A Process for Comparing Dynamics of Distributed Space Systems Simulations

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ABSTRACT: The paper describes a process that was developed for comparing the primary orbital dynamics behavior between space systems distributed simulations. This process is used to characterize and understand the fundamental fidelities and compatibilities of the modeling of orbital dynamics between spacecraft simulations. This is required for high-latency distributed simulations such as NASA’s Integrated Mission Simulation and must be understood when reporting results from simulation executions. This paper presents 10 principal comparison tests along with their rationale and examples of the results.

1. Introduction

The Integrated Mission Simulation (IMSim) (formerly known as the Distributed Space Exploration Simulation (DSES)) is a NASA research and development project focusing on the technologies and processes that are related to the collaborative simulation of complex space systems involved in the exploration of our solar system. Currently, the NASA centers that are actively participating in the IMSim project are the Ames Research Center, the Jet Propulsion Laboratory (JPL), the Johnson Space Center (JSC), the Kennedy Space Center, the Langley Research Center and the Marshall Space Flight Center.

In concept, each center participating in IMSim has its own set of simulation models and environment(s). These simulation tools are used to build the various simulation products that are used for scientific investigation, engineering analysis, system design, training, planning, operations and more. Working individually, these production simulations provide important data to various NASA projects.

To better leverage off of this rich collection of simulation expertise, the IMSim project is investigating methods and technologies to link these resources together through a distributed simulation infrastructure. Currently, this infrastructure is based on the Institute of Electronics and Electrical Engineers (IEEE) High Level Architecture (HLA) [1] [2] [3] [4]. This is also known by its standard number designation IEEE 1516. HLA provides both the interface standard and the Run-Time Infrastructure (RTI) that is used by IMSim for its distributed simulations.

One significant requirement for a distributed or collaborative simulation is that the fidelity of the component simulations be compatible. This requires coordination between the IMSim participants (federates) to assess and compare selected aspects of the fidelities of their simulation models and environment(s). Specifically, this requires a comparison between IMSim participant space environments and space vehicle dynamics. This paper specifies the elements to be compared in the IMSim dynamics comparison test. It also specifies the accuracies required for a successful comparison.
2. Objective

The principal objective of this paper is to outline a means by which to compare the model sets that provide the planetary environment and 6-degree-of-freedom (6-DOF) orbital dynamics for simulations. Of course, the difficulty is in the details, which in this case is the definition of what constitutes a comparison. The need for comparison of the various IMSim federate space environment and space vehicle dynamics models is driven by the desire to maintain a certain level of dynamic consistency in the results of a distributed simulation versus the results of the same scenario in an integrated simulation.

One advantage to an integrated simulation is that, most likely, all the dynamics dependent components share a common dynamics engine. However, this is most often not the case for a distributed simulation. In distributed simulations, each participant (federate) probably uses its own methods for propagating the dynamic state of its elements. This in itself is not a problem, provided each federate propagates its state accurately. However, problems may arise if the federates’ concepts of what constitutes an accurate simulation state are different. This is where the comparison comes in.

For the IMSim federates, the principal information exchanged between federates is vehicle orbital state. For a 6-DOF simulation, that corresponds to the following seven elements or variables:

- time
- position
- attitude
- velocity
- rotation rate
- acceleration
- rotational acceleration

Given a specific epoch and coordinate state for a vehicle trajectory, the three translational and three rotational state elements can be used for a comparative measure of the dynamic state of the federate vehicles.

Sometimes comparisons of these absolute states are not as meaningful as a comparison of some derived states. For instance, it may provide more insight to compute and compare vehicle attitudes in the Local Vertical Local Horizontal (LVLH) reference frame. Another example is the comparison of the Earth Centered Earth Fixed (ECEF) position of the vehicle. This is useful for analyzing values that are inputs to some environment models.

While the accuracy of the vehicle states is a principal concern, it is often necessary to compare certain environmental parameters as well. Various environmental or vehicle systems models use these parameters as inputs to affect the propagation of the vehicle state. Examples would include planetary orientation or the position of the sun, both of which feed into the atmospheric model and the third-body gravitational effects model.

As a result, a number of data items will be generated and logged in addition to time and the six state parameters. A full listing and description of these additional logged items can be viewed in Appendix A.

3. Simulation Configuration Items

When comparing the output of dynamic systems, it is critical to match up the configuration of the models representing the dynamics. The following list shows the principal configuration items that were used as an example for this study; these will be discussed in the following sub-sections:

**Simulation**
- Simulation Duration: 28,800 seconds
- Data Collection Rate: 60 seconds

**Vehicle**
- Orbital State: Highly Elliptical Orbit
- Mass Properties: Mass Properties

**Environmental Models**
- Gravity Model: On
- Planetary Ephemeris: On
- Sun/Moon Perturbations: On
- Gravity Gradient Torque: On
- Atmospheric Model: On
- F10.7: 128.8
- Geomagnetic Index: 15.7
- Aerodynamic Drag Model: On
- Coefficient of Drag: 0.02
- Cross-sectional Area: 1 m²

**Dynamics**
- Rotational Propagation: Yes
- Initial Rotation Rate: LVLH
- External Torques: No
- External Forces: No

3.1 Simulation

This section of configuration items relates to elements that affect the execution of the simulation.

3.1.1 Simulation Duration

This item specifies the duration of the execution of the simulation. In all cases for this study, the simulation runs are 28,800 seconds or 8 hours. At the orbital altitudes in this study, that corresponds to a little more than five orbits. While this value may seem arbitrary, it was selected based on the longest duration that was
allowed for crew interactions during a vehicle rendezvous procedure for the International Space Station (ISS).

3.1.2 Data Collection Rate

This item specifies the rate for systems data collection or data logging. This study uses a default data collection rate of 60 seconds. This limits the amount of data to 481 data points. However, for cases where more detailed characteristics are required, the data collection rate should be increased. For instance, in the cases where there are significant differences in the dynamic behavior of the vehicle between federates, it may be necessary to log data at the dynamics frequency.

3.2 Vehicle

This section of configuration items relates to elements that affect the vehicle.

3.2.1 Orbital State

This item specifies the vehicle initial orbital state. This study uses two reference orbits that roughly correspond to an ISS standard orbit and an ISS highly elliptical orbit. Although the JSC IMSim federate can initialize its orbits in a variety of different ways, this process uses a date and a vehicle state vector (J2000 position and velocity).

3.2.2 Mass Properties

This item specifies the vehicle initial mass properties. In all test cases, the vehicle mass properties are constant. These test cases employ three sets of mass properties: a specialized set to model an idealized 1 m radius solid sphere, a specialized set to model an idealized 12m × 1m radius solid cylinder, and a set that corresponds roughly to ISS mass parameters.

3.3 Environmental Models

This section of configuration items relates to elements that affect the environmental models.

3.3.1 Gravity Model

This is assumed to be a spherical harmonic gravity model for the near-Earth gravity and a simple inverse square model for the third-body perturbations. 

Order: This item represents the order of the gravity model. The JSC model for these tests is the GEM-T1 gravity model and associated coefficients. These test cases investigate, the following three variations: Spherical, 4×4, and 8×8.

Planetary Ephemeris: This item indicates whether the planetary ephemeris model is active. This study only uses the position of the sun and the moon in the Earth J2000 reference frame. The sun position is used for the atmosphere model and gravitational acceleration. The moon position is only used for gravitational acceleration. Non-Earth centered simulation may use different ephemeral bodies.

Sun/Moon Perturbations: This item indicates whether the sun and moon perturbing accelerations are being applied to the dynamics.

3.3.2 Gravity Gradient Torque

This item indicates the computation and application of torques associated with the gravity gradient on a vehicle in Earth orbit. (Only spherical gravity gradient torques are computed.) These torques are applied to the vehicle’s rotational dynamics.

3.3.3 Atmospheric Model

This item is an indicator of the computation of the vehicle atmospheric density at the vehicle’s current location.

F10.7: This item is the value of the solar radio noise flux.

Geomagnetic Index: This item is the value of the geomagnetic variations index.

3.3.4 Aerodynamic Drag Model

This item is an indicator of the computation of drag and its application to the dynamics.

Coefficient of Drag: This item is the value of the coefficient of drag.

Cross-Sectional Area: This item is the value of the aerodynamic cross-sectional area of vehicle.

3.4 Dynamics

This section of configuration items relates to elements that affect the base dynamics.

3.4.1 Rotational Propagation

This section contains configuration items that are used for the rotational dynamics test cases.
**Initial Rotation Rate**: This indicates whether the vehicle has an initial rotation rate with respect to J2000 or LVLH, depending on run scenario.

**External Torques**: This indicates whether the vehicle has external torques applied or not.

**External Forces**: This indicates whether the vehicle has external forces applied. In most cases, externally applied forces will result in an external torque about the vehicle center of mass. However, in rotational test cases for this study (cases 9C and 9D; to be described subsequently), the forces are applied through the center of mass. This is done to ensure that the forces and torques decouple correctly.

### 4. Test Cases

While the objectives in section 2 are simply stated, they are not so simply accomplished. Initially, it may seem sufficient to have each IMSim participant build a 6-DOF space system simulation, agree on initial conditions, execute the simulation, and compare state and environment variable histories. However, the likelihood of getting exact or even numerically equivalent matches between these simulations rapidly approaches zero, given the diversity of model implementations and simulation environments. Therefore, this paper presents a multi-step testing process that facilitates a more systematic approach. This allows for the progressive increase in modeling complexity and for the systematic identification and categorization of the sources and sizes of comparative modeling differences.

The IMSim dynamics comparison test consists of the following 10 principal “unit” comparison test cases and a final fully integrated comparison test case:

- Test Case 1: Earth Modeling Parameters
- Test Case 2: Earth Orientation and Keplerian Propagation
- Test Case 3: Gravity Modeling
- Test Case 4: Planetary Ephemeris
- Test Case 5: Atmospheric Modeling
- Test Case 6: External Force Effects
- Test Case 7: Combined Translational Test
- Test Case 8: Torque-Free Rotation
- Test Case 9 Torque-Driven Rotation
- Test Case 10: Gravity Gradient Torque
- Full Test: Integrated 6-DOF Test

Each test case consists of one or more run scenarios. The test cases and their associated scenarios are designed to test specific contributions to the dynamic propagation of a 6-DOF simulated space vehicle. Each test case has essentially the same configuration differentiated by a select parameter or associated parameters. See section 3 for a listing and description of these configuration items.

In all cases, the IMSim federate simulations will log a specified set of system parameters. A list of example parameters and a description of each may be found in Appendix A. With the exception of Test Case 1 (4.1), these parameters should be logged at a frequency not less than once per minute (every 60 seconds). For trouble-shooting and detailed comparison, higher data logging rates may be required when measured data are available.

#### 4.1 Earth Modeling Parameters

The purpose of this test is to verify the environmental constants associated with the orbital dynamics models. All units are metric (SI) (unless otherwise noted) and referenced in the J2000/FK5 (FK5 stands for Fundamental Katalog, 5th) [5][6].

While this example test case runs for 28,800 seconds (8 hours), its primary purpose is to verify the planetary modeling parameters that are used by the federates. The planetary constants should be logged at the beginning and end of the simulation to verify their values and that those values remain constant across the execution of the simulation.

#### 4.2 Earth Orientation and Keplerian Propagation

The purpose of this test is to verify the fundamental establishment of coordinate frames and the Earth orientation model that is the Rotation, Nutation, and Precession (RNP) matrix values from the IMSim federates, and also to verify the federates’ ability to numerically propagate an orbital state about a spherical planetary body. This test case establishes the base propagation accuracies of the IMSim federates. Each federate will propagate an Earth orientation and a reference orbital state from a prescribed set of initial conditions for a period of 28,800 seconds (8 hours). The log data sets that are generated by each federate can then be used to compare the state histories of the federates to assess comparative propagation accuracies (Appendix A). Since the propagation is about a spherical gravitational body and there are no other perturbing effects, these results should also compare to an analytical Keplerian solution.

The principal comparison parameters that are used for this test case are:
• Vehicle J2000 position and velocity
• Earth attitude (RNP)
• Vehicle planet-fixed position
• Vehicle orbit semi-major axis (which is a measure of conservation of energy of the dynamical state).

At any point along the vehicle trajectory, the following success criteria should be met:

• J2000 position of vehicle must match to $10^{-3}$ m
• RNP coefficients must match to $10^{-9}$
• Vehicle planet-fixed position must match to $10^{-2}$ m
• Semi-major axis must match to $10^{-3}$ m

4.3 Gravity Modeling

The purpose of this test case is to verify the IMSim federates gravity models. Note: The assumption made here is that the gravitational model is based off of a spherical-harmonic expansion. The JSC federate uses a recursive spherical harmonic gravity model that is configured with GEM-T1 coefficients. For the purposes of these tests, only the 4×4 and 8×8 cases will be tested. Each federate will propagate a reference orbital state from a prescribed set of initial conditions for a period of 28,800 seconds (8 hours). The log data sets that are generated by each federate can then be used to compare the state histories of the federates to assess comparative propagation accuracies. In association with the propagation accuracies that are demonstrated in Test Case 2, this should give some assessment of the comparative accuracies of the federates’ gravity models.

This test case consists of two run scenarios for an orbital vehicle. Here, the mass properties are roughly those of the ISS. The vehicle is initially oriented in a (0,0,0) attitude with respect to LVLH. A single distinguishing characteristic differentiates these scenarios: order of the spherical harmonic gravity model. This results in the following two test scenarios:

• Scenario 3A: 4×4 Gravity Model
• Scenario 3B: 8×8 Gravity Model

The purpose of these run scenarios is to verify the IMSim federates’ comparative validity when running with a 4×4 and an 8×8 geopotential model. These are considered to be low-order, accurate gravity models, but they do give corrections to the larger effects of the Earth’s nonspherical nature. This gives a good intermediate data point for assessing the benefits of going to higher-order gravity models.

The principal comparison parameters that are used for this test case are:

• Vehicle J2000 position and velocity
• Vehicle planet-fixed position

At any point along the vehicle trajectory, the following success criteria should be met:

• J2000 position of vehicle must match to $10^{-1}$ m
• Vehicle planet-fixed position must match to $10^{-1}$ m

Note that the success criteria for this test case are less stringent than for the previous test case. This acknowledges the fact that adding in modeling elements provides further variability in modeling due to differences in both formulation and implementation.

4.4 Planetary Ephemeris

The purpose of this test is to verify the output of the IMSim federates planetary ephemeris models. (The atmosphere model uses the geocentric position of the sun. The third-body perturbation model uses the geocentric position of the sun and the moon.) The JSC IMSim federate uses a C version of the JPL DE405 Planetary Ephemeris model [7].

This run scenario is configured to test the contributions of the gravitational perturbations of the sun and the moon on the propagation of a space vehicle’s dynamic state. For this run, the simulation is configured as described above. Note, that unlike the previous test case, this test case uses a simple spherical gravity model. This should remove any differences that might be attributed to differences in aspherical gravity models and that have already been investigated in the previous test case.

The principal comparison parameters that are used for this test case are:

• Vehicle J2000 position and velocity
• Third-body accelerations
• Solar position (J2000)
• Lunar position (J2000)

At any point along the vehicle trajectory, the following success criteria should be met:

• J2000 position of vehicle must match to $10^{-3}$ m

4.5 Atmospheric Modeling

The purpose of this test is to verify the output of the IMSim federates atmosphere models. As a baseline low-Earth orbital atmospheric model, JSC uses a C
version of the Marshall Engineering Thermosphere atmosphere model [8].

This test case consists of three run scenarios for an orbital vehicle. Here, the mass properties are roughly those of the ISS. The vehicle is initially oriented in a (0,0,0) attitude with respect to LVLH. A single distinguishing characteristic differentiates these scenarios, however: solar activity level. This results in the following three test scenarios:

• Scenario A: Minimum Solar Activity
• Scenario B: Mean Solar Activity
• Scenario C: Maximum Solar Activity

The principal comparison parameters that are used for this test case are:

• Vehicle J2000 position and velocity
• Vehicle planet-fixed position
• Vehicle altitude
• Atmospheric density at the vehicle location
• Atmospheric temperature at the vehicle location

At any point along the vehicle trajectory, the following success criteria should be met:

• Vehicle J2000 position of must match to $10^{-3}$ m
• Atmospheric density must compare to $10^{-13}$ kg/m$^3$

Note that the vehicle J2000 position should exactly match the position in Test Case 2 since, at this point, the atmospheric density does not affect vehicle dynamics. In the next test case, atmospheric drag will be applied as an external force on the vehicle; this will affect the vehicle state.

### 4.6 External Force Effects

The purpose of this test is to verify the dynamics in the presence of external forces. Here the run scenarios are separated into two principal force categories: small continuously applied forces (aerodynamic drag) and larger intermittently applied forces (propulsive maneuvers).

This test case consists of four run scenarios for an orbital vehicle. Two test cases are for low-level, continuously applied forces, and two test cases cover larger intermittently applied forces. The two scenarios for low-level, continuously applied forces correspond to an aerodynamic drag test case. The two scenarios for the larger, intermittently applied forces correspond to propulsive orbital maneuvers. This results in the following four test scenarios:

• A: Aerodynamic Drag with Constant Density
• B: Aerodynamic Drag with Dynamic Atmosphere
• C: Plane Change Maneuver
• D: Earth Departure Maneuver

The two continuous force scenarios (A and B) use mass properties of a simple, idealized $1/\sqrt{\pi}$ m radius solid sphere. In both run scenarios, the coefficient of drag is 0.02 with a vehicle cross-sectional area of 1 m$^2$, which corresponds to a Ballistic Coefficient of 50 kg/m$^2$. The vehicle is initially oriented in a (0,0,0) attitude with respect to LVLH. A single distinguishing characteristic differentiates these scenarios: constant density atmosphere versus a dynamic atmosphere.

The two, larger intermittently applied forces scenarios (C and D) use mass properties of the idealized cylinder. In both run scenarios, aerodynamic drag model is disabled. The vehicle is initially oriented in a (0,0,0) attitude with respect to LVLH. This is important since the thrust will be applied in a constant direction relative to the vehicle’s structural reference frame. These two run scenarios were designed to apply forces along two different axes. The first test applies the force along the vehicle’s Y axis to execute a plane change. The second test applies the force along the vehicle’s X axis to execute an Earth-departure trajectory. For simplicity, these tests continue to use the ISS-based initial conditions. The tests do not represent any expected maneuver for vehicles on an ISS orbit; the tests are solely designed to assess the similarity of dynamics under high-thrust conditions. The tests also do not account for loss of propellant mass, although it would be substantial, 60% to 70% of the vehicle’s initial mass. The tests were designed with an ideal end-state that is described in each test.

#### 4.6.A: Aerodynamic Drag with Constant Density

The purpose of this test is to verify the dynamics of the drag force modeling, but without the variability of the dynamic atmosphere model. This is accomplished by setting the atmospheric density to a constant value of $1.4 \times 10^{-12}$ kg/m$^3$.

#### 4.6.B: Aerodynamic Drag with Dynamic Atmosphere

The purpose of this test is to verify the dynamic atmosphere and the drag force modeling. In this run scenario, the atmospheric density is computed in the simulation atmospheric model (see section 4.5). As a result, the drag model formulation and implementation, as well as the atmospheric model affect the drag forces.

#### 4.6.C: Plane Change Maneuver

This test changes the inclination of the orbit by applying thrust along the Y axis of the vehicle’s structural reference frame. The test was designed to change from the inclination from 51.6
degrees to 31.4 degrees under ideal conditions. The final inclination is close to the inclination of the parking orbits that were used during the Apollo missions. The test applies 29,000 N of thrust to the vehicle; this is similar to the thrust that was produced by the L-9 orbital transfer vehicle from Arianespace.

4.6.D: Earth Departure Maneuver: This test brings the vehicle close to escape velocity by applying thrust along the X axis of the vehicle’s structural reference frame. The test was designed to change the speed of the vehicle from the initial 7,673 m/s to 10,800 m/s under ideal conditions. The test applies 66,400 N of thrust to the vehicle; this is similar to the thrust that was produced by the PAM-D orbital transfer vehicle from Boeing.

The principal comparison parameters that are used for this test case are:

• Vehicle J2000 position and velocity
• Vehicle planet-fixed position
• Vehicle external force
• Atmospheric density at the vehicle location
• Orbital elements (scenarios C and D)

At any point along the vehicle trajectory, the following success criteria should be met:

• J2000 position of vehicle must match to 10⁻¹ m

Note that the success criteria for this test case are less stringent than for the previous test cases. This acknowledges the fact that atmospheric density and the associated atmospheric drag models have a fairly high level of variability in both formulation and implementation.

4.7 Combined Translational Test

This is the last dedicated test for translational propagation accuracy. All preceding tests have investigated isolated effects of various models on the propagation of an orbital vehicle. In this test case, all of these modeling effects are combined.

Due to the effects of the 4×4 versus 8×8 gravity model and the variability of the atmosphere and drag models, this test case is broken up into four run scenarios. This results in the following four test scenarios:

• Scenario 7A: No Drag with 4×4 Gravity
• Scenario 7B: No Drag with 8×8 Gravity
• Scenario 7C: All Models with 4×4 Gravity
• Scenario 7D: All Models with 8×8 Gravity

These cases use the mass properties of the idealized sphere. The details of these run scenarios can be found below.

4.7.A: No Drag with 4×4 Gravity: The purpose of this test is to verify that with gravity model, planetary ephemeris, third-body perturbations, and drag the outputs of the simulations meet the user acceptance criteria. In this case, the aerodynamics are turned off and the 4×4 gravity model is selected.

4.7.B: No Drag with 8×8 Gravity: The purpose of this test is to verify that with gravity model, planetary ephemeris, third-body perturbations, and drag the outputs of the simulations meet the user acceptance criteria. In this case, the aerodynamics are turned off and the 8×8 gravity model is selected.

4.7.C: All Models with 4×4 Gravity: The purpose of this test is to verify that with gravity model, planetary ephemeris, third-body perturbation, and drag models are used. In this case, the aerodynamics are turned off and the 4×4 gravity model is selected.

4.7.D: All Models with 8×8 Gravity: The purpose of this test is to verify that with gravity model, planetary ephemeris, third-body perturbation, and drag models are used. In this case, the 8×8 gravity model is selected.

The principal comparison parameters that are used for this test case are:

• Vehicle J2000 position and velocity
• Vehicle planet-fixed position

At any point along the vehicle trajectory, the following success criteria should be met:

• J2000 position of vehicle must match to 10 m

Note that the success criteria for this test case are less stringent than for the previous test cases. This acknowledges the fact that atmospheric density and the associated atmospheric drag models have a fairly high level of variability in both formulation and implementation.

4.8 Torque-Free Rotation

This test case is designed to verify the integration routines and math models that were used in modeling
the propagation of the rotational state of an orbital vehicle with no externally applied torques [9].

This test case consists of two run scenarios for an orbital vehicle with a nontrivial inertia matrix. These cases use mass properties that are roughly those of the ISS. The vehicle is initially oriented in a slightly pitch-down attitude that roughly corresponds to the ISS Docking Torque Equilibrium Attitude. These scenarios are differentiated by one distinguishing characteristic: initial orbital object body rates. This results in the following two test scenarios:

- Scenario 8A: Zero Initial Attitude Rate
- Scenario 8B: Nonzero Initial Attitude Rate

The details of these run scenarios can be found below.

### 4.8.A: Zero Initial Attitude Rate
This scenario represents the simplest torque-free test case. The initial ISS rotational state is at Docking Torque Equilibrium Attitude with zero angular rates. This leads to the following scenario dependent configuration parameter(s): Initial Rotation Rate: 0,0,0. Note that the inertial attitude should be constant and the LVLH attitude will rotate in pitch at the orbital rate.

### 4.8.B: Nonzero Initial Attitude Rate
This scenario represents a more general torque-free test case. For this test scenario, the initial ISS rotational state is at Docking Torque Equilibrium Attitude but the vehicle is rotating at the orbital rate. This is equivalent to the rotation rate of the LVLH vehicle coordinate reference frame (0.065 deg/sec pitch). This leads to the following scenario dependent configuration parameter(s): Initial Rotation Rate: LVLH. Note: In this test case, the LVLH attitude should remain essentially constant and the inertial attitude will change appropriately. Some variation is expected in the LVLH attitude due to the torque-free motion of the orbital body.

The principal comparison parameters that are used for this test case are:

- Vehicle J2000 position and velocity
- Vehicle J2000 to body axis attitude quaternion
- Vehicle J2000 to body axis attitude in Euler angles (roll, pitch, yaw (RPY))

At any point along the vehicle trajectory, the following success criteria should be met:

- Vehicle J2000 to body axis attitude must match to 0.1 degree for each axis

### 4.9 Torque-Driven Rotation
This test case is designed to verify the integration routines and math models that were used in modeling the propagation of the rotational state of an orbital vehicle in the presence of external torques.

This test case consists of four run scenarios for an orbital vehicle with a nontrivial inertia matrix. These cases use mass properties that are roughly those of the ISS. The vehicle is initially oriented in a slightly pitch-down attitude (-11.6 degrees) that roughly corresponds to the ISS Docking Torque Equilibrium Attitude. All scenarios have a small positive 10 Nm external torque applied for 1,000 seconds starting at the time step that is 1,000 seconds into the run. This torque is applied to the vehicle’s X axis (roll axis). These scenarios are differentiated by two distinguishing characteristics: orbital object body rates and externally applied forces. This results in the following four test scenarios:

- 9A: Zero Initial Attitude Rate with Torque
- 9B: Nonzero Initial Attitude Rate with Torque
- 9C: Zero Initial Attitude Rate with Torque and Force
- 9D: Nonzero Initial Attitude Rate with Torque and Force

Details of these run scenarios can be found below.

### 4.9.A Zero Initial Attitude Rate with Torque
This scenario represents a very simple torque-driven test case. The initial inertial rotation rate is set to zero and no external forces are applied.

### 4.9.B Nonzero Initial Attitude Rate with Torque
This test scenario adds in an initial attitude rate corresponding to the orbital vehicle’s orbital rate. This is equivalent to the rotation rate of the LVLH vehicle coordinate reference frame (0.065 deg/sec pitch).

### 4.9.C Zero Initial Attitude Rate with Torque and Force
This test scenario is exactly the same as Scenario A but adds in a 10 N forces applied in the +X direction in the vehicle’s structural reference frame at the vehicle center of mass. This force is applied for 1,000 seconds starting at 1,000 seconds into the simulation run.

Since the force is applied through the vehicle center of mass, it should only affect the translational position and have no affect on the rotational state. The rotational states for this scenario should match Scenario A; however, the translational states should differ from it.
4.9.D Nonzero Initial Attitude Rate with Torque and Force: This test scenario is exactly the same as Scenario B but adds in a 10 N forces applied in the +X direction in the vehicle’s structural reference frame at the vehicle center of mass. This force is applied for 1,000 seconds starting at 1,000 seconds into the simulation run.

The principal comparison parameters that are used for this test case are:

- Vehicle J2000 position and velocity
- Vehicle J2000 to body axis attitude quaternion
- Vehicle J2000 to body axis attitude in Euler angles (RPY)

At any point along the vehicle trajectory, the following success criteria should be met:

- Vehicle J2000 to body axis attitude must match to 0.1 degree for each axis.

4.10 Gravity Gradient Torque

This test case is designed to verify the numerical integration routines and mathematical models that were used in computation of the effects of gravity gradient torque on a free-flying object in low-Earth orbit. In this case, the free flying object is modeled as a solid cylinder with uniform density.

There are four run scenarios for this test case. These scenarios are separated into two sets, one with a near-circular orbit and one with a significantly elliptic orbit. Each set has a scenario for a zero LVLH state, and one set also has an initial LVLH rate. In all cases, the initial orbital rotational state is set with respect to the LVLH frame. The initial attitude is set to an in-plane displacement (pitch) of 5 degrees and an out-of-plane displacement of 1 degree.

4.10.A Initial Rotation Rate: (0,0,0) LVLH: The analytic solution for this cylinder is that the amplitude of the gravity gradient libration will be a max and min of 5 degrees in plane with a period of 3257.94 seconds and an out-of-plane libration max and min of 1 degree with a period of 2821.46 seconds.

4.10.B Initial Rotation Rate: 0.01 deg/s LVLH Pitch: The analytic solution for this cylinder is that the amplitude of the gravity gradient libration will be a max and min of 5 degrees in plane with a period of 3257.94 seconds and an out-of-plane libration max and min of 1 degree with a period of 2821.46 seconds without an initial rate. The initial pitch rate will result in a small increase in the pitch libration amplitude and a phase shift in the libration period.

4.10.C Initial Rotation Rate: (0,0,0) LVLH: The analytic solution for this cylinder is that the amplitude of the gravity gradient libration will be a max and min of 5 degrees in plane with a period of 3257.94 seconds and an out-of-plane libration max and min of 1 degree with a period of 2821.46 seconds.

4.10.D Initial Rotation Rate: 0.01 deg/s LVLH Pitch: An initial LVLH pitch rate of 0.01 deg/sec is given to the vehicle in this case. The analytic solution for this cylinder is that the amplitude of the gravity gradient libration will be a max and min of 5 degrees in plane with a period of 3257.94 seconds and an out-of-plane libration max and min of 1 degree with a period of 2821.46 seconds without an initial rate. The initial pitch rate will result in a small increase in the pitch libration amplitude and a phase shift in the libration period.

The principal comparison parameters that are used for this test case are:

- Vehicle J2000 position and velocity
- Vehicle J2000 to body axis attitude quaternion
- Vehicle J2000 to body axis attitude in Euler angles (pitch, yaw, roll (PYR))

Note, this comparison uses a pitch-yaw-roll sequence for the Euler angles since the vehicle’s initial attitude is near 90 degrees in pitch, which is near a singularity in the RPY sequence.

At any point along the vehicle trajectory, the following success criteria should be met:

- Vehicle J2000 to body axis attitude must match to 0.1 deg/axis

4.11 Integrated 6-DOF Test

The purpose of this test is to verify the outputs of the simulations meet the acceptance criteria with gravity, planetary ephemeris, third-body perturbations, and drag.

This test case is run with all models turned on and configured to their highest fidelity. This run should give a good comparison between IMSim federate dynamics and Earth environment implementations.

The principal comparison parameters that are used for this test case are:
• Vehicle J2000 position and velocity
• Vehicle planet-fixed position
• Vehicle J2000 to body axis attitude quaternion
• Vehicle J2000 to body axis attitude in Euler angles (PYR)

At any point along the vehicle trajectory, the following success criteria should be met:

• Vehicle J2000 position must match to 10 m
• Vehicle J2000 to body axis attitude must match to 1 degree for each axis

Note that the success criteria for this test case are less stringent than for the previous test cases. This acknowledges the fact that the variability of the combined effects of the models adds into the expected differences.

5. Gravity Gradient Torque Example

To provide a quantitative example, the gravity gradient test case listed in section 4.10 above is described here. The purpose of this test is to verify the numerical integration routines and math model for the propagation of the gravitationally perturbed rotational state of an orbital vehicle.

As defined in section 4.10, the orbital vehicle is a special “cylinder-like” object with a mass of 1,000 kg, 12 m long, and 1 m in radius. It is placed in a circular orbit of 100 minutes (758.5 km). The inertia matrix in the structural reference frame has its center of mass at 6.0, 0.0, 0.0 m. The cylinder has the corresponding diagonal inertia matrix: Ixx = 500.0, Iyy = 12250.0, and Izz = 12250.0 (kgM²). Note the cylindrical symmetry where Iyy=Izz.

The initial rotational state is at a special attitude that is set with respect to the LVLH frame. The in-plane displacement (pitch) is 5 degrees, the out-of-plane displacement (yaw) is 1 degree. This allows for the use of a linearized analytic solution as a check. Specifically, the amplitude of the in-plane gravity gradient libration will have an amplitude of 5 degrees and a period of 3257.94 seconds. The out-of-plane libration will have an amplitude of 1 degree and a period of 2821.46 seconds. The motion in the roll axis will be irregular as long as the initial yaw motion is small because the yaw and roll motion are slightly coupled.

These libration periods and amplitudes can be computed from the linearized analytic theory of small amplitude gravity gradient dynamics [9]. The amplitudes can be given as expressions in elliptic functions, but if the motion is small enough, it reduces to simple trigonometric functions. The libration period in the LVLH pitch axis has the analytic expression

\[ P_{\text{pitch}} = \frac{2\pi \omega \sqrt{3(1 - \frac{I_{yy}}{I_{xx}})}} \]

(where \(\omega\) is the orbital angular rate). The libration period about the LVLH yaw axis has the analytic expression

\[ P_{\text{yaw}} = \frac{2\pi \omega \sqrt{1 - \frac{I_{zz}}{I_{xx}}}} \]

Figure 1

Note from the simulation results shown in figure 1 that the libration in pitch has an amplitude of 5 degrees and a period of 3257.94 seconds. Note also that the libration in yaw has an amplitude of 1 degree and a period of 2821.46 seconds. These match the analytical linearized gravity gradient dynamics in [9].

This kind of test not only validates gravity gradient modeling, but also the numerical integrator's propagation of a Keplerian orbit and the rotational dynamics modeling. Thus, this test case uses an analytic model to validate several aspects of classical physical dynamics of the simulation modeling.
6. Conclusion

Simulation has historically been an important support element in NASA’s mission of human space flight. It continues to be today and, if anything, is even more important. In recent years, NASA began employing collections of collaborative or distributed simulations.

One significant requirement for a distributed or collaborative simulation is that the fidelity of the component simulations be compatible. However, these individual simulations often vary dramatically in implementation. This requires coordination between participants to assess and compare selected aspects of the fidelities of their simulation models and environment(s). Specifically, this requires a comparison between participant space environments and space vehicle dynamics.

This paper describes a process for assessing dynamical compatibility between different space systems simulations. In doing so, it specifies the elements to be compared in the dynamics comparison tests and also specifies the accuracies that are required for successful comparison in this specific use.

Due to space constraints, results for only the gravity gradient torque test case are presented. However, this provides a good combined effect test case with an analytical comparison. The authors plan to present results from all of the test cases in subsequent papers.

7. References


Appendix A. Data Logging

As the IMSim project advances and the participating federates become more complex, data comparisons and regression testing will be important elements of ensuring overall simulation validity and fidelity. To ensure the validity of these simulations as development proceeds, it will be necessary to create sets of data parameters that represent the “state” of the simulation. These parameters will be used as a reference point for simulation comparisons. This includes most of the important data items that are needed for meaningful comparison of dynamic test results. An overview of these parameters can be viewed in Table A.1.

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<th>Reference Frame</th>
<th>Description</th>
<th>Units</th>
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<td>Position vector of vehicle</td>
<td>m</td>
</tr>
<tr>
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<td>Position vector of vehicle</td>
<td>m</td>
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<td>Acceleration vector of vehicle</td>
<td>m/s²</td>
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**R, N and P Matrices**

Earth attitude matrices for computing ECEF and related parameters

**Acknowledgments**

Much of the process outlined in this document originated in a cooperative simulation project between NASA JSC and the Japanese Aerospace eXploration Agency (JAXA). Specifically, the Distributed Simulation (DIS) project developed a simulation of the ISS and the H-II Transfer Vehicle (HTV). Many of the test cases and scenarios that are presented in this document have their origins in the dynamics comparison tests formulated for DIS.

Thanks also to Michael Madden for contributing run scenarios 6C and 6D. Thanks as well to Jason Neuhaus for his diligent proof reading and suggestions on rewording, clarification, and general improvement.

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**EDWIN Z. CRUES, Ph.D.** has supported the Automation, Robotics, and Simulation Division at NASA JSC for the past 14 years. Since 2004, he is a member of the Simulation and Graphics Branch where he leads the research and development of distributed simulations technologies. In this capacity, he has led the development of the HTV Flight Controller Training (FCT) and the NASA Integrated Mission Simulation (IMSim). Dr. Crues also leads the JSC Engineering Orbital Dynamics (JEOD) development team.

**ALBERT A. JACKSON, Ph.D.** came to JSC (then the Manned Spacecraft Center) in 1966 and has been involved with crew training during Apollo, flight planning software, planetary science, orbit debris and engineering simulation software for NASA and several contractors. He is currently with Jacobs Engineering on the ESCG contract. He is Chair of the AIAA Houston Section Astrodynamics technical committee and is an Associate Fellow of the AIAA. Dr. Jackson received his Ph.D. in physics at the University of Texas in 1975.

**JEFFERY C. MORRIS** has more than 7 years’ experience with modeling, simulation, and related technologies. Since 2006, he has worked for Odyssey Space Research, providing support to NASA JSC’s Automation, Robotics, and Simulation Division (Simulation and Graphics Branch) via participation in both the JEOD and IMSim development projects.
A Process for Comparing Dynamics of Distributed Space Systems Simulations

SISO Spring 2009 SIW: Paper# 059
Edwin Z. Crues, Albert A. Jackson and Jeffery C. Morris

Edwin Z. Crues
AR&SD ER7
NASA Johnson Space Center
March 25, 2009
Overview

• Introduction

• Objectives

• Simulation Configuration Items

• Test Cases

• Gravity Gradient Torque Example

• Conclusions

• Questions?
Introduction

- Simulation plays an important role in NASA’s engineering process.
  - Simulation is getting particular emphasis since the Columbia accident.
  - Verification, validation and the appropriate use of simulation was a particular finding in the Diaz report.

- Engineering simulation software validation requires rigorous and often varied testing methods.
  - Simulation to simulation data comparison.
  - Comparison to appropriate analytical solutions when available.
  - Comparison to measured empirical data such as fits to precession satellite orbit.

- Distributed simulations introduce unique challenges.
  - Differing dynamics engines
  - Differing integration rates
  - Differing environment models
  - Significant communication latencies between simulations
Objectives

• Provide a basis for comparing dynamics models that provide the planetary environment and 6-degree-of-freedom (6-DOF) orbital dynamics for distributed simulations.

• Principally compare “states”:
  – Time, position, velocity, acceleration, attitude, rotation rate and rotational acceleration.
  – Sometimes need derived states
    • Local Vertical Local Horizontal (LVLH)
    • Earth Centered Earth Fixed (ECEF)

• Also need to compare environmental parameters
  – Gravity
  – Atmosphere
  – Planetary Orientation
  – Sun and Moon Position
Simulation Configuration Items

- **Simulation**
  - Simulation Duration
  - Data Collection Rate
- **Vehicle**
  - Orbital State
  - Mass Properties
- **Environment Force and Torque Modeling**
  - Gravity Model: Degree and Order, Planetary Ephemeris, Sun/Moon Perturbations
  - Gravity Gradient Torques
  - Atmosphere Model: Solar Activity (F10.7, Geomagnetic Index)
  - Aerodynamic Drag Model: Coefficient of Drag, Cross-sectional Area
- **Dynamics**
  - Translational Dynamics: External Forces
  - Rotational Dynamics: External Torques
Test Cases

- Test Case 1: Earth Modeling Parameters
- Test Case 2: Earth Orientation and Keplerian Propagation
- Test Case 3: Gravity Modeling
  - Scenario 3A: 4x4 Gravity Model
  - Scenario 3B: 8x8 Gravity Model
- Test Case 4: Planetary Ephemeris
- Test Case 5: Atmospheric Modeling
  - Scenario 5A: Minimum Solar Activity
  - Scenario 5B: Mean Solar Activity
  - Scenario 5C: Maximum Solar Activity
- Test Case 6: External Force Effects
  - Scenario 6A: Atmospheric Drag with Constant Density
  - Scenario 6B: Atmospheric Drag with Dynamics Atmosphere
  - Scenario 6C: Plane Change Maneuver
  - Scenario 6D: Earth Departure Maneuver

Introduction

Objectives

Configuration

Test Cases

Example

Conclusions

Questions
Test Cases

Continued

- Test Case 7: Combined Translational Test
  - Scenario 7A: No Drag with 4x4 Gravity
  - Scenario 7B: No Drag with 8x8 Gravity
  - Scenario 7C: All Models with 4x4 Gravity
  - Scenario 7D: All Models with 8x8 Gravity

- Test Case 8: Torque-Free Rotation
  - Scenario 8A: Zero Initial Attitude Rate
  - Scenario 8B: Nonzero Initial Attitude Rate

- Test Case 9 Torque-Driven Rotation
  - Scenario 9A: Zero Initial Attitude Rate with Torque
  - Scenario 9B: Nonzero Initial Attitude Rate with Torque
  - Scenario 9C: Zero Initial Attitude Rate with Torque and Force
  - Scenario 9D: Nonzero Initial Attitude Rate with Torque and Force
Test Cases

Continued

- Test Case 10: Gravity Gradient Torque
  - Common states:
    - 5 deg In-Plane Displacement (Pitch)
    - 1 deg Out-of-Plane Displacement (Yaw)
  - Scenario 10A: Circular Orbit with Zero Initial Rate
    - Circular Orbit
    - Initial Rotation Rate: (0,0,0) LVLH
  - Scenario 10B: Circular Orbit with Nonzero Initial Rate
    - Circular Orbit
    - Initial Rotation Rate: 0.01 deg/s LVLH Pitch
  - Scenario 10C: Elliptic Orbit with Zero Initial Rate
    - Elliptic Orbit
    - Initial Rotation Rate: (0,0,0) LVLH
  - Scenario 10D: Elliptic Orbit with Nonzero Initial Rate
    - Elliptic Orbit
    - Initial Rotation Rate: 0.01 deg/s LVLH Pitch
- Full Test: Integrated 6-DOF Test
Gravity Gradient Torque Example

- Cannot cover all test cases here but need something to ground the discussion.
- Test Case 10: Gravity Gradient Torque
  - Has an accurate analytical approximation for circular orbits
  - Scenario 10A provides for a good numerical comparison
- Verifies the numerical integration routines and math models for the propagation of the gravitationally perturbed rotational state of an orbital vehicle.
- Shows how states compare closely to the analytic approximate solution.
- Not only validates gravity gradient modeling, but also the numerical integrator’s propagation of a Keplerian orbit and the rotational dynamics modeling.
Gravity Gradient Torque Example

Pitch Libration

Analytic Equations

Pitch Angle: \( \theta = \theta_0 \sin[\sqrt{3k_2} \omega t] \)

Inertia Ratio: \( k_2 = \frac{I_{yy} - I_{zz}}{I_{xx}} \)

Pitch Period: \( P_{pitch} = 2\pi / \sqrt{\omega \sqrt{3(1 - \frac{I_{zz}}{I_{xx}})}} \)
**Gravity Gradient Torque Example**

**Yaw Libration**

---

**Analytic Equations**

**Yaw Angle:**
\[ \psi = \psi_0 \sin[2\sqrt{k_1} \omega t] \]

**Inertia Ratio:**
\[ k_1 = \frac{I_{xx} - I_{zz}}{I_{yy}} \]

**Yaw Period:**
\[ P_{yaw} = 2\pi / \omega \sqrt{(1 - \frac{I_{zz}}{I_{xx}})} \]
Conclusion

- NASA relies heavily on simulations
  - Simulation has become even more important and more closely scrutinized since the Columbia accident.
  - NASA is now using distributed simulation in many activities.

- Distributed simulation has additional challenges
  - Simulation components often have different implementations.
  - Requires coordination between participating groups to ensure compatible fidelities.
  - Requires comparison between participating simulation dynamics and environment models.

- This paper presents a dynamics comparison process for space systems simulations to address these challenges
  - Specifies elements to be compared in dynamics tests.
  - Specifies accuracies required for success.
  - Provides an example for one particular application.
Questions?
## Introduction

- Need to define a sets of data parameters that represent the “state” of the simulation.
- These parameters will be used as a reference point for simulation comparisons.
- Include most important data items that are needed for meaningful comparison of dynamic test results.

## Objectives

## Configuration

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<td>Earth attitude matrices for computing ECEF and related parameters</td>
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## Example

## Conclusions

## Questions