Analysis and Design of Launch Vehicle Flight Control Systems

Bong Wie* and Wei Du†

Iowa State University, Ames, IA 50011-2271

and

Mark Whorton‡

NASA Marshall Space Flight Center, Huntsville, AL 35812

This paper describes the fundamental principles of launch vehicle flight control analysis and design. In particular, the classical concept of “drift-minimum” and “load-minimum” control principles is re-examined and its performance and stability robustness with respect to modeling uncertainties and a gimbal angle constraint is discussed. It is shown that an additional feedback of angle-of-attack or lateral acceleration can significantly improve the overall performance and robustness, especially in the presence of unexpected large wind disturbances. Non-minimum-phase structural filtering of “unstably interacting” bending modes of large flexible launch vehicles is also shown to be effective and robust.

I. Introduction

Note to Session Organizer/Reviewers: This draft manuscript summarizes very preliminary results obtained during an early phase of a project for the launch vehicle control systems analysis and design as applied to Ares-I Crew Launch Vehicle. During the next several months, a more detailed, rigorous study will be conducted in the areas of drift-minimum vs load-minimum control, flexible-body stabilization and analysis, gain scheduling vs. adaptive control, etc. A companion paper on dynamic modeling of large flexible launch vehicles is also being submitted to this Space Exploration and Transportation GNC session.

II. Rigid-Body Control Analysis

Consider a simplified linear dynamical model of a launch vehicle [15], as illustrated in Fig. 2, as follows:

\[
\begin{align*}
\dot{\theta} &= M_{\alpha}\alpha + M_{\delta}\delta \\
\dot{Z} &= -\frac{F}{m}\theta - \frac{N}{m}\alpha + \frac{T}{m}\delta \\
\alpha &= \theta + \frac{\dot{Z}}{V} + \alpha_w \\
F &= T_o + T - D
\end{align*}
\]

where \( \theta \) is the pitch attitude, \( \alpha \) the angle of attack, \( Z \) the inertial Z-axis drift position of the center-of-mass, \( \dot{Z} \) the inertial drift velocity, \( m \) the vehicle mass, \( T_o \) the unimbled sustained thrust, \( T \) the gimbaled thrust, \( N = N_o\alpha \) the aerodynamic normal (lift) force acting on the center-of-pressure, \( D \) the aerodynamic axial (drag) force, \( F \) the total
x-axis force, $\delta$ the gimbal deflection angle, $V$ the vehicle velocity, $\alpha_w = V_w/V$ the wind-induced angle of attack, $V_w$ the wind disturbance velocity, and

$$M_\alpha = x_c N_\alpha / I_y$$
$$M_\delta = x_c T / I_y$$
$$N_\alpha = \frac{1}{2} \beta V^2 SC N_\alpha$$

where $I_y$ is the pitch moment of inertia. For an effective thrust vector control of a launch vehicle, we need

$$M_\delta \delta_{\text{max}} > M_\alpha \alpha_{\text{max}}$$

where $\delta_{\text{max}}$ is the gimbal angle constraint and $\alpha_{\text{max}}$ is the maximum wind-induced angle of attack.

The open-loop transfer functions from the control input $\delta(s)$ can then be obtained as

$$\theta(s) \frac{s}{\delta(s)} = s \frac{M_\delta (s + N_\alpha / mV) + M_\alpha T / mV}{\Delta(s)}$$
$$Z(s) \frac{s}{\delta(s)} = \frac{1}{\Delta(s)} \left[ \frac{T}{m} (s^2 - M_\alpha) - \frac{M_\alpha (F + N_\alpha)}{m} \right]$$
$$\alpha(s) \frac{s}{\delta(s)} = \frac{s}{\Delta(s)} \left[ \frac{T}{mV} s^2 - M_\delta s + \frac{M_\alpha F}{mV} \right]$$

where

$$\Delta(s) = s \left[ s^3 + \frac{N_\alpha}{mV} s^2 - M_\alpha s + \frac{M_\alpha F}{mV} \right]$$

Consequently, the 4th-order system described by Eq. (1) - (3) is completely controllable by $\delta$ and is observable by $Z$; however, the system is not observable by $\theta$ and $\alpha$.

In 1959, Hoelkner introduced the “drift-minimum” and “load-minimum” control concepts as applied to the launch vehicle flight control system [6]. The concepts have been further investigated in [7-14]. Basically, Hoelkner’s controller utilizes a full-state feedback control of the form

$$\delta = -K_1 \theta - K_2 \dot{\theta} - K_3 \dot{\alpha}$$

Figure 1. Comparison of Space Shuttle, Ares I, Ares V, and Saturn V Launch Vehicles [1].
for a 3rd-order dynamical model of the form

\[
\frac{d}{dt} \begin{bmatrix} \theta \\ \dot{\theta} \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & M_\alpha \\ -F/(mV) & 1 - N_\alpha/(mV) \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ \alpha \end{bmatrix} + \begin{bmatrix} 0 \\ M_\delta \\ T/(mV) \end{bmatrix} \delta + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \dot{\alpha}_w
\]  

(14)

This 3rd-order system is observable by \( \theta \) or \( \alpha \). The feedback gains are to be properly selected to minimize the lateral drift velocity \( \dot{Z} = V(\alpha - \theta - \alpha_w) \) or the bending moment caused by the angle of attack. Note that

\[
\frac{\dot{Z}}{V} \equiv \gamma = \alpha - \theta - \alpha_w
\]

(15)

where \( \gamma \) is often called the flight-path angle.

Instead of measuring the angle-of-attack, we may employ a body-mounted accelerometer, as illustrated in Fig. 2, as follows:

\[
\delta = -K_1 \theta - K_2 \dot{\theta} + K_a \ddot{z}_m
\]

\[
= -K_1 \theta - K_2 \dot{\theta} + K_a \left( -\frac{N_\alpha}{m} \alpha + \frac{T}{m} \delta + \frac{x_a}{m} \dot{\theta} \right)
\]

\[
= -K_1 \theta - K_2 \dot{\theta} + K_a \frac{x_a}{m} \dot{\theta} - K_a \frac{N_\alpha}{m} \alpha + K_a \frac{T}{m} \delta
\]

Because the resulting effect of \( \ddot{z}_m \) feedback is basically the same as the \( \alpha \) feedback, we consider here only the control logic described by Eq. (13).

Substituting Eq. (13) into Eq. (1) - (2) or Eq. (14), we obtain the closed-loop transfer function from the wind disturbance \( \alpha_w(s) \) to the drift velocity \( \dot{Z}(s) \) as

\[
\frac{\dot{Z}}{\alpha_w V} = -\frac{A_2 s^2 + A_1 s + A_0}{s^3 + B_2 s^2 + B_1 s + B_0}
\]  

(16)
where

\[
\begin{align*}
B_2 &= M_{\delta}K_2 + \frac{T}{mV} \left( K_3 + \frac{N_{\alpha}}{T} \right) \\
B_1 &= M_{\delta}(K_1 + K_3) - M_\alpha + \frac{K_2T}{mV} \left( M_\alpha + \frac{M_{\delta}N_{\alpha}}{T} \right) \\
B_0 &= \frac{T}{mV} \left( K_3 + \frac{N_{\alpha}}{T} \right) - \frac{F}{mV} (M_{\delta}K_3 - M_\alpha) \\
A_2 &= \frac{T}{mV} \left( K_3 + \frac{N_{\alpha}}{T} \right) \\
A_1 &= \frac{K_2T}{mV} \left( M_\alpha + \frac{M_{\delta}N_{\alpha}}{T} \right) \\
A_0 &= B_o
\end{align*}
\]

For a unit-step wind disturbance of \( \alpha_w(s) = 1/s \), the steady-state value of \( \dot{Z} \) can be found as

\[
\frac{\dot{Z}_{ss}}{V} = \lim_{s \to 0} \frac{-(A_2s^2 + A_1s + A_0)}{s^3 + B_2s^2 + B_1s + B_o} = -\frac{A_0}{B_o} = -1
\]

(17)

The launch vehicle drifts along the wind direction with \( \dot{Z}_{ss} = -V_w \) and also with \( \theta = \dot{\theta} = \alpha = \delta = 0 \) as \( t \to \infty \). It is interesting to notice that the steady-state drift velocity (or the flight path angle) is independent of feedback gains provided an asymptotically stable closed-loop system with \( B_o \neq 0 \).

If we choose the control gains such that \( B_o = 0 \) (i.e., one of the closed-loop system roots is placed at \( s = 0 \)), the steady-state value of \( \dot{Z} \) becomes

\[
\frac{\dot{Z}_{ss}}{V} = \lim_{s \to 0} \frac{-(A_2s + A_1)}{s^2 + B_2s + B_1} = -\frac{A_2}{B_1} = -1 + C
\]

(18)

where

\[
C = \frac{mV[M_{\delta}(K_1 + K_3) - M_\alpha]}{M_{\delta}K_2T + M_{\delta}N_{\alpha}/T}
\]

(19)

For a stable closed-loop system with \( M_{\delta}(K_1 + K_3) - M_\alpha > 0 \), we have \( C > 1 \) and

\[
|\dot{Z}_{ss}| < V_w
\]

(20)

when \( B_o = 0 \). The drift-minimum condition, \( B_o = 0 \), can be rewritten as

\[
\frac{M_{\delta}K_3 - M_\alpha}{M_{\delta}K_1} = \frac{N_{\alpha}}{F} \left( 1 + \frac{x_{cg}}{x_{cg}} \right)
\]

(21)

Consider the following closed-loop transfer functions:

\[
\frac{\alpha}{\alpha_w} = -\frac{s(s^2 + M_{\delta}K_2s + M_{\delta}K_1)}{s^3 + B_2s^2 + B_1s + B_o}
\]

(22)

\[
\frac{\delta}{\alpha_w} = -\frac{s(K_3s^2 + M_\alpha K_2s + M_\alpha K_1)}{s^3 + B_2s^2 + B_1s + B_o}
\]

(23)

For a unit-step wind disturbance of \( \alpha_w(s) = 1/s \), we have \( \alpha = \delta = 0 \) as \( t \to \infty \). However, for a unit-ramp wind disturbance of \( \alpha_w(s) = 1/s^2 \), we have

\[
\begin{align*}
\lim_{t \to \infty} \alpha(t) &= M_{\delta}K_1 \\
\lim_{t \to \infty} \delta(t) &= M_{\alpha}K_1
\end{align*}
\]

Consequently, the bending moment induced by \( \alpha \) and \( \delta \) can be minimized by choosing \( K_1 = 0 \), which is the "load-minimum" condition introduced by Hoelkner [6]. The closed-loop system with \( K_1 = 0 \) is unstable because

\[
B_o = -\frac{F}{mV} (M_{\delta}K_3 - M_\alpha) < 0
\]

(24)

---

4 of 11
However, the load-minimum control for short durations has been known to be acceptable provided a deviation from the nominal flight trajectory is permissible.

A set of full-state feedback control gains, \((K_1, K_2, K_3)\), can be found by using a pole-placement approach or the linear-quadratic-regulator (LQR) control method [21-22], as follows:

\[
\min_{\delta} \int_0^\infty (x^T Q x + \delta^2) dt
\]

subject to \(\dot{x} = Ax + B\delta\) and \(\delta = -Kx\) where \(x = [\theta \ \dot{\theta} \ \phi]^T\) and \(K = [K_1 \ K_2 \ K_3]\).

### III. Rigid-Body Control Example

Consider a launch vehicle control design example discussed by Greensite in [15]. Its basic parameters are given as in [15]

\[
\begin{align*}
I_y &= 2.43E6 \text{ slug-ft}^2, \quad m = 5830 \text{ slug}, \quad T = 341,000 \text{ lb} \\
F &= 375,000 \text{ lb}, \quad x_{cp} = 38 \text{ ft}, \quad x_{cg} = 32.3 \text{ ft} \\
V &= 1320 \text{ ft/sec}, \quad V_w = 132 \text{ ft/sec}, \quad \alpha_w = 5.73 \text{ deg} \\
N_a &= 240,000 \text{ lb/rad}, \quad M_{ax} = 3.75 \text{ s}^{-2}, \quad M_5 = 4.54 \text{ s}^{-2}
\end{align*}
\]

The open-loop poles of this example vehicle are: -1.9767, 0.0488, 1.8967

Note that the wind-induced angle of attack of 5.73 deg considered for this example in [15] is somewhat unrealistic because it will require a maximum gimbal deflection angle of

\[
\delta_{\text{max}} > \frac{M_5}{M_3} \alpha_w = 4.73 \text{ deg}
\]

Most practical thrust vector control systems have a maximum gimbal angle constraint of about ±5 deg. In this paper, we also assume a second-order gimbal actuator dynamics of the form

\[
\delta(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \delta_c(s)
\]

where \(\zeta = 1\) and \(\omega_n = 50 \text{ rad/s}\).

### Table 1. Summary of rigid-body control analysis and design

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Controller Type</th>
<th>Feedback Gains ((K_1, K_2, K_3))</th>
<th>Closed-Loop Poles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((\theta, \dot{\theta}))-Feedback Control [15]</td>
<td>(2, 0.8, 0)</td>
<td>-1.7488±1.3934j, -0.1596</td>
</tr>
<tr>
<td>2</td>
<td>Drift-Minimum Control [15]</td>
<td>(2, 0.8, 3.614)</td>
<td>-1.9087±4.2774j, 0.0</td>
</tr>
<tr>
<td>3</td>
<td>Load-Minimum Control [15]</td>
<td>(0, 0.8, 3.614)</td>
<td>-1.9323±3.0533j, 0.0471</td>
</tr>
<tr>
<td>4</td>
<td>LQR Control ((Q = 0))</td>
<td>(0.6852, 0.8491, 0.9542)</td>
<td>-1.9767, -1.8967, -0.0488</td>
</tr>
<tr>
<td>5</td>
<td>Drift-Minimum Control</td>
<td>(0.3220, 0.8352, 1.2765)</td>
<td>-1.9767, -1.8967, 0.0</td>
</tr>
<tr>
<td>6</td>
<td>Load-Minimum Control</td>
<td>(0, 0.8352, 1.2765)</td>
<td>-3.1323, -0.7816, 0.0405</td>
</tr>
</tbody>
</table>

### IV. Flexible-Body Control Analysis

More detailed control and stability analysis results for Figs. 10 and 11 will be included in the final manuscript.
Figure 3. $(\theta, \dot{\theta})$-feedback control (Case 1).

Figure 4. Drift-minimum control (Case 2).
Figure 5. Load-minimum control (Case 3).

Figure 6. LQR control (Case 4).
Figure 7. Drift-minimum control (Case 5).

Figure 8. Load-minimum control (Case 6).
Figure 9. Case 5 (drift-minimum control) with $\delta_{\text{max}} = \pm 5 \, \text{deg}$.

Figure 10. Illustrations of dominant bending modes and sensor locations (Ref. 2).
Figure 11. Nichols plot for a baseline pitch-axis flight control system (Ref. 2).
V. Conclusions

References