Analysis and Design of Launch Vehicle Flight Control Systems

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This paper describes the fundamental principles of launch vehicle flight control analysis and design. In particular, the classical concept of “drift-minimum” and “load-minimum” control principles is re-examined and its performance and stability robustness with respect to modeling uncertainties and a gimbal angle constraint is discussed. It is shown that an additional feedback of angle-of-attack or lateral acceleration can significantly improve the overall performance and robustness, especially in the presence of unexpected large wind disturbance. Non-minimum-phase structural filtering of “unstably interacting” bending modes of large flexible launch vehicles is also shown to be effective and robust.

I. Introduction

Note to Session Organizer/Reviewers: This draft manuscript summarizes very preliminary results obtained during an early phase of a project for the launch vehicle flight control systems analysis and design as applied to Ares-I Crew Launch Vehicle. During the next several months, a more detailed, rigorous study will be conducted in the areas of drift-minimum vs load-minimum control, flexible-body stabilization and analysis, gain scheduling vs. adaptive control, etc. A companion paper on dynamic modeling of large flexible launch vehicles is also being submitted to this Space Exploration and Transportation GNC session.

II. Rigid-Body Control Analysis

Consider a simplified linear dynamical model of a launch vehicle [15], as illustrated in Fig. 2, as follows:

\[
\begin{align*}
\dot{\theta} &= M_\alpha \alpha + M_\delta \\
\dot{Z} &= -\frac{F}{m} \dot{\theta} - \frac{N_\alpha}{m} \alpha + \frac{T}{m} \delta \\
\alpha &= \theta + \frac{\dot{Z}}{V} + \alpha_w \\
F &= T_\alpha + T - D
\end{align*}
\]

where \( \theta \) is the pitch attitude, \( \alpha \) the angle of attack, \( Z \) the inertial \( Z \)-axis drift position of the center-of-mass, \( \dot{Z} \) the inertial drift velocity, \( m \) the vehicle mass, \( T_\alpha \) the ungimbaled sustainer thrust, \( T \) the gimballed thrust, \( N = N_\alpha \alpha \) the aerodynamic normal (lift) force acting on the center-of-pressure, \( D \) the aerodynamic axial (drag) force, \( F \) the total

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x-axis force, $\delta$ the gimbal deflection angle, $V$ the vehicle velocity, $\alpha_w = V_w/V$ the wind-induced angle of attack, $V_w$ the wind disturbance velocity, and

$$M_\alpha = x_c N_\alpha / I_y$$  
$$M_\delta = x_c g T / I_y$$  
$$N_\alpha = \frac{1}{2} \rho V^2 S C_{N_\alpha}$$

where $I_y$ is the pitch moment of inertia. For an effective thrust vector control of a launch vehicle, we need

$$M_\delta \delta_{\text{max}} > M_\alpha \alpha_{\text{max}}$$

where $\delta_{\text{max}}$ is the gimbal angle constraint and $\alpha_{\text{max}}$ is the maximum wind-induced angle of attack.

The open-loop transfer functions from the control input $\delta(s)$ can then be obtained as

$$\frac{\theta(s)}{\delta(s)} = \frac{s}{\Delta(s)} \left[ M_\delta \left( \frac{s + N_\alpha}{mV} \right) + M_\alpha \frac{T}{mV} \right]$$

$$\frac{Z(s)}{\delta(s)} = \frac{1}{\Delta(s)} \left[ \frac{T}{mV} \left( s^2 - M_\alpha \right) - \frac{M_\alpha (F + N_\alpha)}{m} \right]$$

$$\frac{\alpha(s)}{\delta(s)} = \frac{s}{\Delta(s)} \left[ \frac{T}{mV} s^2 - M_\delta s + \frac{M_\alpha F}{mV} \right]$$

where

$$\Delta(s) = s \left[ s^2 + \frac{N_\alpha}{mV} s^2 - M_\alpha s + \frac{M_\alpha F}{mV} \right]$$

Consequently, the 4th-order system described by Eq. (1) - (3) is completely controllable by $\delta$ and is observable by $Z$; however, the system is not observable by $\theta$ and $\alpha$.

In 1959, Hoelkner introduced the “drift-minimum” and “load-minimum” control concepts as applied to the launch vehicle flight control system [6]. The concepts have been further investigated in [7-14]. Basically, Hoelkner’s controller utilizes a full-state feedback control of the form

$$\delta = -K_1 \theta - K_2 \dot{\theta} - K_3 \alpha$$

2 of 11
for a 3rd-order dynamical model of the form

\[
\frac{d}{dt} \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & M_\alpha \\ -F/(mV) & 1 & -N_\alpha/(mV) \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \alpha \end{bmatrix} + \begin{bmatrix} 0 \\ M_\delta \\ T/(mV) \end{bmatrix} \dot{\delta} + \begin{bmatrix} 0 \\ 0 \\ \dot{\alpha}_w \end{bmatrix}
\]  

(14)

This 3rd-order system is observable by \( \theta \) or \( \alpha \). The feedback gains are to be properly selected to minimize the lateral drift velocity \( \ddot{Z} = V(\alpha - \dot{\theta} - \alpha_w) \) or the bending moment caused by the angle of attack. Note that

\[
\frac{\ddot{Z}}{V} \equiv \gamma = \alpha - \dot{\theta} - \alpha_w
\]  

(15)

where \( \gamma \) is often called the flight-path angle.

Instead of measuring the angle-of-attack, we may employ a body-mounted accelerometer, as illustrated in Fig. 2, as follows:

\[
\dot{\delta} = -K_1 \theta - K_2 \dot{\theta} + K_a \ddot{x}_m
\]

\[
= -K_1 \theta - K_2 \dot{\theta} + K_a \left( -\frac{N_\alpha}{m} \alpha + \frac{T}{m} \delta + \frac{x_a}{m} \dot{\theta} \right)
\]

\[
= -K_1 \theta - K_2 \dot{\theta} + K_a \frac{x_a}{m} \ddot{\theta} - K_a \frac{N_\alpha}{m} \alpha + K_a \frac{T}{m} \delta
\]

Because the resulting effect of \( \ddot{x}_m \) feedback is basically the same as the \( \alpha \) feedback, we consider here only the control logic described by Eq. (13).

Substituting Eq. (13) into Eq. (1) - (2) or Eq. (14), we obtain the closed-loop transfer function from the wind disturbance \( \alpha_w(s) \) to the drift velocity \( \ddot{Z}(s) \) as

\[
\frac{\ddot{Z}}{\alpha_w V} = -\frac{A_2 s^2 + A_1 s + A_0}{s^3 + B_2 s^2 + B_1 s + B_0}
\]  

(16)
where
\[
B_2 = M_b K_2 + \frac{T}{mV} \left( K_3 + \frac{N_\alpha}{T} \right)
\]
\[
B_1 = M_b (K_1 + K_3) - M_\alpha + \frac{K_3 T}{mV} \left( M_\alpha + \frac{M_b N_\alpha}{T} \right)
\]
\[
B_\alpha = \frac{TK_1}{mV} \left( M_\alpha + \frac{M_b N_\alpha}{T} \right) - \frac{F}{mV} (M_b K_3 - M_\alpha)
\]
\[
A_2 = \frac{T}{mV} \left( K_3 + \frac{N_\alpha}{T} \right)
\]
\[
A_1 = \frac{K_3 T}{mV} \left( M_\alpha + \frac{M_b N_\alpha}{T} \right)
\]
\[
A_\alpha = B_\alpha
\]

For a unit-step wind disturbance of \( \alpha_w(s) = 1/s \), the steady-state value of \( \dot{Z} \) can be found as
\[
\dot{Z}_{ss} = \lim_{s \to 0} \frac{- (A_2 s^2 + A_1 s + A_\alpha)}{s^3 + B_2 s^2 + B_1 s + B_\alpha} = \frac{-A_\alpha}{B_\alpha} = -1
\]  \( \text{(17)} \)

The launch vehicle drifts along the wind direction with \( \dot{Z}_{ss} = -V_w \) and also with \( \theta = \dot{\theta} = \alpha = \delta = 0 \) as \( t \to \infty \). It is interesting to notice that the steady-state drift velocity (or the flight path angle) is independent of feedback gains provided an asymptotically stable closed-loop system with \( B_\alpha \neq 0 \).

If we choose the control gains such that \( B_\alpha = 0 \) (i.e., one of the closed-loop system roots is placed at \( s = 0 \)), the steady-state value of \( \dot{Z} \) becomes
\[
\dot{Z}_{ss} = \lim_{s \to 0} \frac{- (A_2 s + A_1)}{s^2 + B_2 s + B_1} = \frac{-A_1}{B_1} = \frac{-1}{1 + C}
\]  \( \text{(18)} \)

where
\[
C = \frac{mV [M_b (K_1 + K_3) - M_\alpha] \right]
\[
M_b K_3 - M_\alpha \right)
\]
\[
\left( 1 + \frac{x_{cg}}{x_{cg}} \right)
\]  \( \text{(19)} \)

For a stable closed-loop system with \( M_b (K_1 + K_3) - M_\alpha > 0 \), we have \( C > 1 \) and
\[
|\dot{Z}_{ss}| < V_w
\]  \( \text{(20)} \)

when \( B_\alpha = 0 \). The drift-minimum condition, \( B_\alpha = 0 \), can be rewritten as
\[
\frac{M_b K_3 - M_\alpha}{M_b K_1} = \frac{N_\alpha}{F} \left( 1 + \frac{x_{cg}}{x_{cg}} \right)
\]  \( \text{(21)} \)

Consider the following closed-loop transfer functions:
\[
\frac{\alpha}{\alpha_w} = -\frac{s(s^2 + M_b K_3 s + M_b K_1)}{s^3 + B_2 s^2 + B_1 s + B_\alpha}
\]  \( \text{(22)} \)
\[
\frac{\delta}{\alpha_w} = -\frac{s(s^2 + M_b K_2 s + M_b K_1)}{s^3 + B_2 s^2 + B_1 s + B_\alpha}
\]  \( \text{(23)} \)

For a unit-step wind disturbance of \( \alpha_w(s) = 1/s \), we have \( \alpha = \delta = 0 \) as \( t \to \infty \). However, for a unit-ramp wind disturbance of \( \alpha_w(s) = 1/s^2 \), we have
\[
\lim_{t \to \infty} \alpha(t) = M_b K_1
\]
\[
\lim_{t \to \infty} \delta(t) = M_b K_1
\]

Consequently, the bending moment induced by \( \alpha \) and \( \delta \) can be minimized by choosing \( K_1 = 0 \), which is the “load-minimum” condition introduced by Hoelkner [6]. The closed-loop system with \( K_1 = 0 \) is unstable because
\[
B_\alpha = -\frac{F}{mV} (M_b K_3 - M_\alpha) < 0
\]  \( \text{(24)} \)
However, the load-minimum control for short durations has been known to be acceptable provided a deviation from the nominal flight trajectory is permissible.

A set of full-state feedback control gains, \((K_1, K_2, K_3)\), can be found by using a pole-placement approach or the linear-quadratic-regulator (LQR) control method [21-22], as follows:

\[
\min_{\delta} \int_{0}^{\infty} (x^T Q x + \delta^2) dt
\]  

subject to \(\dot{x} = Ax + B\delta\) and \(\delta = -Kx\) where \(x = [\theta \ \dot{\theta} \ \alpha]^T\) and \(K = [K_1 \ K_2 \ K_3]\).

### III. Rigid-Body Control Example

Consider a launch vehicle control design example discussed by Greensite in [15]. Its basic parameters are given as in [15]

\[
\begin{align*}
I_y &= 2.43E6 \text{ slug-ft}^2, \quad m = 5830 \text{ slug}, \quad T = 341,000 \text{ lb} \\
F &= 375,000 \text{ lb}, \quad x_{cp} = 38 \text{ ft}, \quad x_{cg} = 32.3 \text{ ft} \\
V &= 1320 \text{ ft/sec}, \quad V_w = 132 \text{ ft/sec}, \quad \alpha_w = 5.73 \text{ deg} \\
N_a &= 240,000 \text{ lb/rad}, \quad M_\alpha = 3.75 \text{ s}^{-2}, \quad M_\delta = 4.54 \text{ s}^{-2}
\end{align*}
\]

The open-loop poles of this example vehicle are: -1.9767, 0.0488, 1.8967

Note that the wind-induced angle of attack of 5.73 deg considered for this example in [15] is somewhat unrealistic because it will require a maximum gimbal deflection angle of

\[
\delta_{\text{max}} > \frac{M_\alpha}{M_\delta} \alpha_w = 4.73 \text{ deg}
\]

Most practical thrust vector control systems have a maximum gimbal angle constraint of about \(\pm 5 \text{ deg}\). In this paper, we also assume a second-order gimbal actuator dynamics of the form

\[
\delta(s) = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \delta_c(s)
\]

where \(\zeta = 1\) and \(\omega_n = 50 \text{ rad/s}\).

### Table 1. Summary of rigid-body control analysis and design

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Controller Type</th>
<th>Feedback Gains ((K_1, K_2, K_3))</th>
<th>Closed-Loop Poles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((\theta, \dot{\theta}))-Feedback Control [15]</td>
<td>(2, 0.8, 0)</td>
<td>-1.7488±1.3934j, -0.1596</td>
</tr>
<tr>
<td>2</td>
<td>Drift-Minimum Control [15]</td>
<td>(2, 0.8, 3.614)</td>
<td>-1.9087±4.2774j, 0.0</td>
</tr>
<tr>
<td>3</td>
<td>Load-Minimum Control [15]</td>
<td>(0, 0.8, 3.614)</td>
<td>-1.9323±3.0533j, 0.0471</td>
</tr>
<tr>
<td>4</td>
<td>LQR Control ((Q = 0))</td>
<td>(0.6852, 0.8491, 0.9542)</td>
<td>-1.9767, -1.8967, -0.0488</td>
</tr>
<tr>
<td>5</td>
<td>Drift-Minimum Control</td>
<td>(0.3220, 0.8352, 1.2765)</td>
<td>-1.9767, -1.8967, 0.0</td>
</tr>
<tr>
<td>6</td>
<td>Load-Minimum Control</td>
<td>(0, 0.8352, 1.2765)</td>
<td>-3.1323, -0.7816, 0.0405</td>
</tr>
</tbody>
</table>

### IV. Flexible-Body Control Analysis

More detailed control and stability analysis results for Figs. 10 and 11 will be included in the final manuscript.
Figure 3. \((\theta, \dot{\theta})\)-feedback control (Case 1).

Figure 4. Drift-minimum control (Case 2).
Figure 5. Load-minimum control (Case 3).

Figure 6. LQR control (Case 4).
Figure 7. Drift-minimum control (Case 5).

Figure 8. Load-minimum control (Case 6).
Figure 9. Case 5 (drift-minimum control) with $\delta_{\text{max}} = \pm 5$ deg.

Figure 10. Illustrations of dominant bending modes and sensor locations (Ref. 2).
Figure 11. Nichols plot for a baseline pitch-axis flight control system (Ref. 2).
V. Conclusions

References