RESEARCH MEMORANDUM

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AN ANALYTICAL INVESTIGATION OF THE HEAT LOSSES FROM A

U.S. NAVY K-TYPE AIRSHIP

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SUMMARY

The heat losses from the envelope surface of a U.S. Navy K-type airship are evaluated to determine if the use of heat is a feasible means of preventing ice and snow accumulations on lighter-than-air craft during flight and when moored uncovered. Consideration is given to heat losses in clear air (no liquid water present in the atmosphere) and in probable conditions of icing and snow.

The results of the analysis indicate that the amount of heat required in flight to raise the surface temperature of the entire envelope to the extent considered adequate for ice protection, based on experience with tests of heavier-than-air craft, is very large. Existing types of heating equipment which could be used to supply this quantity of heat would probably be too bulky and heavy to provide a practical flight installation.

The heat requirements to provide protection for the nose and stern regions in assumed mild to moderate icing conditions appear to be within the range of the capacity of current types of heating equipment suitable for flight use. The amount of heat necessary to
prevent snow accumulations on the upper surface of the airship
envelope when moored uncovered under all conditions appears to be
excessive for the heating equipment presently available for flight
use, but could possibly be achieved with auxiliary ground heating
equipment.

INTRODUCTION

This analysis is a part of a general research program being
conducted to evaluate the problems associated with the operation of
lighter-than-air craft in icing conditions, and to devise means for
the solution of those problems considered critical. One of the
chief difficulties encountered during operations in inclement
weather is the possible accumulation of ice or snow on the envelope
surface during flight, and when moored uncovered. A solution to the
danger of icing of heavier-than-air craft, which has met with consid-
erable experimental success, has been found in the use of heat to
keep the surfaces subject to icing at a temperature above freezing.
The purpose of this investigation is to determine, analytically,
to what extent heat can be used as a feasible method of ice preven-
tion for lighter-than-air craft. The analysis is concerned only
with the heat losses from the airship envelope and no consideration
is given to the design of a specific system. The possibility is
noted that for lighter-than-air craft the danger caused by the
added weight of an ice accumulation may be alleviated by heating
the gas contained in the envelope to provide additional lift and,
therefore, complete removal or prevention of ice or snow may not be
The present analysis first considers the heat losses from the envelope of a U.S. Navy K-type airship in clear air (no liquid water present) for a range of temperature rises of the surface above ambient-air temperature and a range of airspeeds which cover the flight and moored conditions. The heat losses in clear air are considered important because experience with heavier-than-air craft has shown that the amount of heat required to provide satisfactory protection against icing has produced a temperature rise of the heated surface above free-air temperature in clear air of from $75^\circ F$ to $100^\circ F$.

A method of determining the heat losses under icing conditions is then applied to the airship. The latter method provides a rational approach to the probable effect of the icing condition on the heat requirements, but has not been experimentally verified because of a lack of meteorological data associated with icing. For the present calculations, several icing conditions are assumed and the heat requirements are determined for the nose and the stern regions of the airship, which were found by flight test to be subject to the most severe accumulations of ice or snow. Consideration is also given to the heat losses associated with maintaining the envelope surface above freezing temperature when the airship is moored uncovered in snow conditions.
SYMBOLS

\( a \)  
one-half the length of the airship, feet

\( a_1 \)  
one-half the major axis of a prolate spheroid, feet

\( A \)  
surface area, square feet

\( A_s \)  
total surface area of the airship envelope, square feet

\( c_p \)  
specific heat of air, Btu per pound, °Fahrenheit

\( c_{p_w} \)  
specific heat of water, Btu per pound, °Fahrenheit

\( c_s \)  
specific heat of snow, Btu per pound, °Fahrenheit

\( d \)  
distance longitudinally from the maximum section of the airship to a station (plus aft and minus forward of maximum section), feet

\( d_1 \)  
distance longitudinally from the maximum section of the prolate spheroid, feet

\( e_s \)  
vapor pressure saturated air at temperature \( t_s \), millimeters of mercury

\( e_a \)  
vapor pressure saturated air at temperature \( t_a \), millimeters of mercury

\( F \)  
latent heat of fusion of ice, Btu per pound

\( h_L \)  
coefficient of heat transfer by convection between the airship surface and the atmosphere at a point in the laminar boundary layer, Btu per hour, square foot, °Fahrenheit

\( h_T \)  
coefficient of heat transfer by convection between the airship surface and the atmosphere at a point in the turbulent boundary layer, Btu per hour, square foot, °Fahrenheit
weighted coefficient of convective heat transfer for the entire airship surface, Btu per hour, square foot, °Fahrenheit

thermal conductivity of air, Btu per hour, square foot, °Fahrenheit per foot

over-all length of the airship measured along the longitudinal axis (2a), feet

latent heat of vaporization of water, Btu per pound

rate of catch of water per unit area, pounds per hour, square foot

ambient-air pressure, millimeters of mercury

rate of heat transfer, Btu per hour, square foot

rate of heat transfer by convection, Btu per hour, square foot

rate of heat transfer by evaporation, Btu per hour, square foot

rate of heat transfer required to melt snow, Btu per hour, square foot

rate of heat transfer in laminar boundary layer, Btu per hour, square foot

rate of heat transfer by radiation, Btu per hour, square foot

rate of heat transfer required to heat snow to melting point, Btu per hour, square foot

rate of heat transfer required to heat water to temperature $t_s$, Btu per hour, square foot

rate of heat transfer in turbulent boundary layer, Btu per hour, square foot

total quantity of heat lost from the surface of the airship by convection, Btu per hour

total quantity of heat lost from the surface of the airship by radiation, Btu per hour
r \quad \text{radial distance from the longitudinal axis to the outer surface of the envelope, feet}

r_1 \quad \text{radial distance from the longitudinal axis to the surface of the prolate spheroid, feet}

R_b \quad \text{boundary-layer Reynolds number } (V_b/V), \text{ dimensionless}

R_{b_c} \quad \text{critical boundary-layer Reynolds number } (V_{b_c}/V), \text{ dimensionless}

R_L \quad \text{Reynolds number based on airship length } (V_cL/V), \text{ dimensionless}

s \quad \text{distance along the surface of the airship from the forward stagnation point, feet}

t_a \quad \text{ambient-air temperature, } ^\circ\text{Fahrenheit}

T_a \quad \text{ambient-air temperature, } ^\circ\text{Rankine}

t_s \quad \text{average temperature of the envelope surface, } ^\circ\text{Fahrenheit}

T_s \quad \text{average temperature of the envelope surface, } ^\circ\text{Rankine}

T_{av} \quad \text{arithmetic average of the absolute temperatures of the air and envelope surface, } ^\circ\text{Rankine}

u \quad \text{local air velocity in the boundary layer, feet per second}

V \quad \text{local air velocity outside the boundary layer, feet per second}

V_0 \quad \text{free-stream air velocity, feet per second}

w \quad \text{weight density of air, pounds per square foot}

W \quad \text{rate of evaporation of water per unit area, pound per hour, square foot}

y \quad \text{distance perpendicular to surface of the airship, feet}

x \quad \text{distance from the nose of the airship measured along the longitudinal axis, feet}
\( \delta_L \) distance from the surface to the point in the laminar boundary layer where the velocity head is one-half its local value outside the boundary layer \((u = 0.707V)\), feet

\( \delta_c \) the value of \( \delta_L \) at the point of transition between laminar and turbulent flow, feet

\( \delta_T \) distance from the surface to the point in the turbulent boundary layer where the velocity head is one-half its local value outside the boundary layer \((u = 0.707V)\), feet

\( \Delta_T \) heat-transfer characteristic length for a turbulent boundary layer

\( \epsilon \) emissivity factor for the airship envelope surface, dimensionless

\( \theta \) boundary-layer momentum thickness

\( \nu \) kinematic viscosity of air, square foot per second

\( \sigma \) Stefan-Boltzmann radiation constant \(1.73 \times 10^{-9}\), Btu per \((hr)\), \((sq \ ft)\), \((^\circ R)^4\)

\( \zeta \) turbulent boundary-layer parameter

ANALYSIS

In order to determine the heat losses from the airship in clear air or icing conditions, it is necessary to evaluate the heat-transfer coefficients which pertain to the transfer of heat from the airship surface to the atmosphere by convection. For the determination of the rate of heat transfer from a streamlined body due to convection the method developed in reference 1 is used for the region of laminar boundary layer. This method is based on theories of
previous investigators for incompressible flow along a flat plate maintained at a constant temperature. The method presented in reference 1 is complex when applied to the turbulent region, and therefore is presented for one airspeed only. An approximate method having satisfactory agreement with that of reference 1 has been employed to complete the calculations.

K-Type Airship Velocity Boundary Layer Characteristics

In order to apply the heat-transfer equations presented herein it is necessary first to determine the characteristics of the velocity boundary layer over the surface of the airship.

Velocity distribution over the airship.—The profile of the airship is represented by the following equation from reference 2:

\[ r = R \left[ 1 - \left( \frac{d}{a} - \frac{1}{b} \right)^{3/2} \right] \left( 1.0206 - 0.2126 \frac{d}{a} \right) \]  

(1)

where

\[ R = 28.9275 \text{ feet} \]
\[ a = \text{length of envelope } \div 2 = 123 \text{ feet} \]
\[ d = \text{distance longitudinally from the maximum section to station} \]

(plus aft and minus forward of origin; max. section is at 40 percent of airship length), feet

Table I contains the profile offsets calculated from equation (1).

Since no data are available on the velocity distribution over the K-type airship, the profile of the forward portion of the airship is approximated by a prolate spheroid:
\[ d_1 = a_1 \left( 1 - \frac{r_1^2}{R^2} \right)^{1/2} \]  

(2)

where

\( a_1 = 90 \text{ feet} \)

\( R = 28.9275 \text{ feet} \)

The spheroid profile offsets thus determined appear in table II and are almost identical with those of the airship back to 90 feet.

Reference 3 shows that a theoretical calculation of the velocity distribution by the following equation for a nonviscous, incompressible fluid over the forward portion of quadrics such as prolate spheroids is in good agreement with experimental data.

\[ V = V_0 \left( 1 + k_a \right) \sin \theta \]  

(3)

where

\[ k_a = \frac{\log \frac{1 + e}{1 - e} - 2e}{\log \frac{1 + e - 2e}{1 - e} \frac{1}{1 - e^2}} \]

\[ c = \frac{1}{a_1} \sqrt{a_1^2 - R^2} \]

\[ \sin^2 \theta = \frac{a_1^2 r_1^2}{R^4 - c^2 r_1^2} \]

\[ c^2 = a_1^2 - R^2 \]

The velocity distribution as calculated by equation (3) appears in table II. Since the profiles of the spheroid and the airship are very nearly identical from the nose back to 90 feet, the velocity
distribution thus calculated is assumed to apply in this region of 
the airship and is listed for the corresponding profile offsets of 
the airship in table I.

The experimental data presented in reference 3 for spheroids 
and in reference 4 for the C-7 airship in flight show that the 
velocity distribution is nearly uniform between about the 35-percent 
point and the 85-percent point of the airship length. Between the 
85-percent point and the aft end of the airship the velocity drops 
to about 60 percent of free stream velocity, and tests of airship 
models (reference 5) show that while the boundary layer thickens 
rapidly turbulent separation does not occur. It appears justified 
to assume that the velocity distribution over the K-type airship 
follows the same trend, and the values obtained thereby are presented 
in table I for the entire airship.

**Laminar boundary-layer thickness**.— Millikan in reference 6 
assumes power-series expressions for the boundary-layer profiles 
in laminar and turbulent flow and substitutes into the Kármán 
"integral relations," which are essentially first integrals of the 
Prandtl boundary-layer equations for a figure of revolution. The 
expressions which are derived apply to incompressible flow in which 
turbulent separation does not occur.

The equation for the thickness of the laminar boundary layer 
thus derived appears in reference 1 in the following form:

\[
\delta_L = \sqrt{\frac{2.3}{R_L} \left( \frac{L}{V} \right)^2 \left( \frac{V}{V_0} \right)^{0.17}} \int_0^{s/L} \left( \frac{R}{L} \right)^2 \left( \frac{V}{V_0} \right)^{0.17} \, \text{d} \left( \frac{R}{L} \right)
\]
Using this formula the thickness of the laminar boundary layer is calculated for airspeeds of 10 and 50 miles per hour and is plotted in figure 1. The calculations of boundary-layer thickness, convective heat-transfer coefficients, and heat losses in clear air are made for standard sea-level atmospheric pressure and atmospheric temperature of 32°F.

**Location of point of transition between laminar and turbulent boundary layers.**—The position at which the laminar boundary layer becomes unstable and turbulent flow sets in is defined as the transition point. The critical Reynolds number $R_{\theta c}$ is based on the thickness of the laminar boundary layer at this point.

The magnitude of $R_{\theta c}$ is dependent on the level of turbulence in the air stream and the characteristics of the airship nose. Experiments with flat plates and airship models as summarized in reference 6 indicate that the most forward point at which transition will occur corresponds to a value of $R_{\theta c}$ of about 1200.

The nature of the bow mooring assembly on the K-type airship is such as to induce early transition and it is assumed that the magnitude of $R_{\theta c}$ will coincide with the lower limit, 1200. The choice of this value yields heat-transfer data which are conservative, since the heat lost in a region of laminar flow is less than that lost in one of turbulent flow.

**Turbulent boundary-layer thickness.**—Model tests reported in reference 7 have shown that the formula

$$u = V \left( \frac{V}{\delta} \right)^{\alpha}$$

(5)
used in reference 6 is quite accurate in representing the profile of the turbulent boundary layer of the airship back to about 85 percent of the length. Aft of 85 percent the boundary layer thickens and changes profile rapidly in the presence of an adverse pressure gradient and the equation no longer applies.

The thickness of the turbulent boundary layer is represented by the following expression from reference 6:

\[ \delta_T = \frac{R_6 V}{V} \]  

where

\[ R_6 = (R_L)^{4/5} 0.0327 \left( \frac{V}{V_c} \right)^{16/7} L \int_{s_c/L}^{s/L} \left( \frac{V}{V_c} \right)^{27/7} \left( \frac{r}{L} \right)^{5/4} d \left( \frac{s}{L} \right) \]

\[ + 71.8 R_L^{1/4} \left( \frac{S_c}{L} \right)^{5/4} \left( \frac{V_c}{V_c} \right)^{115/28} \left( \frac{r_0}{L} \right)^{5/4} \] \[ \times \left( \frac{r_0}{L} \right)^{4/5} \]

Figure 1 presents the turbulent boundary-layer thickness of the airship at 50 miles per hour and for transition at \( R_{6c} = 1200 \), obtained from the preceding equation.

In order to express the turbulent boundary-layer thickness in the form required by reference 1 two new relationships are introduced:

1. According to reference 8, \( \delta_T \) is related to the momentum thickness \( \theta \) in a turbulent boundary layer of the form of equation (5) by

\[ \delta_T = 0.907 \theta \]  

where \( \theta \) is defined as

\[ \theta = \int_0^\delta \frac{u}{V} (1 - u/V) dy \]
and $\delta$ is the distance from the surface to the point where $u = V$. The initial value of $\theta$ at the transition point is determined from the relationship $\theta = 0.2898L$ for the laminar profile.

2. The following relationship was derived in reference 9 from von Kármán's formula for the skin friction experienced by a flat plate with a fully developed turbulent boundary layer:

$$ \zeta = 2.557 \log_4 \left( \frac{0.075 \frac{V_o}{V}}{1} \right) $$

Coefficients of Convective Heat Transfer

From the Airship Surface

Heat-transfer coefficients in the laminar region.—Reference 1 develops the following equation for the rate of heat transfer in the laminar boundary layer:

$$ q_L = 0.700 \frac{k}{h_L} \left( t_s - t_a \right) \text{ Btu/hr, ft}^2 $$

where the heat-transfer coefficient is

$$ h_L = 0.70 \frac{k}{\delta_L} \text{ Btu/hr, ft}^2, ^\circ \text{F} $$

and $\delta_L$ is expressed by equation (4).

The values of $h_L$ are calculated for several airspeeds and plotted in figure 2 back to the transition point where $R_{\delta_c} = 1200$.

Heat-transfer coefficients in the turbulent region by method of reference 1.—The heat-transfer equations presented in reference 1 for the turbulent region are similar to those presented in the previous section:
The characteristic heat-transfer length $\Delta_T$ in reference 1 is related to the velocity boundary-layer expression by the relationship

$$\Delta_T = \frac{s^2 L}{R_L \left( \frac{V}{V_o} \right)}$$

(13)

The heat-transfer coefficients thus determined are plotted for an airspeed of 50 miles per hour in figure 3.

Heat-transfer coefficients in the turbulent region by method of reference 10.— Because of the complexities involved in the calculation of the heat-transfer coefficients in the turbulent region by the method of reference 1, an alternative method was sought. In reference 10 a method is developed for calculating heat transfer along a flat plate from the equations of Colburn in reference 11.

The expression for the turbulent region is:

$$h_T = 0.52 \left( \frac{T_{av}}{T_a} \right)^{0.296} \left( \frac{V_w}{s} \right)^{0.50}$$

(14a)

If the temperature difference between the airship surface and the atmosphere is less than $100^\circ F$ the error introduced by replacing $T_{av}$ by $T_a$ is less than 3 percent and the expression becomes

$$h_T = 0.52 \left( T_a \right)^{0.296} \left( \frac{V_w}{s} \right)^{0.50}$$

(14b)
The agreement between the methods of reference 1 and reference 10 is shown in figure 3 for an airspeed of 50 miles per hour. It is assumed that the agreement is equally satisfactory at other airspeeds and so equation (14b) was used in calculating heat-transfer coefficients in the turbulent region of the curves in figure 2 for a range of airspeeds between 10 and 65 miles per hour.

Heat Loss In Clear Air

Experience with heavier-than-air craft has shown that satisfactory ice protection was obtained by heating the wing leading edge back to 10 percent chord to a temperature rise of approximately 100°F in clear air and allowing the heated air to circulate through the aft portion of the wing which heats the wing surface in this region to a rise of from about 30°F to 10°F. It is not known if this heat distribution would apply directly to an airship, but it is believed that heating the entire surface would ensure protection. Therefore, the heat requirements for raising the entire envelope surface temperature above the atmospheric temperature are calculated for various airspeeds and temperature rises.

An indication of the regions of the airship envelope which are susceptible to icing was obtained during a flight conducted in icing and snow conditions with a K-type Navy airship. The results of this test are reported in reference 12 and show that the nose region extending aft for a distance of about 30 feet, and the upper surface of the airship stern accumulated the greatest amount of ice and snow. These test results indicate that heating of the nose and
stern regions only may provide satisfactory ice protection. Therefore, after the heat requirements to maintain the entire envelope at various temperatures are presented, the computation of the heat loss from the nose and stern regions only is given.

**Total heat loss from the entire envelope.**—The total heat loss from the airship is made up of the heat losses by convection and by radiation. To determine the convective heat loss, an average heat-transfer coefficient for the entire surface was calculated. For this computation the surface of the envelope was divided into 25 segments. The heat-transfer coefficient at the center of each segment was weighted by the ratio of the area of that segment to the total surface area and the average taken of the 25 values thus obtained. The resultant average convective heat-transfer coefficient is plotted against indicated airspeed in figure 4 and, for indicated airspeeds above about 7 miles per hour, may be expressed as

$$h_{av} = 0.263 V^{0.8} \text{ Btu/hr ft}^2 \text{ °F}$$

(15)

where $V$ is in miles per hour. Below an indicated airspeed of 7 miles per hour the curve is fairied to an approximate value of the heat-transfer coefficient for free convection at zero airspeed calculated from equations given in reference 13 for flat plates in horizontal and vertical positions. For the airship calculation an average value of the factors expressing the heat-transfer coefficient for the different plate positions was used. The heat-transfer coefficient for free convection is a direct function of the temperature difference between the airship surface and the atmosphere;
therefore, values for the various temperature differences are included in figure 4.

For the calculations of the heat lost from the airship envelope by convection, the entire surface is assumed to be at a uniform temperature. Employing the average coefficient from figure 4 the heat loss is represented by:

$$Q_c = h_{av} A_s (t_s - t_a), \text{ Btu/hr}$$  \hspace{1cm} (16)

and is plotted for a range of values of \((t_s - t_a)\) between 1°F and 100°F in figure 5. The lower portion of figure 5 is expanded in figure 6.

Because of the large surface area of the envelope (36,000 sq ft) radiation losses, even for small temperature differences, are significant. According to the Stefan-Boltzmann's equation (reference 13)

$$Q_R = \sigma e A_s (T_s^4 - T_a^4), \text{ Btu/hr}$$  \hspace{1cm} (17)

For an aluminum surface \(\epsilon = 0.5\).

The subject of heat transfer by radiation is complicated, particularly in the case where solar radiation is present. However, on overcast days it may be assumed that the cloud base acts as a black body at the temperature of the air at that altitude, and that the ground acts as a black body at the temperature of the air at the ground level. Further, in the case where the ground is covered with snow and the cloud base is at freezing temperatures it may be assumed that the entire area to which heat is radiating from the airship is at 32°F. It is for this idealized case that the radiation losses appearing in figures 5 and 6 are calculated over a range of
temperature differences from $1^\circ$ to $100^\circ$ F.

These calculations are presented to indicate the order of magnitude of radiation effects which are present for the specific conditions assumed. During other conditions, particularly where solar radiation is present, other assumptions are required and further concepts of radiant heat transfer introduced which are beyond the scope of the present analysis.

Total heat loss from nose and stern regions.—The nose region considered to require heating extends aft for 30 feet. The stern region requiring protection was estimated to be the upper half extending aft for a distance of 20 feet from the section located 225 feet from the nose of the airship. The heat lost from the nose and stern regions by convection and radiation is computed for surface temperature rises of $100^\circ$ F and $50^\circ$ F at a free-stream air velocity of 50 miles per hour.

The total heat loss from the nose region by convection is obtained by dividing the heated region into nine segments and taking the average heat-transfer coefficient for each segment from figure 2. The summation of the products of the heat-transfer coefficients, segment areas, and the surface temperature rise gives the total heat loss by convection from the nose region. The convective heat loss from the stern region is calculated in the same manner, but using a value of the convective heat-transfer coefficient of 5 Btu per hour, square foot, °F. This value was obtained by extrapolating the curve of figure 2 for 50 miles per hour because it is difficult to calculate the heat-transfer coefficients for this region of the
airship due to uncertainties in the type of flow existing over the stern portion.

The heat loss by radiation from the nose and stern regions is computed by equation (17), using the heated surface area of these regions and assuming that the region to which heat is radiating is at $32^\circ F$, as was done in the calculation for the radiation losses from the entire airship envelope. The results of the calculations to determine the total heat loss from the nose and stern regions are tabulated below:

<table>
<thead>
<tr>
<th>Temperature rise, $^\circ F$</th>
<th>Heat loss, Btu/hr.</th>
<th>Convection</th>
<th>Radiation</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>From nose</td>
<td>From stern</td>
<td>From nose</td>
<td>From stern</td>
</tr>
<tr>
<td>50</td>
<td>1,274,900</td>
<td>149,600</td>
<td>79,300</td>
<td>14,500</td>
</tr>
<tr>
<td>100</td>
<td>2,562,900</td>
<td>299,200</td>
<td>182,600</td>
<td>33,400</td>
</tr>
</tbody>
</table>

Heat Loss in Wet Air (Icing Conditions) or Snow

A method of calculating the heat losses from the envelope surface during flight when liquid water is present in the atmosphere at temperatures below freezing is presented. Consideration is then given to the heat requirements for preventing accumulations of snow during flight and when the airship is moored uncovered on the ground.

Icing conditions during flight. For the calculation of the heat required to provide protection from icing during flight, the method given in reference 14 is used. The regions which are considered to require heating in order to provide satisfactory protection for the
airship are the nose and stern areas, which were found to be susceptible to icing in the test reported in reference 12 and are the areas used in the last part of the clear-air analysis.

The method of calculation given in reference 14 considers the total heat loss to be made up of the heat required to bring the temperature of the water caught on the surface up to the surface temperature, the heat loss by evaporation of the water from the wetted surface, and the heat transferred from the surface to the atmosphere by convection. In this analysis the heat loss by radiation is added because of the large surface area involved. The amount of heat lost to the liquid water caught on the surface will depend upon the rate of catch of water. A method for calculating the rate of catch of water on cylinders or spheres, which includes the determination of the region of catch and the distribution of water over this region, is presented in reference 15. The various factors involved in ascertaining the rate of water catch are water drop size, concentration of liquid water in the atmosphere, wind velocity, and the geometry of the body concerned. The application of the method given in reference 15 to the airship indicated that, at a velocity of 50 miles per hour and considering the nose of the airship as a sphere with a radius of 11 feet, water will not be caught if the drop diameter is less than about 80 microns (0.00315 inch). The limited amount of data obtained to date indicates that in icing conditions the diameter of the water drops varies from 6 to 50 microns. However, the flight test reported in reference 12 showed that the nose of the airship accumulated ice and snow. It is,
therefore, evident that the idealized conditions set up to calculate the rate of catch, and the available information on the meteorological conditions existing in icing, cannot be applied to the airship. This may be attributed in part to the probability that the bow mooring attachment on the airship causes disturbances in the flow over the nose which departs from the ideal conditions assumed in the calculations. A smooth fairing over the bow of the airship might improve the flow conditions to the extent that very little, if any, ice would accumulate on the nose. Also, it is possible that the water drops encountered during the flight test of the airship were considerably larger than reported to date because the information on meteorological conditions which is presently available is admittedly limited in scope.

For the present calculations, therefore, the region of catch at the nose was determined from the results of the flight test of reference 12 to extend back 7\(\frac{1}{2}\) feet along the longitudinal axis from the nose of the airship. In order to determine the rate of catch of water it is assumed that the surface over the nose and extending back 30 feet along the longitudinal axis is wetted. This region is assumed to require heating and the total rate of catch is taken to be equal to the total rate of evaporation from the wetted surface.

In order to calculate the rate of heat loss per unit area of the airship surface, the following equations are used:

1. The rate of heat loss to the liquid water required to raise the water from atmospheric to heated surface temperature is given by

\[
q_w = m c_p w (t_s - t_a)
\]  

(18)
In this equation the temperature of the water before it is heated is assumed to be the same as that of the atmosphere because the magnitude of kinetic heating is very small. The variation of the rate of catch over the region of catch was approximated by the method of reference 15 and is a maximum at the nose of the airship and zero at the point 7\(\frac{1}{2}\) feet back from the nose.

2. The rate of heat loss by evaporation of water from the wetted surface is computed from

\[ q_e = WI_w \]  

where \( W \), the rate of evaporation, is obtained from

\[ W = 0.622 \frac{h_c}{c_p} \left( \frac{e_s - e_a}{p} \right) \]  

The coefficient of heat transfer by convection \( h_c \) has been previously calculated for the clear-air analysis, and for this calculation the values are taken from figure 2.

3. The rate of heat transfer from the heated surface of the airship to the atmosphere by convection is given by

\[ q_c = h_c (t_s - t_a) \]  

where \( h_c \), the convective heat-transfer coefficient, is obtained from figure 2. The temperature rise of the surface due to kinetic heating is neglected in equation (21) because it is only 0.2\(^\circ\) F to 0.3\(^\circ\) F. Information covering the calculation of the kinetic temperature rise in wet air (icing conditions) is presented in references 16 and 17.
4. The heat loss by radiation is calculated by the same method used in the clear-air analysis, which utilizes the following equation:

\[ q_r = \sigma \epsilon (T_s^4 - T_a^4) \]  

(22)

5. For the region of catch of liquid water the total rate of heat transfer per unit area is the sum of the various heat losses computed by equations (18) through (22), or

\[ q = q_w + q_e + q_c + q_r \]  

(23)

For the wetted area aft of the region of catch the loss due to heating the liquid water \( q_w \) is not included in the total values. The calculation of the heat loss from the airship surface is based on the following conditions:

- Free-stream air velocity \( V_o \), 50 miles per hour
- Pressure altitude, 1000 feet
- Ambient-air temperature \( t_a \), 30° F and 10° F

For the calculations of the heat losses from the nose region, the surface is assumed to be heated to 40° F. The four components of the total rate of heat transfer per unit area are calculated for several stations on the longitudinal axis from the nose back to the 30-foot point. The calculated rates of heat transfer are plotted in figure 7 to show their variation over the heated nose region of the airship. The variation of the total rate of heat transfer per unit area, obtained by summing the values shown in figure 7, is presented in figure 8. The variation of the total rate of heat transfer per unit area obtained for the clear-air condition for surface
temperature rises of 50°F and 100°F is included in figure 8 for comparison with the wet-air values. The values of the rate of heat transfer at \( x/L = 0 \), shown on figure 8, have been estimated because the nose of the airship is obstructed by the mooring attachment which makes it impossible to evaluate the heat losses in that region.

An indication of the total rate of heat loss from the nose region of the airship was obtained in the same manner as was used in the clear-air analysis by dividing the heated region into nine segments and obtaining the average rate of heat transfer per unit area for each segment from figure 8. The summation of the products of these average heat-transfer rates and segment surface areas is considered as the total rate of heat loss from the nose region. This procedure gives values for the total rate of heat loss of about 441,000 Btu per hour for \( t_o \) of 30°F, and 1,165,000 Btu per hour for \( t_o \) of 10°F.

The heat losses from the upper surface of the airship envelope at the stern are calculated in the same manner as outlined for the nose region and for the same conditions. The stern region requiring protection was taken to be the same as used in the clear-air analysis, extending aft for a distance of 20 feet from the section located 225 feet from the nose of the airship. The area of catch was estimated to extend between these longitudinal stations and to cover the upper one-third of the stern region. The entire upper surface of the stern is considered to be wetted and, therefore, requires heating. For the calculation of the heat losses due to
evaporation and convection an average value of the convective heat-
transfer coefficient \( h_c \) of 5 Btu per hour, square foot, \( ^\circ F \)
is used. This value of the heat-transfer coefficient is the same as
used in the clear-air analysis and, as explained in that part of the
report, was obtained by extrapolating the curve of figure 2 for 50
miles per hour. The heat losses calculated for the stern region are
tabulated below:

<table>
<thead>
<tr>
<th>Ambient-air temperature ( t_a (^\circ F) )</th>
<th>Heat water</th>
<th>Evaporation</th>
<th>Convection</th>
<th>Radiation</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>220</td>
<td>23,700</td>
<td>29,900</td>
<td>2,600</td>
<td>56,420</td>
</tr>
<tr>
<td>10</td>
<td>1,430</td>
<td>51,800</td>
<td>89,700</td>
<td>7,100</td>
<td>150,030</td>
</tr>
</tbody>
</table>

The total heat losses from the nose and stern regions are 497,420 Btu
per hour for 30\(^\circ\) F atmospheric temperature and 1,315,030 Btu per hour
for 10\(^\circ\) F atmospheric temperature.

Snow during flight.—The results of the flight test reported in
reference 12 indicated that the only serious accumulations of snow
during flight at cruising speed of about 50 miles per hour occurred
at the stern of the airship. Therefore, only the stern region,
previously defined for the icing calculations, is considered to
require heating. For the snow condition the heat losses from the area
of catch are considered to be made up of the heat required to raise
the temperature of the snow to its melting point of 32\(^\circ\) F, the heat
required to melt the snow, the heat necessary to bring the melted
snow up to the surface temperature, and the heat losses due to
evaporation, convection and radiation which are the same as calcu-
lated for the icing condition. The heat losses from the wetted
region are made up of evaporation, convection, and radiation only.

The rate of heat loss per unit area of the surface to the snow and water is given by the following equations:

1. The heat required to raise the temperature of the snow to the melting point is obtained from

\[ q_s = mc_s (32 - t_a) \]  \hspace{1cm} (24)

where \( c_s \), the specific heat of snow, is taken as 0.5 Btu per pound, °F.

2. The heat required to melt the snow is given by

\[ q_f = mF \]  \hspace{1cm} (25)

where \( F \), the heat of fusion, is 144 Btu per pound.

3. The heat required to raise the temperature of the water (melted snow) up to the surface temperature is computed by

\[ q_w = mc_{pw} (t_s - 32) \]  \hspace{1cm} (26)

The total rate of heat transfer from the stern region is obtained by multiplying the preceding rates of heat transfer by the area of catch and adding the total to the evaporation, convection, and radiation losses.

The conditions for which the heat losses were calculated are the same as assumed for the icing computations, with the exception that instead of determining the rate of catch from the rate of evaporation, rates of catch were assumed for the two atmospheric temperatures. At the atmospheric temperature of 30° F the rate of
catch was considered to be 0.624 pound per hour per square foot of horizontal surface. This value is based upon the average rate of precipitation encountered during the flight test reported in reference 12, which was about 1.2 inches of snow per hour, and is considered to be representative of moderate snow conditions. For the calculation of the heat losses at an atmospheric temperature of 10°F, the rate of precipitation was taken as one-half the value for 30°F, or 0.312 pound per hour per square foot of horizontal surface. The values of the heat losses calculated for the snow condition are as follows:

<table>
<thead>
<tr>
<th>Ambient air temperature (°F)</th>
<th>Rate of heat loss from the airship stern (area = 600 ft²), Btu/hr.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Heat snow</td>
</tr>
<tr>
<td>30</td>
<td>200</td>
</tr>
<tr>
<td>10</td>
<td>1,140</td>
</tr>
</tbody>
</table>

Snow on moored airship.— In order to determine the heat requirements when the airship is moored uncovered in snow conditions, the entire upper half of the envelope was considered to be heated. The area of catch was defined in the same manner previously used for the stern region and the same method of calculation is used. The rates of precipitation are taken to be the same as for the preceding computation, but the other conditions are as follows:

Free-stream air velocity \( V_0 \) zero mile per hour

Pressure altitude, standard sea level pressure, 760 millimeters of mercury
Ambient-air temperature \( t_a \), 30°F and 10°F

Since the free-stream air velocity is assumed to be zero, a new value of the convective heat-transfer coefficient is calculated by the following equation:

\[
h_c = 0.34 \left( t_s - t_a \right)^{0.25}
\]

(27)

which is based on information given in reference 13 for heat transfer by natural or free convection from flat plates in horizontal and vertical positions. The values of the heat losses calculated are tabulated below:

<table>
<thead>
<tr>
<th>Ambient-air temperature ( t_a ) (°F)</th>
<th>Rate of heat loss from upper surface of airship (area = 12,000 ft²), Btu/hr.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Heat snow</td>
</tr>
<tr>
<td>30</td>
<td>6,200</td>
</tr>
<tr>
<td>10</td>
<td>34,300</td>
</tr>
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</table>

**DISCUSSION**

The heat requirements to provide an appreciable temperature rise of the entire surface of the airship envelope are shown by figures 5 and 6 to be extremely high, and the existing types of heating equipment of sufficient capacity to supply this quantity of heat probably would be too bulky and heavy to make a practical installation for flight use. An indication of the magnitude of the calculated heat losses may be obtained by comparing them with the heat available in the engine exhaust gas, which has been successfully employed as a
source of heat for ice-prevention systems installed on heavier-than-air craft. For the K-type airship, which has two engines rated at about 450 horsepower each, the total amount of heat available in the exhaust gas at maximum power is approximately 2,000,000 Btu per hour for a gas temperature drop of 1200°F. Actually, a maximum of about 50 percent of this value could be recovered by heat exchangers for use in an ice-prevention system. By reference to figure 6 it is apparent that this amount of heat would provide a surface temperature rise of about 25°F at zero airspeed, and of about 4°F at a cruising speed of 50 miles per hour.

The heat requirements of the nose and stern regions calculated for clear-air conditions to give surface temperature rises of 50°F and 100°F are high. The calculations for the wet-air conditions indicate that the heat requirements of the nose and stern regions for the icing condition assumed in the computations for an atmospheric temperature of 30°F could be met with existing types of heating equipment. The validity of assuming that satisfactory protection would be afforded by heating only the nose and stern regions would have to be ascertained by flight tests.

A comparison of the rates of heat transfer in clear air and wet air shown in figure 8 for the nose region indicates that the amount of heat required to produce a surface temperature rise of 50°F in clear air is about the same as that required to maintain the surface temperature at 40°F in wet air at 10°F. These results show that, if a system were designed to provide a 50°F surface temperature rise in clear air, the limiting icing condition in which the system would
afford protection would be that represented by the wet-air calculations for an atmospheric temperature of 10° F.

The amount of heat calculated to be necessary to prevent snow accumulations on the airship stern during flight appears to be within the range of capacity of existing types of heating equipment suitable for flight use. Flight tests would be required, however, to determine if supplying heat only to the upper surface of the stern region would provide satisfactory protection to allow the airship to be operated safely in snow conditions.

The heat requirements to prevent accumulations of snow on the upper surface of the airship when moored uncovered could probably be supplied with heating equipment suitable for flight use for the snow conditions assumed for the calculations at an ambient-air temperature of 30° F. Under more severe conditions auxiliary ground equipment could be utilized to provide additional heat. In the calculations of the heat requirements for the moored airship, and for the stern of the airship during flight, heat losses from only the upper surfaces were considered. Actually, a portion of the lower surface may require heating to prevent the freezing of water which may run onto the lower surface from the heated upper surface. Tests would be necessary to determine the severity of ice accumulations which might form in this manner.

CONCLUDING REMARKS

The results of this analysis indicate that the amount of heat required in flight to raise the surface temperature of the entire
envelope to the extent considered adequate for ice protection probably cannot be met by any present type of heating equipment suitable for flight use. This conclusion is based upon an airplane wing leading-edge heating requirement of a surface temperature rise of 75°F to 100°F in clear air which has proved adequate in flight tests in natural icing conditions. The use of heat to protect the nose and stern regions of the airship from ice or snow formation during flight appears to be feasible for mild to moderate icing and snow conditions. However, flight tests will be necessary to determine if heating only these regions will provide satisfactory protection for the airship. The heat required to prevent the accumulation of snow on the upper surface of the airship when moored uncovered under all conditions appears to be greater than could be supplied by existing heating equipment adaptable for flight, but could probably be met by the use of auxiliary ground equipment.

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Wesley H. Hillendahl,
Aeronautical Engineer.

Ralph E. George,
Mechanical Engineer.

Approved:

[Signature]
John F. Parsons,
Aeronautical Engineer.
REFERENCES


### Table I. Profile Ordinates and Velocity Distribution for U.S. Navy K-Type Airship

<table>
<thead>
<tr>
<th>$x$ (ft)</th>
<th>$r/L$</th>
<th>$x/L$</th>
<th>$V/V_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0183</td>
<td>0.00407</td>
<td>0.505</td>
</tr>
<tr>
<td>2.5</td>
<td>0.02765</td>
<td>0.0102</td>
<td>0.702</td>
</tr>
<tr>
<td>5</td>
<td>0.0386</td>
<td>0.0203</td>
<td>0.845</td>
</tr>
<tr>
<td>10</td>
<td>0.0550</td>
<td>0.0407</td>
<td>0.980</td>
</tr>
<tr>
<td>15</td>
<td>0.0659</td>
<td>0.061</td>
<td>1.045</td>
</tr>
<tr>
<td>25</td>
<td>0.0822</td>
<td>0.102</td>
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<tr>
<td>40</td>
<td>0.0985</td>
<td>0.163</td>
<td>1.135</td>
</tr>
<tr>
<td>60</td>
<td>0.1107</td>
<td>0.244</td>
<td>1.155</td>
</tr>
<tr>
<td>80</td>
<td>0.1166</td>
<td>0.3255</td>
<td>1.160</td>
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<tr>
<td>90</td>
<td>0.1176</td>
<td>0.366</td>
<td>1.160</td>
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<tr>
<td>105</td>
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<td>140</td>
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<td>170</td>
<td>0.0976</td>
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<td>190</td>
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</tr>
<tr>
<td>220</td>
<td>0.061</td>
<td>0.895</td>
<td>0.860</td>
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<tr>
<td>230</td>
<td>0.0468</td>
<td>0.935</td>
<td>0.710</td>
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<tr>
<td>240</td>
<td>0.244</td>
<td>0.976</td>
<td>0.60</td>
</tr>
<tr>
<td>246</td>
<td>0</td>
<td>1.0</td>
<td>0.58</td>
</tr>
</tbody>
</table>

### Table II. Profile Ordinates and Velocity Distribution of Forward Half of Prolate Spheroid

<table>
<thead>
<tr>
<th>$x$ (ft)</th>
<th>$r/L$</th>
<th>$x/L$</th>
<th>$V/V_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0181</td>
<td>0.00407</td>
<td>0.505</td>
</tr>
<tr>
<td>2.5</td>
<td>0.0278</td>
<td>0.0102</td>
<td>0.702</td>
</tr>
<tr>
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<td>0.0381</td>
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<td>0.845</td>
</tr>
<tr>
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<td>0.0537</td>
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<td>0.980</td>
</tr>
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<td>0.0652</td>
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<td>1.045</td>
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<tr>
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<td>1.105</td>
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<td>40</td>
<td>0.0976</td>
<td>0.163</td>
<td>1.135</td>
</tr>
<tr>
<td>60</td>
<td>0.1108</td>
<td>0.244</td>
<td>1.155</td>
</tr>
<tr>
<td>80</td>
<td>0.1165</td>
<td>0.3255</td>
<td>1.160</td>
</tr>
<tr>
<td>90</td>
<td>0.1174</td>
<td>0.366</td>
<td>1.160</td>
</tr>
</tbody>
</table>

**Note:**

- $L =$ length of airship, 246 feet
- $x =$ longitudinal distance from nose of airship, feet
- $r =$ radial distance from longitudinal axis to envelope surface, feet
FIGURE LEGENDS

Figure 1. — Profiles of the laminar and turbulent boundary layers for the K-type airship.

Figure 2. — Variation over length of K-type airship of convective heat-transfer coefficients for several airspeeds (where $R_{0c} = 1200$).

Figure 3. — Comparison of convective heat transfer coefficients obtained by equations given in reference 1 and reference 10 for the turbulent region of the airship at 50 miles per hour.

Figure 4. — Variation of the over-all average external convective heat-transfer coefficient with indicated airspeed for K-type airship.

Figure 5. — Variation with airspeed of total heat loss from K-type airship at several values of temperature difference between ambient air and airship surface, $\Delta t$; clear air conditions (no liquid water present).

Figure 6. — Lower region of figure 5 expanded.

Figure 7. — Rate of heat transfer from surface of U. S. Navy K-type airship due to losses to liquid water caught on the surface, evaporation of water from the surface, convection, and radiation, when the surface temperature is maintained at $40^\circ F$, for ambient-air temperatures of $30^\circ F$ and $10^\circ F$, during flight at 1000 feet pressure altitude at 50 miles per hour.

Figure 8. — Rate of heat transfer from surface of U. S. Navy K-type airship to maintain the surface temperature at $100^\circ F$ and $50^\circ F$ above ambient-air temperature in clear air, and to maintain the surface at $40^\circ F$ when liquid is present in the atmosphere at temperatures of $30^\circ F$ and $10^\circ F$, during flight at 1000 feet pressure altitude at 50 miles per hour.
Figure 1 - Profiles of the laminar and turbulent boundary layers for the K-type airship.
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Figure 6 - Lower region of figure 5 expanded.
Figure 7. - Rate of heat transfer from surface of U.S. Navy K-type airship due to losses to liquid water caught on the surface, evaporation of water from the surface, convection, and radiation, when the surface temperature is maintained at 40°F. For ambient-air temperatures of 50°F to 10°F, during flight at 1000-feet pressure altitude at 50 miles per hour.
Figure 8. Rate of heat transfer from surface of U.S. Navy K-type airship to maintain the surface temperature at 100°F and 50°F above ambient-air temperature in clear air, and to maintain the surface at 40°F when liquid water is present in the atmosphere at temperatures of 30°F and -10°F, during flight at 1000 feet pressure altitude at 60 miles per hour.