} \]

\[ \text{Schmidt No.} = Sc = \frac{\mu}{\rho D} \]

\[ \text{Prandtl No.} = Pr = \frac{c_p \mu}{k} \]

- Homogeneous Flow
- Low Velocity
- No Significant External Forces

First Damköhler Group = \( Da,i = \frac{L}{v \tau_i} \)

Third Damköhler Group = \( Da,iii = \frac{q'L}{v c_p T \tau_i} \)

Heat Transfer to the Chamber Walls

- \( Re \) and \( Pr \) are fixed

\[ \text{Reynolds No.} = Re = \frac{\rho v L}{\mu} \quad \text{Prandtl No.} = Pr = \frac{c_p \mu}{k} \]

- Therefore Nusselt number \( Nu \) is fixed

\[ \text{Nusselt No.} = Nu \sim \text{Constant} \times Re^{x} Pr^{y} \]

- Therefore, heat transfer characteristics are scaled properly since \( Re \) and \( Pr \) are scaled properly
**Typical Timescales from SP-194**

**COMPARISON OF CHARACTERISTIC TIMES**

**TIME LAG CORRELATION:** \( \tau \sim \frac{D_t^m}{M^m R_t^n} \)

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**DYKEMA ANALYSIS:**

\[ \frac{\tau}{\tau_i} \sim \frac{D_r R_i}{V_i} \]

**STRAHLE ANALYSIS:**

\[ t_{ch} = \frac{C \rho_i R_i^2}{A} \sim \rho R_i^3 \]

**HEIDMANN-WIEBER ANALYSIS:**

\[ \tau \sim \frac{R_t^4}{M c^2 \rho_i^2} \]

**DROP SIZE (INGEBO):**

\[ R_L \sim \frac{D_v}{V_i^{0.4}} \text{ (APPROX.)} \]

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**The Meaning of**

\[ \left( \frac{\tau_{i,S}}{\tau_{i,F}} \right) = \left( \frac{L_S}{L_F} \right)^2 \]

- At constant chamber pressure and temperature, as the length scales are reduced, the chemical conversion times must be reduced as the *square* of the length scales
  - For example, as \( L_S = \frac{1}{2} L_F \) (half geometric scale)
    then \( \tau_{i,S} = \frac{1}{4} \tau_{i,F} \) (chemical times quartered)

- Note that because of \( Re = \) constant, then
  as \( L_S = \frac{1}{2} L_F \) (half geometric scale)
  then \( V_S = 2 V_F \) (velocities doubled)
The Meaning of $\left( \frac{\tau_{i,S}}{\tau_{i,F}} \right) = \left( \frac{L_S}{L_F} \right)^2$, cont.

- Note that injector orifice diameter = scale ratio
- Thus if $L_S = \frac{1}{2} L_F$, then $d_S = \frac{1}{2} d_F$ and $A_S = \frac{1}{4} A_F$, $v_S = 2 v_F$
  - Element flow continuity $m_s = (\rho_F)(\frac{1}{4} A_F)(2 v_F) = \frac{1}{2} m_F$
  - Note that with geometric half-size element, $m_s = \frac{1}{4} m_F$
  - Element pressure drop $\Delta P_S \sim \rho_F v_S^2 \sim \rho_F 4 v_F^2 \sim 4 \Delta P_F$
- Therefore, through a half-sized element, have to increase the flowrate to achieve 4 times $\Delta P$
  - High velocity sprays with enhanced atomization
  - Note that $Re$ are still matched
  - How are flow rates doubled but chamber pressures constant?
    - $M$ not constant – change chamber contraction ratio
- But is $\tau_{i,S} = \frac{1}{4} \tau_{i,F}$ as required? Not clear...

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The Meaning of $\left( \frac{\tau_{i,S}}{\tau_{i,F}} \right) = \left( \frac{L_S}{L_F} \right)^{2m/(m+1)}$

- For $m = 1$ (i.e., $\tau \sim 1/p$, from Crocco)
  $\left( \frac{\tau_{i,S}}{\tau_{i,F}} \right) = \left( \frac{L_S}{L_F} \right)$
  - As the length scales are reduced, the chemical conversion times must be reduced proportionally
  - For example, as $L_S = \frac{1}{2} L_F$ (half geometric scale)
    then $\tau_{i,S} = \frac{1}{2} \tau_{i,F}$
  - However, the chamber pressure is increased, in this case, since $(p_S / p_F)^m = (\frac{\tau_{i,F}}{\tau_{i,S}})$,
    or $p_S = 2 p_F$
  - Also, $v_S = v_F$, or $M = constant$
The Meaning of \( \left( \frac{\tau_{i,S}}{\tau_{i,F}} \right) = \left( \frac{L_S}{L_F} \right)^{2m/(m+1)} \), cont.

- Continuing for \( m=1 \) and \( L_S = \frac{1}{2} L_F \),
  - then \( d_S = \frac{1}{\sqrt{2}} d_F \) and \( A_S = \frac{1}{2} A_F \), \( v_S = v_F \)
  - Element flow continuity \( m_S = (\rho F)(\frac{1}{2} A_F)(v_F) = \frac{1}{2} m_F \)
    - Note that through half-size element, \( m_S = \frac{1}{4} m_F \) normally
  - Element pressure drop \( \Delta P_S \sim \rho_S v_S^2 \sim \rho_F v_F^2 \sim \Delta P_F \)

- Therefore, element flow rate is doubled but element area is doubled so pressure drop is constant
  - Equal velocity sprays
  - Note that \( Re \) are still matched

- Also, is \( \tau_{i,S} = \frac{1}{2} \tau_{i,F} \) as required?

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Five Sub-Elements of Combustor Performance

1. Multi-element inefficiency of all core elements
2. Multi-element inefficiency of all barrier elements
3. Boundary losses
4. Unintentional maldistribution of mass and velocity across the injector face
5. Intentional maldistribution of mass and velocity across the injector face.
Multi-element Efficiencies of Core and Barrier are Comprised of Many Parts

1. Single element mixing inefficiency for each element type
2. Single element vaporization inefficiency for each element type
3. Inter-element mixing inefficiency
4. Inter-element vaporization inefficiency
5. Losses due to two-dimensional effects of the flowstream
6. Losses due to reaction kinetics
7. Losses due to the radiation energy from various combustion species

Boundary Losses

- Heat energy losses from the fluids to the injector and chamber walls
- Boundary layer losses (effect of wall boundaries on the flow streams)
Maldistribution Losses

- Unintentional
  - Non-uniform mass, velocity, and pressure distributions at the injector inlets
  - Non-uniform mass, velocity, and pressure distributions from the injector manifolding
  - Manufacturing tolerance variations on injector metering features

- Intentional
  - Fuel film coolant (FFC) injected into the chamber periphery
  - Deliberate mass flow rate bias of various elements across the injector face (mixture ratio bias)
  - Local element mass flow bias (e.g., off-set, angled or scarfed coaxial posts)
  - Deliberate burning rate variations across the injector face, due to different elements used in the pattern

Scaling the Combustion Chamber – Nomenclature

\[ L_b, L', D_{ch}, d \]
Scaling with Constant Element Dimensions

Fullscale

Element size \( d \) is the same

Subscale

Scaling with Constant Element Dimensions – Maintain Constant Mach No.

\[ L_b = \text{constant} \]
Typical Subscale Chamber Configurations

3-D Chamber
- Subscale Chamber
- Full Scale Injection Elements
- Subscale Transverse Mode for Full Scale Transverse Mode

2D Chamber
- Chamber Width Mode to Simulate Full Scale Mode
- Full Scale Injection Elements
- Variable Width and No. of Elements to Simulate Different Full Scale Modes

Subscale Chamber
- Subscale Injection Elements
- Subscale Transverse Mode for Full Scale Transverse

Transverse Excitation Chamber
- Pie Shape of Full Scale Chamber Diameter
- Full Scale Injection Elements
- Throat at Centerline
- Two Dimensional
- Radial Flow

Longitudinal Chamber
- Subscale Chamber
- Full Scale Injection Elements
- Subscale Longitudinal Mode for Full Scale Transverse Mode

Wedge Chamber
- A Segment of Full Scale Chamber
- Full Scale Injector

Scaling with Constant Element Dimensions – Maintain Constant $M$ Even for Single Element

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Scaling with Photoscaled Element Dimensions

Fullscale \[ d \]

Element size is reduced

Geometric Photoscaling

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**Hewitt d/V for Scaling**

![Graph showing frequency vs. d/V](image)

**Re-evaluate the Required Similarity Groups**

Example: Primary Atomization

![Graph showing oxidizer jet Weber number](image)

**Oxidizer Jet Weber Number, \( \rho_g (V_g - V_j)^2 d/\sigma \)**

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Significant Reduction of Scales Does Not Change Primary Atomization Regimes

Marshall Space Flight Center

Oxidizer Jet Reynolds Number, \( \frac{\rho V d/\mu}{\text{Rayleigh Type Region}} \)

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M-1 Subscale Combustor

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M-1 Subscale Main Injectors

13.7 cm

Fine Element

Coarse Element

M-1 Unielement Combustor

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NASA LeRC Thrust/Element Studies

Chamber pressure tap
Oxygen Injector tubes
Oxygen

High-frequency pressure transducers
Hydrogen temperature probe
Hydrogen temperature probe
Hydrogen

Combustion chamber

NASA LeRC Thrust/Element Injectors

89 N (20 lbf)/Element
4.4 kN (1000 lbf)/Element

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