Black Hole Mergers as Probes of Structure Formation

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ABSTRACT

Intense structure formation and reionization occur at high redshift, yet there is currently little observational information about this very important epoch. Observations of gravitational waves from massive black hole (MBH) mergers can provide us with important clues about the formation of structures in the early universe. Past efforts have been limited to calculating merger rates using different models in which many assumptions are made about the specific values of physical parameters of the mergers, resulting in merger rate estimates that span a very wide range ($0.1 - 10^4$ mergers/year). Here we develop a semi-analytical, phenomenological model of MBH mergers that includes plausible combinations of several physical parameters, which we then turn around to determine how well observations with the Laser Interferometer Space Antenna (*LISA*) will be able to enhance our understanding of the universe during the critical $z \sim 5 - 30$ structure formation era. We do this by generating synthetic *LISA* observable data (total BH mass, BH mass ratio, redshift, merger rates), which are then analyzed using a Markov Chain Monte Carlo method. This allows us to constrain the physical parameters of the mergers. We find that our methodology works well at estimating merger parameters, consistently giving results within 1-σ of the input parameter values. We also discover that the number of merger events is a key discriminant among models. This helps our method be robust against observational uncertainties. Our approach, which at this stage constitutes a proof of principle, can be readily extended to physical models and to more general problems in cosmology and gravitational wave astrophysics.

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1. Introduction

The non-linear universe is a new frontier in cosmology, especially at relatively high redshifts ($z \sim 5-30$) because that is when baryonic structure formation begins (e.g., Barkana & Loeb 2001). The first objects to form, dark matter halos, undergo hierarchical mergers and assemble into larger structures (Rees & Ostriker 1977; White & Rees 1978; Peebles 1993). The first galaxies formed inside these halos; however, these primordial galaxies (with $M \sim 10^8 M_\odot$) are very dim and hence not readily observed. Thus we are (quite literally) left in the dark regarding this crucial era in the history of the universe. Therefore, new techniques must be developed that will allow us to probe the physical processes that drive the formation of structures in the early universe. One such technique involves studying the mergers of massive black holes (MBHs), which trace the mergers of their parent halos. MBH mergers can be observed at high redshifts even if their host galaxies are not bright enough to be detected (e.g., Volonteri et al. 2003; Wyithe & Loeb 2003; Sesana et al. 2004; Hughes & Menou 2005). Consequently, observations of the gravitational waves emitted by MBH mergers at high-z can further our understanding of structure formation in the early universe.

Due to their importance in the study of structure formation, many research groups have calculated merger rates of MBHs (e.g., Haehnelt 2003; Enoki et al. 2004; Islam et al. 2004; Sesana et al. 2004, 2005; Rhook & Wyithe 2005). However, the results span a very broad range ($0.1 - 10^4$ mergers/year). The large discrepancies that arise in the assessment of merger rates result from the choice of different physical parameters in the models used to perform the calculations. Preliminary results suggest that the minimum mass for a halo to host a black hole is of special importance in the determination of merger rates.

Here we develop a semi-analytical, phenomenological model that includes plausible combinations of several physical parameters involved in MBH mergers. As a proof of principle we use statistical methods to generate synthetic *LISA* observable data (total BH mass, BH mass ratio, redshift, merger rates), which are then run through a Markov Chain Monte Carlo (MCMC) algorithm to constrain the physical parameters associated with the mergers. Because *LISA* distributions of observable parameters are expected to be broad, our method is relatively robust against uncertainties in the observations. Indeed, our results suggest that even if there are measurement biases, the total number of *LISA*-detected events will be a strong discriminant among models. Our method is also general enough that it can be used with a number of different observational data constraints.

In § 2 we establish a framework for understanding the dynamics of halo mergers using the standard Press-Schechter and Extended Press-Schechter approaches. In § 3 we discuss the relationship between MBHs and their host halos. This is the source of most of the uncertainty

in merger models. We present our simulations and results in § 4, then summarize in § 5.

2. Dynamics of Halo Mergers

The theory of hierarchical structure formation is based on the growth of linear perturbations in an initial cold dark matter (CDM) density field. Specifically, the Press-Schechter formalism (Press & Schechter 1974) gives the number density of dark matter halos as a function of mass and redshift. We use the Sheth & Tormen (1999) modification of the original Press-Schechter theory in our calculations, as it provides a better fit with numerical simulations. A nicely detailed summary of these standard procedures is given by Mo & White (2002).

The Press-Schechter formalism does not, however, allow us to follow the merger history of any particular halo. For this we use the Extended Press-Schechter formalism of Lacey & Cole (1993). This formalism has been criticized because of mathematical inconsistencies that arise in the calculations of merger trees (Somerville & Kolatt 1999; Benson et al. 2005). However, it is important to note that we do not construct full merger trees. Instead, we recalculate the Press-Schechter halo distribution at each redshift interval. This approach eliminates cumulative errors, since the inconsistencies disappear for small redshift intervals.

Throughout our calculations we use cosmological parameters determined by the WMAP 3rd-year results (Spergel et al. 2007). Although newer results are available, our proof of principle method does not need such high precision to work properly.

3. The Relationship Between Massive Black Holes and their Host Halos

The largest uncertainty in merger rate calculations involves the relationship between MBHs and their host dark matter halos. Here we describe some of these issues.

3.1. Minimum Halo Mass and the Black Hole Occupation Fraction

Suppose no black holes can form in dark matter halos that have less than some minimum mass $M_{\rm min}$. We motivate this with the standard assumption that black holes need to form from baryons. That is because baryons can cool, allowing for the formation of stars which can then evolve into black holes. If the baryons cannot be kept within the halo, then this does not happen and no black holes can form. The minimum mass of a halo that can host a

black hole therefore depends on the depth of the potential well compared to the temperature of the baryons.

There are various possible mechanisms for the formation of massive black holes (Rees 1984). The most common scenario is the collapse of Population III stars (Abel et al. 2002; Bromm et al. 2002; Heger et al. 2003). These are very massive ($\sim 100 M_\odot$) metal-free stars which form at high redshifts in halos with masses $10^5 - 10^6 M_\odot$. Thus halos that contain Population III stars are likely to be populated with a MBH after the star dies. Here we assume for simplicity that all halos with mass above a certain $M_{\rm min}$ are already occupied with black holes by $z = 20$, and that halos with mass below $M_{\rm min}$ have a decreasing non-zero probability of being occupied by a black hole, $P_{\rm occ} = (M/M_{\rm min})^p$, with $p > 0$.

3.2. Black Hole-Halo Mass Correlation

Even if we do have a halo that can form a black hole, there is uncertainty about the relation of the mass of the central black hole to the mass of the dark matter halo. Though a link has been shown between the central velocity dispersion of stars and the mass of a supermassive black hole (see Ferrarese & Ford 2005 for a review), there has been less work establishing a link between dark matter halo mass and black hole mass. However, Ferrarese (2002) found the following relation in the local universe:

$$M_{\rm BH} = 10^7 M_\odot \left(\frac{M_{\rm halo}}{10^{12} M_\odot} \right)^{5/3} . \tag{1}$$

This can be generalized to include a redshift dependence:

$$M_{\rm BH} = 10^7 M_\odot \left(\frac{M_{\rm halo}}{10^{12} M_\odot} \right)^{5/3} (1+z)^n , \tag{2}$$

where scaling arguments suggest $n = 5/2$ (e.g., Rhook & Wyithe 2005).

3.3. Other issues not treated in the current model

There are various other issues which are important in a full physical model of black hole mergers. However, these are not essential in our proof of principle, since our goal here is to demonstrate the method. Thus we make certain assumptions about them without parameterizing. Two of these issues, which will be treated in more detail in later work, are dynamical friction and gravitational recoil.

In order for a black hole merger to occur, there must first be a merger between two halos with black holes at their centers. Dynamical friction plays an important role in this process, as its drag brings the two halos together. Once the halos have merged, dynamical friction causes the central black holes to get closer and form a binary. Note, however, that dynamical friction between halos becomes inefficient for very large halo mass ratios. Therefore, in our work we assume that for any two halos with masses M_1 and M_2, where $M_1 > M_2$, there is a mass ratio cutoff $M_1/M_2 = 50$ above which no mergers happen. Additionally, if a merger takes too long, there is the probability of a third object coming in and disrupting the binary, thus ejecting one or more black holes in the process (Hoffman & Loeb 2007). Thus we assume that for $M_1/M_2 < 50$ mergers happen immediately, thus not allowing for three-body interactions.

After a black hole binary hardens, gravitational wave emission will make the black holes plunge and merge. However, if the black holes have unequal masses or spins there will be a gravitational recoil effect (e.g., Baker et al. 2006; Campanelli et al. 2007; Herrmann et al. 2007; Koppitz et al. 2007; Brügmann et al. 2008) that can eject the merger remnant out of the halo if the kick velocity is high enough (Madau & Quataert 2004). In our work we assume that if the kick speed of a merger is greater than the escape speed of the host halo ($v_{\rm kick} > v_{\rm esc}$), then there is a high probability of ejection of the merger remnant. For simplicity, we consider only non-spinning black holes. Black holes with spin will be treated in a future work.

4. Simulations and Results

We want to answer the question of what would *LISA* observations tell us about the process of structure formation in the early universe. To do this, we generate synthetic *LISA* observable data (total BH mass, BH mass ratio, redshift, merger rates) based on a simple model of MBH mergers that involves various assumptions about a handful of merger parameters. We choose input values for these parameters within a range of plausible, physically realistic extremes. Using these inputs, we calculate the number density of halos using the procedures described in § 2. We then sample observables from the probability distribution thus generated by using the rejection method (Gentle 2003). The generated number of events is determined by the observation time. For our simulations the assumed observation time is three years.

Our statistical goal is to find and explore regions of high likelihood in parameter space. The synthetic data is analyzed to determine how well we can recover the values of the input parameters used in the data generation process. The Markov Chain Monte Carlo (MCMC)

method gives us a computationally robust yet inexpensive way of doing this. A useful description of the general MCMC algorithm is given by Verde et al. (2003).

We are currently working with a four-parameter model of merger events. These parameters, and their explored ranges, are:

(i) **Minimum halo mass ($M_{\rm min}$).** This is the mass above which all halos are occupied with black holes at $z = 20$. The explored range of this parameter is $5.0 < \log_{10}(M_{\rm min}/M_\odot) < 12.0$.

(ii) **Power law index (p).** Indicates the probability of occupation of a halo with mass $M < M_{\rm min}$ at $z = 20$ as $P_{\rm occ} = (M/M_{\rm min})^p$, for $0.1 < p < 4.0$.

(iii) **Redshift dependence (n).** Indicates the redshift dependence of the BH-halo mass relationship, and it is given by $M_{\rm BH,0} = [M_{\rm BH}(M_{\rm halo})](1+z)^n$, for $0.0 < n < 5.0$.

(iv) **BH mass spread (σ).** This is the spread in the BH mass as a function of halo mass, defined as $P(\log_{10}(M_{\rm BH}/M_\odot)) \propto \exp[(\log_{10}(M_{\rm BH}/M_\odot) - \log_{10}(M_{\rm BH,0}/M_\odot))^2/2\sigma^2]$, for $0.1 < \sigma < 1.0$.

We generated synthetic data for five combinations of input parameter values. The top portion of Table 1 shows the input values and the results of the simulations, and Figure 1 illustrates a representative sample. Additionally, we simulated the effects of observational uncertainty by adding errors to the synthetic *LISA* data. This is done by scrambling the values of the total BH mass, BH mass ratio and redshift independently of each other. That is, for some parameter x with unaltered value x_0, we created a biased data set by altering the parameter value to $(1.0 - 1.4)x_0$, and an unbiased data set by altering the parameter value to $(0.8 - 1.2)x_0$. The bottom portion of Table 1 shows the results of these tests, and Figure 2 shows a representative sample.

Table 1 (top section) shows the parameter values at the point of maximum likelihood obtained from our MCMC procedure in each simulation. The error bars represent the 68% confidence level, or a 1-σ deviation. A quick comparison between the results and the input parameter values reveals that for Runs 1, 2, 3 and 5 our points of maximum likelihood fall within 1-σ of their input value, while Run 4 has one resultant parameter value lie just outside the 1-σ margin. Of the 20 parameter ranges explored (five simulations with four parameters each), only one is outside the 68% confidence level. This suggests a slightly non-Gaussian distribution.

Figure 1 illustrates the results from one of these simulations (Run 1). Each plot shows the relationship between a pair of parameters (left column: $\log_{10}(M_{\rm min}/M_\odot) - p$,

$\log_{10}(M_{\rm min}/M_\odot) - n$, $\log_{10}(M_{\rm min}/M_\odot) - \sigma$; right column: $p - n$, $p - \sigma$, $n - \sigma$). Notice the nicely constrained correlations, particularly in the $\log_{10}(M_{\rm min}/M_\odot) - p$ plot. This is because the number of merger events is critical in determining the shape of the confidence regions.

The bottom part of Table 1 (Runs 1b, 1u, 2b, 2u) shows the parameter values at the point of maximum likelihood obtained from our MCMC procedure on each simulation done with data altered in a biased or unbiased way. From the table data it can quickly be assessed that simulations carried out with altered unbiased data give us results as accurate as those for which the data were not altered, i.e., all resultant parameter values lie within 1-σ of their input values. However, for altered biased data we can see that the results lie outside the 1-σ region.

Figure 2 shows the results of a representative altered-data simulation. All three plots show the $\log_{10}(M_{\rm min}/M_\odot) - p$ correlation; the top panel resulting from altered unbiased data (Run 2u) and the bottom panel resulting from altered biased data (Run 2b). The middle panel (Run 2) shows the results from the control simulation, for which the data was not altered in any way. Notice that the results from altered unbiased data keep the point of maximum likelihood within the 68% confidence level, while the results from the altered biased data do not. Additionally, altered unbiased data results in a wider 1-σ region than unaltered data. Note however that we obtain a fairly reasonable parameter estimation even from altered unbiased data, and biased data as well but to a lesser extent, because in our simple scenario the number of merger events can discriminate between models with different parameter values.

5. Summary and Discussion

Our method works well at estimating merger parameters even when there are observational uncertainties involved. Our simulations run with altered unbiased data give us results of comparable accuracy to the results obtained from simulations where the data was unaltered, albeit with slightly less tight constraints. Results from simulations run with altered biased data are expectedly less accurate, with some parameter values estimated outside of the 1-σ margin from the input values. The general shape of the confidence regions, however, remains consistent throughout our results, even when altered biased data was used. This is because the number of merger events determines the shape of the confidence region, thus allowing us to distinguish models with different parameter values.

We have thus established a flexible framework which works well at estimating MBH merger parameters in our phenomenological model. In future work we will explore more

physically driven models with additional merger and cosmological parameters. One especially important parameter is the redshift of reionization. Reionization can increase the temperature of the baryons at the centers of dark matter halos, effectively resulting in an increase in the minimum mass for a halo to form a black hole. Another essential model component is the *LISA* detection sensitivity, since not every merger that happens in the universe is detectable. This can also lead to degeneracies between models, i.e., very different models resulting in similar numbers of detected merger events (Sesana et al. 2007). Furthermore, the statistical method we use is sufficiently general that in the future it will be possible to incorporate additional sources of parameter constraints, such as currently available observational data of high redshift galaxies and future observations carried out with the Atacama Large Millimeter Array (*ALMA*) and the James Webb Space Telescope (*JWST*) once they become operational.

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Table 1. Simulations

		Input Parameters				Max Likelihood			
Run	N_{ev}	m	p	n	σ	m	p	n	σ
1	169	8.5	1.5	0.5	0.2	8.62 ± 0.36	1.42 ± 0.45	0.55 ± 0.20	0.19 ± 0.10
2	245	9.3	1.0	1.5	0.4	9.43 ± 0.36	0.94 ± 0.11	1.68 ± 0.27	0.39 ± 0.08
3	219	9.3	1.0	1.5	0.2	9.13 ± 0.28	1.07 ± 0.12	1.59 ± 0.23	0.22 ± 0.08
4	16	9.3	1.5	1.5	0.4	9.08 ± 0.46	1.75 ± 1.36	1.45 ± 0.54	0.18 ± 0.21
5	155	9.3	1.0	0.5	0.4	9.19 ± 0.46	1.07 ± 0.21	0.68 ± 0.28	0.39 ± 0.09
1b	169	8.5	1.5	0.5	0.2	8.54 ± 0.24	1.57 ± 0.35	0.99 ± 0.17	0.11 ± 0.07
1u	169	8.5	1.5	0.5	0.2	8.63 ± 0.36	1.41 ± 0.41	0.52 ± 0.20	0.22 ± 0.10
2b	245	9.3	1.0	1.5	0.4	8.93 ± 0.21	1.16 ± 0.12	1.85 ± 0.20	0.35 ± 0.07
2u	245	9.3	1.0	1.5	0.4	9.51 ± 0.38	0.92 ± 0.12	1.74 ± 0.28	0.39 ± 0.08

Note. — Col. (1): Simulation number. Col. (2): Number of generated merger events over a three year period. Cols. (3, 7): Minimum halo mass, $m = \log_{10}(M_{\rm min}/M_\odot)$. Cols. (4, 8): Power law index, p. Cols. (5, 9): Redshift dependence, n. Cols. (6, 10): BH mass spread, σ. Columns $3-6$ are the input parameters for the simulations. Columns $7-10$ show the maximum likelihood results for each simulation (i.e., the best estimate parameters). Error bars represent the 68% confidence level. Simulations labeled 'b' and 'u' represent biased and unbiased data, respectively. Results show our method works well at estimating merger parameters.

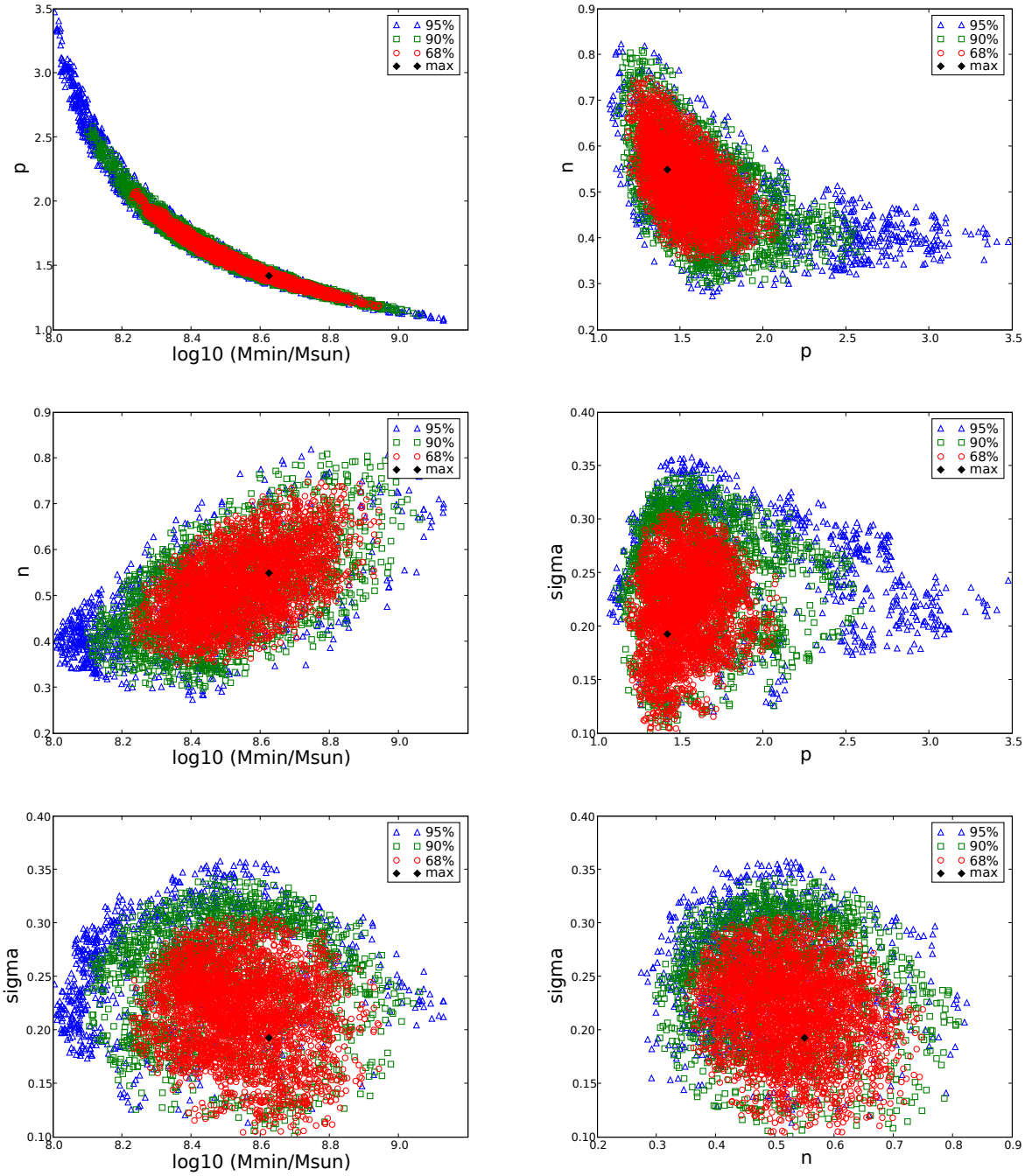

Fig. 1.— Confidence regions in parameter estimation for 169 merger events over a 3-year observation period. The actual values of the parameters are $\log_{10}(M_{\rm min}/M_\odot) = 8.5$, $p = 1.5$, $n = 0.5$, $\sigma = 0.2$ (Run 1). Notice the nicely constrained correlation on the first panel (top-left).

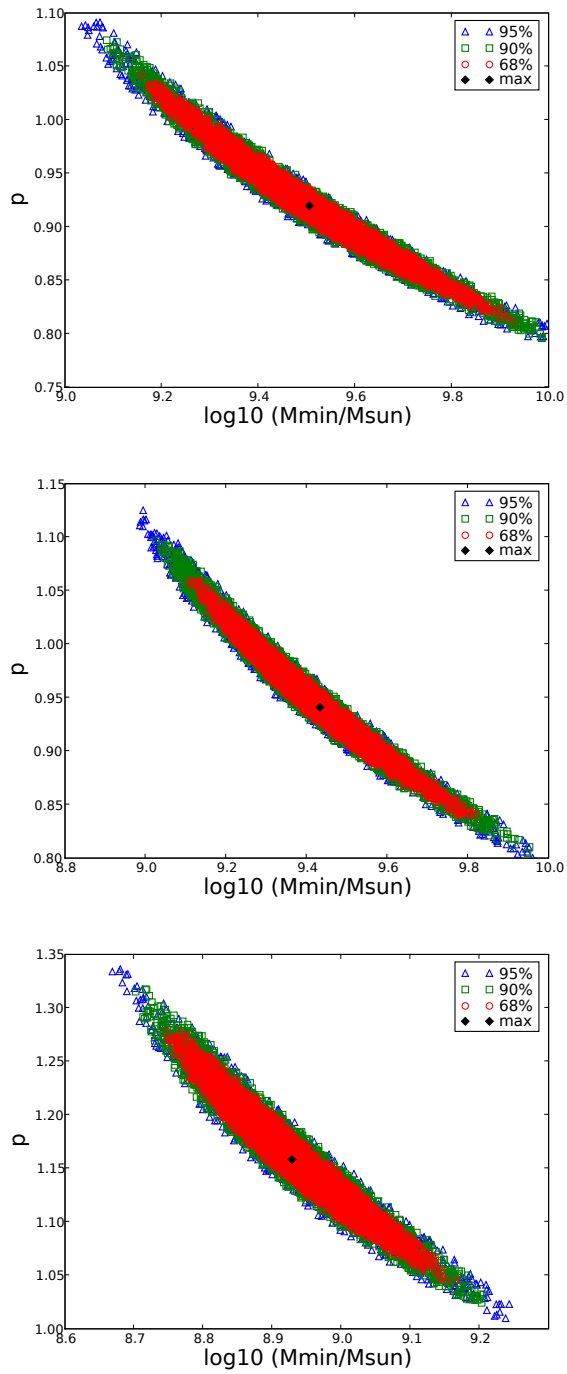

Fig. 2.— Simulating observational errors. Top panel: altered unbiased data (Run 2u). Middle panel: no alterations (Run 2). Bottom panel: altered biased data (Run 2b). Actual values of the parameters shown here are $\log_{10}(M_{\rm min}/M_\odot) = 9.3$ and $p = 1.0$, with 245 total merger events over a 3-year observation run. Note that the parameter estimation is not severely affected by either biased or unbiased errors in the data.