

Expanding the Bandwidth of Slow and Fast Pulse Propagation in Coupled Micro-resonators

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Abstract—Coupled resonators exhibit coherence effects which can be exploited for the delay or advancement of pulses with minimal distortion. The bandwidth and normalized pulse delay are simultaneously enhanced by proper choice of the inter-resonator couplings.

I. INTRODUCTION

The strongly modified dispersion associated with whispering-gallery-mode resonances in coherently coupled micro-resonators can be used to slow or advance pulses of light with minimal loss and pulse distortion, via effects such as coupled-resonator induced transparency [1] and gain-assisted superluminality [2], analogous to effects in atomic systems [3, 4]. These structures are therefore promising for applications such as all-optical buffers and delay lines [5-8], differential sensing and laser gyroscopy [9-11], high fidelity image processing [12], and optical computing schemes [13], but tend to suffer due to their small bandwidth and normalized pulse time delay. Recently attention has focused on expanding the useable bandwidth of slow and fast light materials [7, 14, 15]. In this paper, we demonstrate that proper choice of the couplings between resonators can simultaneously increase bandwidth and pulse delay, resulting in devices that can better accommodate the high bit content and bandwidth demands of modern communications systems.

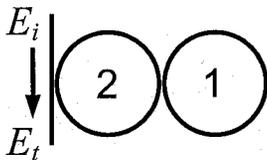


Fig. 1. Two coupled ring-resonators

II. RESPONSE FUNCTION FOR TWO RESONATORS

The response of two coupled-resonators, one of which is coupled to an excitation waveguide as shown in Fig. 1, can be readily solved by iterative or matrix techniques [16-18]. The complex electric-field transmission

(E_t/E_i) across such a structure is given by the Airy expression

$$\tilde{\tau}_2(\phi_1, \phi_2) = \frac{r_2 - a_2 \tilde{\tau}_1 e^{i\phi_2}}{1 - r_2 a_2 \tilde{\tau}_1 e^{i\phi_2}} = \tau_2 \exp[i\phi_2^{(\text{eff})}], \quad (1)$$

where

$$\tilde{\tau}_1(\phi_1) = \frac{r_1 - a_1 e^{i\phi_1}}{1 - r_1 a_1 e^{i\phi_1}} = \tau_1 \exp[i\phi_1^{(\text{eff})}] \quad (2)$$

is the complex transmittivity through the first resonator, $\phi_j = 2\pi n_j L_j / \lambda_0$ are the single-pass phase-shifts, r_j are the coupler reflection coefficients, $a_j = \exp(-\alpha_j L_j / 2)$ are the single-pass attenuation coefficients, n_j are the refractive indices, α_j are the loss coefficients, L_j are the circumferences of the resonators, and $j=1,2$ specifies the first (furthest from the excitation waveguide) or second (closest to the waveguide) resonator. We will assume the resonators have identical optical path lengths (they are co-resonant) so that we can drop the subscript from the single-pass phase shifts. The transmittance is given by $T_2(\phi) = \tau_2^2$.

If the complex transmittivity $\tilde{\tau}$ is expressed as a phasor, i.e., $\tilde{\tau} = \tau \cos \phi^{(\text{eff})} \hat{e}' + \tau \sin \phi^{(\text{eff})} \hat{e}''$, then the response can be plotted on an Argand diagram. It can be shown that extremes in transmission occur when

$$\frac{d\tau}{d\phi} = \frac{1}{\tau} \left[\tilde{\tau} \cdot \frac{d\tilde{\tau}}{d\phi} \right] = 0, \quad (3)$$

i.e., when $\tilde{\tau} \perp d\tilde{\tau}/d\phi$, while dispersion reversals occur when

$$\frac{d\phi^{(\text{eff})}}{d\phi} = \frac{1}{\tau^2} \left| \tilde{\tau} \times \frac{d\tilde{\tau}}{d\phi} \right| = 0 \text{ or } \infty, \quad (4)$$

i.e., when $\tilde{\tau} \parallel d\tilde{\tau}/d\phi$, or when the structure is critically-coupled to the excitation waveguide, i.e., $T=0$. The derivative $d\phi^{(\text{eff})}/d\phi$ (the phase time) thus determines the amount of pulse advancement or delay that occurs. The difference in transmittance at $\phi=0$ and at the extremes of Eq. 3, $\phi = \phi_{\text{sp}}$, determines whether induced

transparency or induced absorption occurs on-resonance. The transition between the two effects occurs at $\Delta T = T_2(0) - T_2(\phi_{sp}) = 0$, which corresponds to

$$r_1 = \frac{2r_2(1-a_1^2a_2^2)}{2a_1r_2(1-a_2^2) + a_2(1-a_1^2)(1+r_2^2)}. \quad (5)$$

At this value of the inter-resonator coupling, neither induced transparency nor induced absorption occur, and instead the transmission spectrum is flattened as shown in Fig. 2. Moreover, for two resonators there are two loops, rather than one, in the Argand diagram, and the inner loop forms an almost perfect circle, indicative of the flattened spectrum. One can readily verify that $\tau_2(0) = \tau_2(\phi_{sp})$ from this graph because $\phi^{(eff)}(\phi_{sp}) \approx \pi$ for this particular case. The broadening of the spectrum is also accompanied by an increase in the time delay, as evidenced by the increased slope $d\phi^{(eff)}/d\phi$ in the case of the coupled resonators.

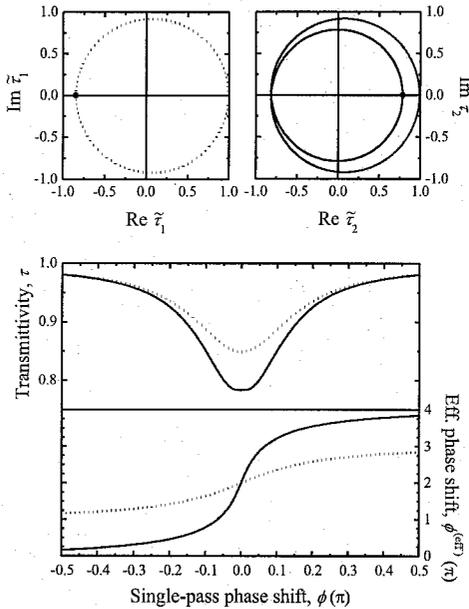


Fig. 2. Spectral response for a single resonator (dashed lines), and two coupled resonators (solid lines). Top: transmission Argand diagrams. The plots are parametric, starting on-resonance ($\phi = 0$), as indicated by the dot, and proceeding counterclockwise around the loop over a range of 2π . Bottom: spectra of the transmission and phase.

Finally, we note that this same principal can be used to broaden and flatten the spectrum when the resonators incorporate an amplifying medium. In this case, the dispersion is anomalous and a pulse advancement rather than delay occurs.

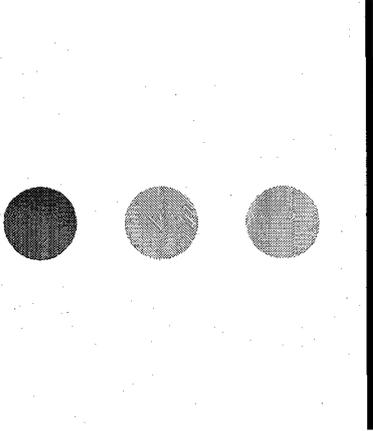
III. CONCLUSION

The response of two coupled resonators exhibits coherence phenomena such as induced transparency and absorption and gain-assisted superluminality in analogy with multilevel atomic systems, which can be exploited

for the delay or advancement of pulses in optical systems with minimal pulse distortion. Proper choice of the inter-resonator coupling can simultaneously increase the bandwidth and the normalized pulse delay (or advancement) which is crucial to increasing the storage capacity of these devices for the resolution of data packet contention issues in communications systems.

REFERENCES

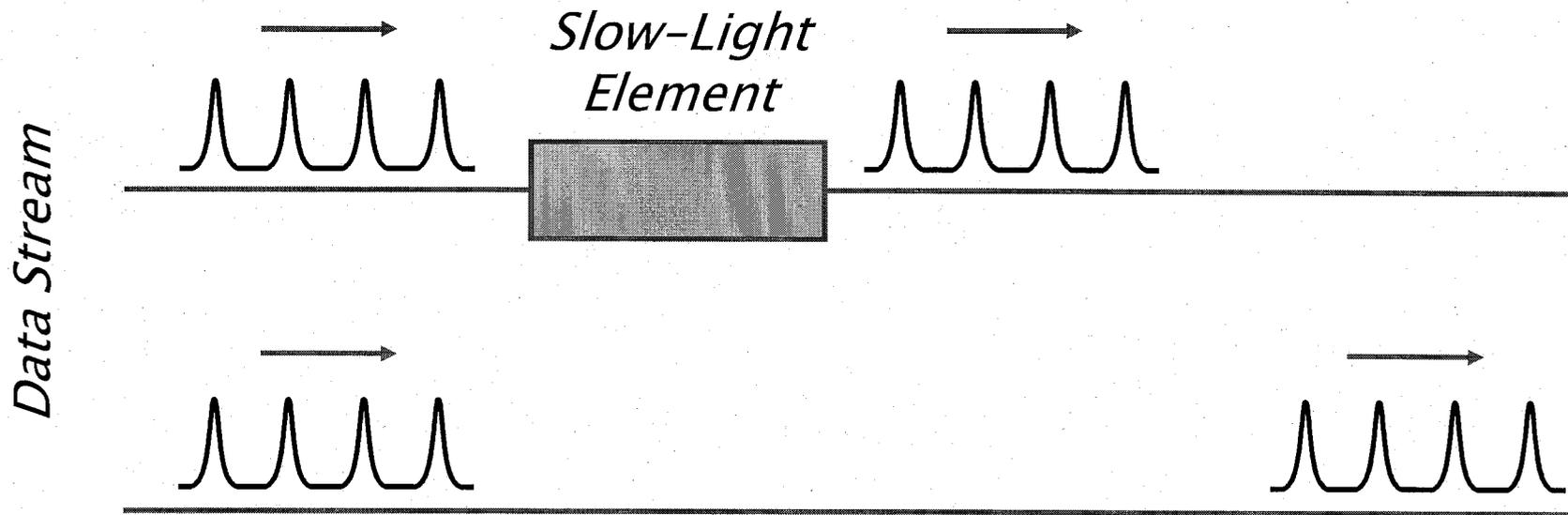
- [1] D. D. Smith, H. Chang, K. A. Fuller, A. T. Rosenberger, and R. W. Boyd, "Coupled-resonator-induced transparency," *Phys. Rev. A* 69, 63804 (2004).
- [2] H. Chang and D. D. Smith, "Gain-assisted superluminal propagation in coupled optical resonators," *J. Opt. Soc. Am. B* 22, 2237 (2005).
- [3] S. E. Harris, J. E. Field, and A. Imamoglu, "Nonlinear optical processes using electromagnetically induced transparency," *Phys. Rev. Lett.* 64, 1107, (1990).
- [4] L. J. Wang, A. Kuzmich, and A. Dogariu, "Gain-assisted superluminal light propagation," *Nature* 406, 277-279 (2000).
- [5] L. Maleki, A. B. Matsko, A. A. Savchenkov, and V. S. Ilchenko, "Tunable delay line with interacting whispering-gallery-mode resonators," *Opt. Lett.* 29, 626-628 (2004).
- [6] J. K. S. Poon, J. Scheuer, Y. Xu, and A. Yariv, "Designing coupled-resonator optical waveguide delay lines," *J. Opt. Soc. Am. B* 21, 1665-1673 (2004).
- [7] M. D. Stenner, M.A. Neifeld, Z. Zhu, A. M. C. Dawes, D. J. Gauthier, "Distortion management in slow light pulse delay," *Opt. Exp.* 13, 9995 (2005).
- [8] M. F. Yanik and S. Fan, "Stopping light all optically," *Phys. Rev. Lett.* 92, 083901(1-4) (2004).
- [9] R. J. C. Spreeuw, R. Centeno, N. J. van Druen, E. R. Eliel, and J. P. Woerdman, "Mode coupling in a He-Ne ring laser with backscattering," *Phys. Rev. A* 42, 4315-4324 (1990).
- [10] S. A. Diddams, J. C. Diels, and B. Atherton, "Differential intracavity phase spectroscopy and its application to a three-level system in samarium," *Phys. Rev. A* 58, 2252-2264 (1998).
- [11] R. Quintero-Torres, M. Navarro, M. Ackerman, and J. C. Diels, "Scatterometer using a bidirectional ring laser," *Opt. Comm.* 241, 179-183 (2004).
- [12] R. M. Camacho, C. J. Broadbent, I. Ali-Khan, and J. C. Howell, "All-optical delay of images using slow light," *Phys. Rev. Lett.*, *Phys. Rev. Lett.* 98, 043902 (2007)
- [13] T. Opatrný and D. G. Welsch, "Coupled cavities for enhancing the cross-phase-modulation in electromagnetically induced transparency," *Phys. Rev. A* 64, 23805 (2001).
- [14] E. Shumakher, N. Orbach, A. Nevet, D. Dahan, and G. Eisenstein, "On the balance between delay, bandwidth and signal distortion in slow light systems based on stimulated Brillouin scattering in optical fibers" *Opt. Exp.* 14, 5877-5884 (2007)
- [15] M. G. Herráez, K. Y. Song, and J. Thévenaz, "Arbitrary-bandwidth Brillouin slow light in optical fibers," *Opt. Exp.* 14, 1395 (2006)
- [16] J. Capmany and M. A. Muriel, "A new transfer matrix formalism for the analysis of fiber ring resonators: compound coupled structures for FDMA demultiplexing," *J. Lightwave Technol.* 8, 1904-1919 (1990).
- [17] D. D. Smith, H. Chang, and K. A. Fuller, "Whispering-gallery mode splitting in coupled microresonators," *J. Opt. Soc. Am. B*, 20, 1967-1974 (2003).
- [18] J. K. S. Poon, J. Scheuer, S. Mookherjee, G. T. Paloczi, Y. Huang, and A. Yariv, "Matrix analysis of microring coupled-resonator optical waveguides," *Opt. Exp.* 12, 90-103 (2004).



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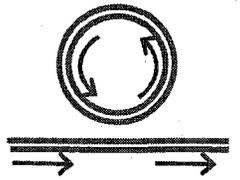
Slow-Light Optical Buffer (SLOB)



- Resolution of data packet contention via slow (or fast) light buffer
- Requires broadband controllable delay (or advancement) of at least 1 packet length, i.e., thousands of pulse widths, with minimal pulse distortion and attenuation.
- What sort of element can be used? No current material meets need.

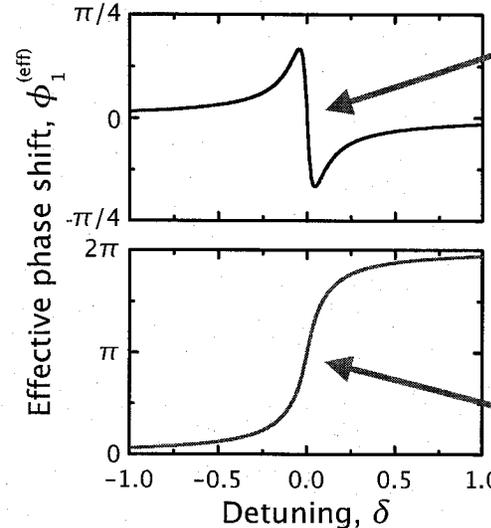
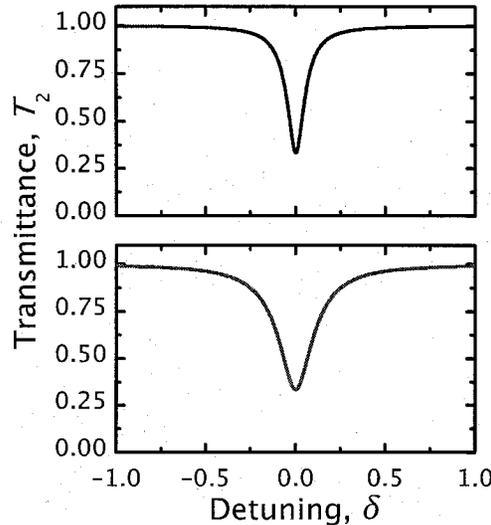


Optical Micro-Resonators



Under-Coupled

Over-Coupled



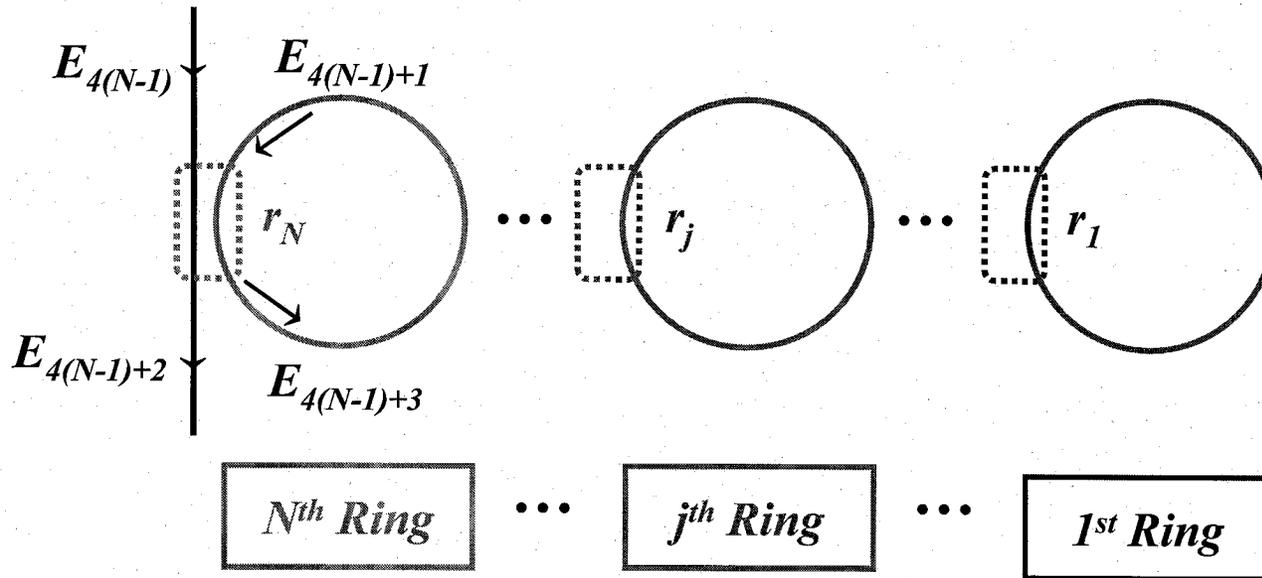
Fast light

$$\frac{d\phi_1^{(\text{eff})}}{d\delta} : \text{Group delay}$$

Slow light

- *Single optical resonators can delay or advance pulses of light, albeit with some absorption.*
- *Advantage: Periodic resonances from tunable all-solid-state structures. Not restricted to atomic absorption lines.*
- *Problems: Narrow bandwidth. Delay limited to about one pulse width when critically-coupled. Advancement less than delay.*
- *Question: How to increase the bandwidth w/o sacrificing delay?*

Multiple Coupled Resonators



Coupling

$$\begin{pmatrix} E_{4(j-1)+2} \\ E_{4(j-1)+3} \end{pmatrix} = \begin{pmatrix} r_j & it_j \\ it_j & r_j \end{pmatrix} \begin{pmatrix} E_{4(j-1)+1} \\ E_{4(j-1)+2} \end{pmatrix}$$

Feedback

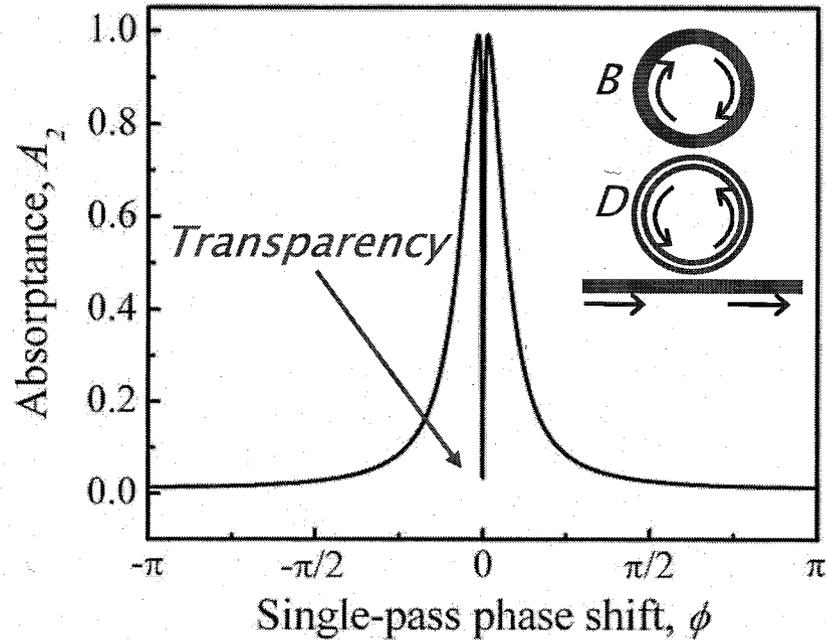
$$E_{4(j-1)+1} = a_j e^{i\phi_j} E_{4(j-1)+3}$$

+

$i = e^{i\pi/2}$

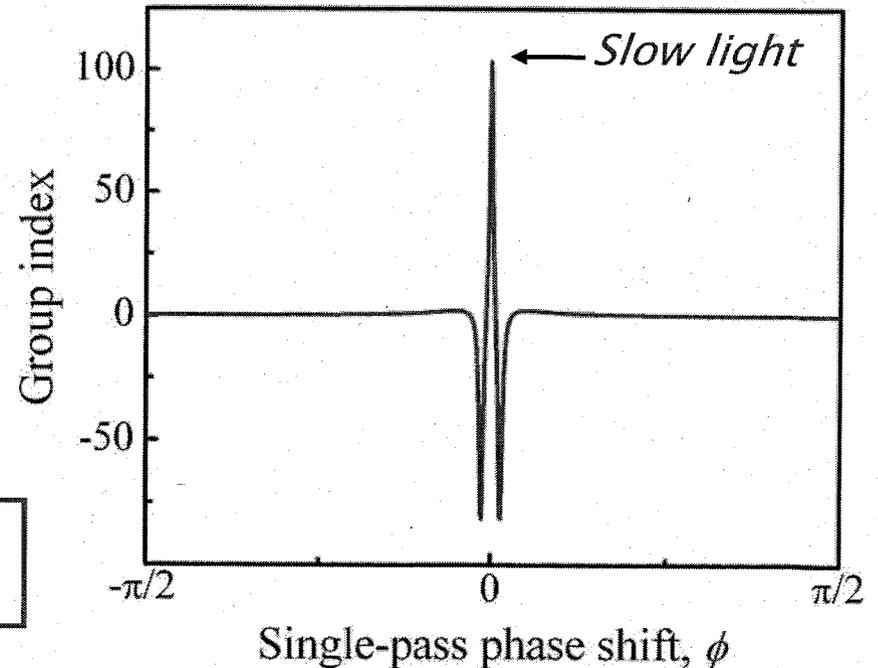
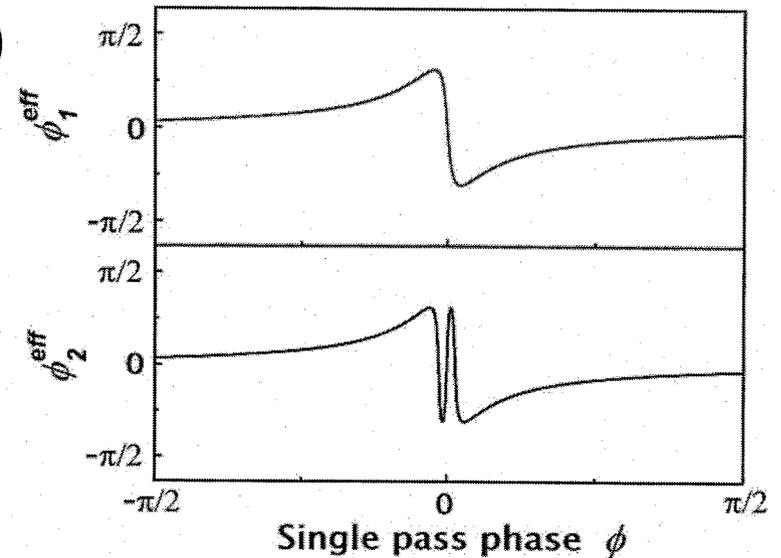
π phase shift results after 2 passes across coupler!

Coupled-Resonator-Induced Transparency (CRIT)



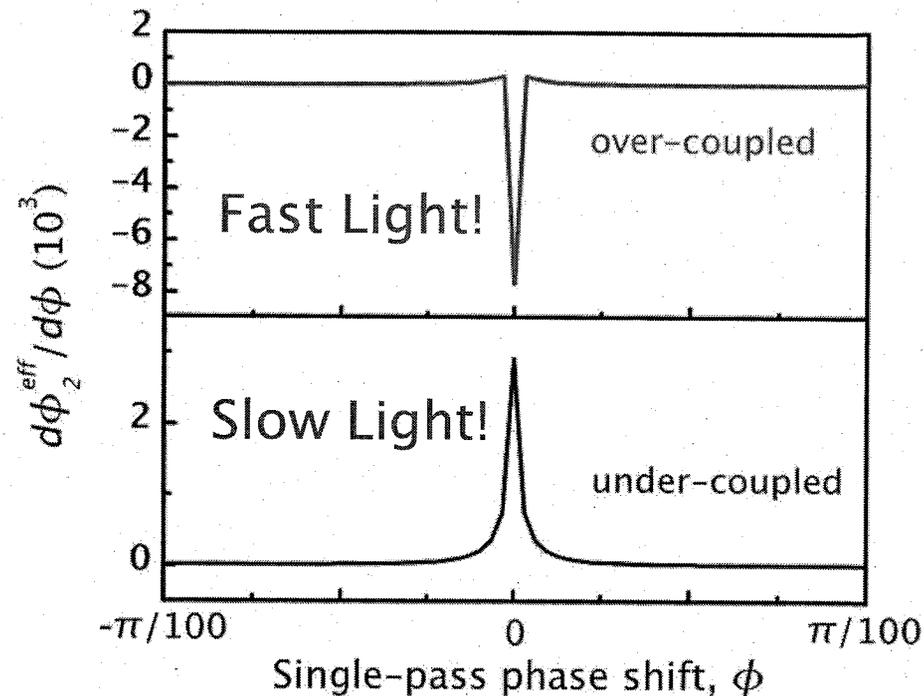
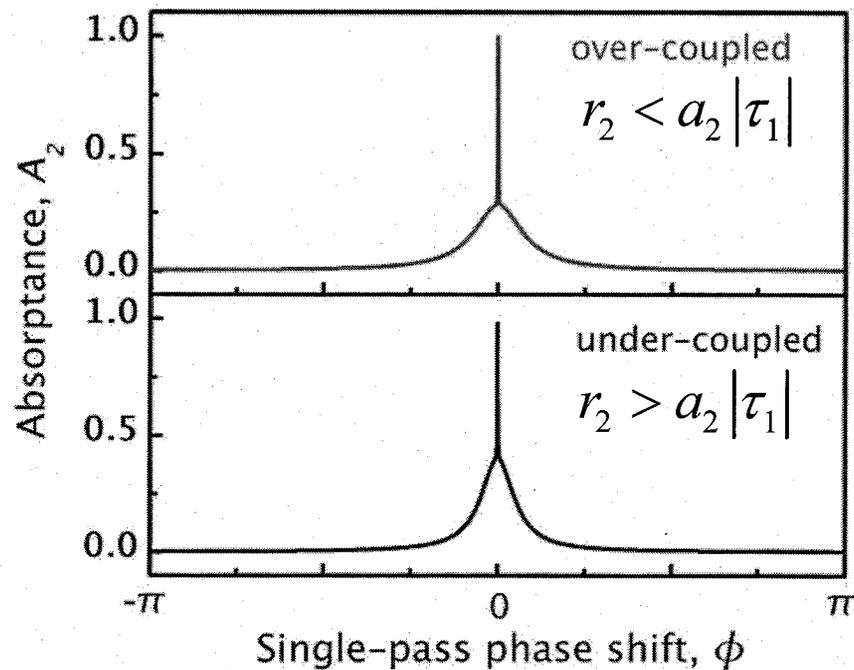
- ◆ Like EIT, but Photon Trapping!
- ◆ Slow light with no absorption!

D.D. Smith, H. Chang, K. Fuller, A.T. Rosenberger, R.W. Boyd, *PRA* **69**, 63804 (2004).



Coupled-Resonator-Induced Absorption (CRIA)

D.D. Smith, H. Chang, *JMO* 51, 2503 (2004).

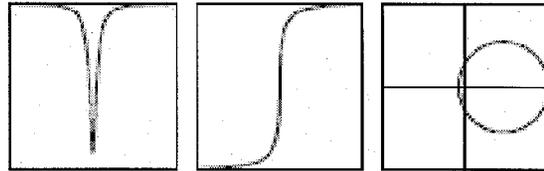


- ◆ Like EIA, but can be fast or slow depending on coupling.
- ◆ Considerable absorption
- ◆ Requires waveguide-coupled resonator to be strongly over-coupled $r_2 < a_1 a_2$

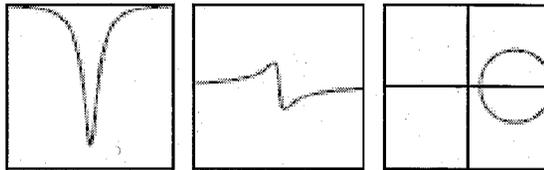
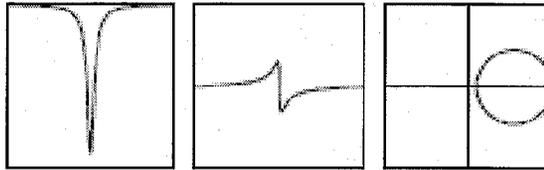


Routes to CRIT

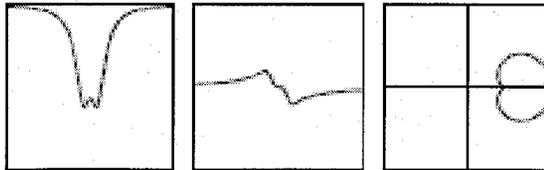
Weakly over-coupled single:
 $r_2 > a_1 a_2$



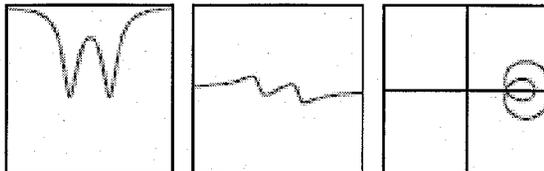
Under-coupled single resonator



Anomalous CRIT



CRIT

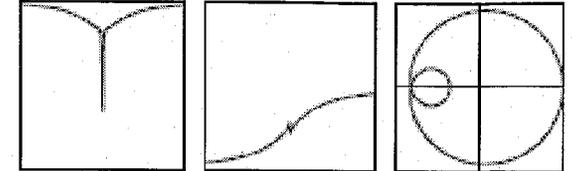


➤ CRIA not possible

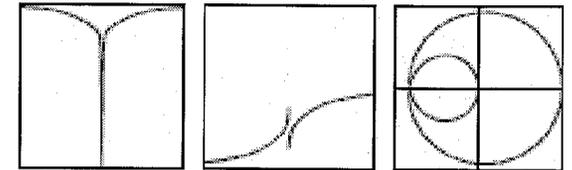
Strongly over-coupled single:
 $r_2 < a_1 a_2$



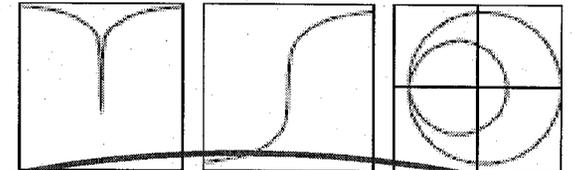
Under-coupled CRIA



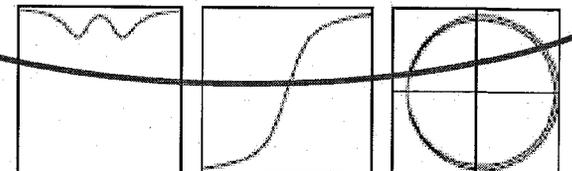
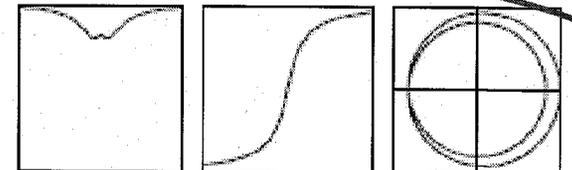
Critically-coupled CRIA



Over-coupled CRIA



CRIT



➤ Anom. CRIT not possible

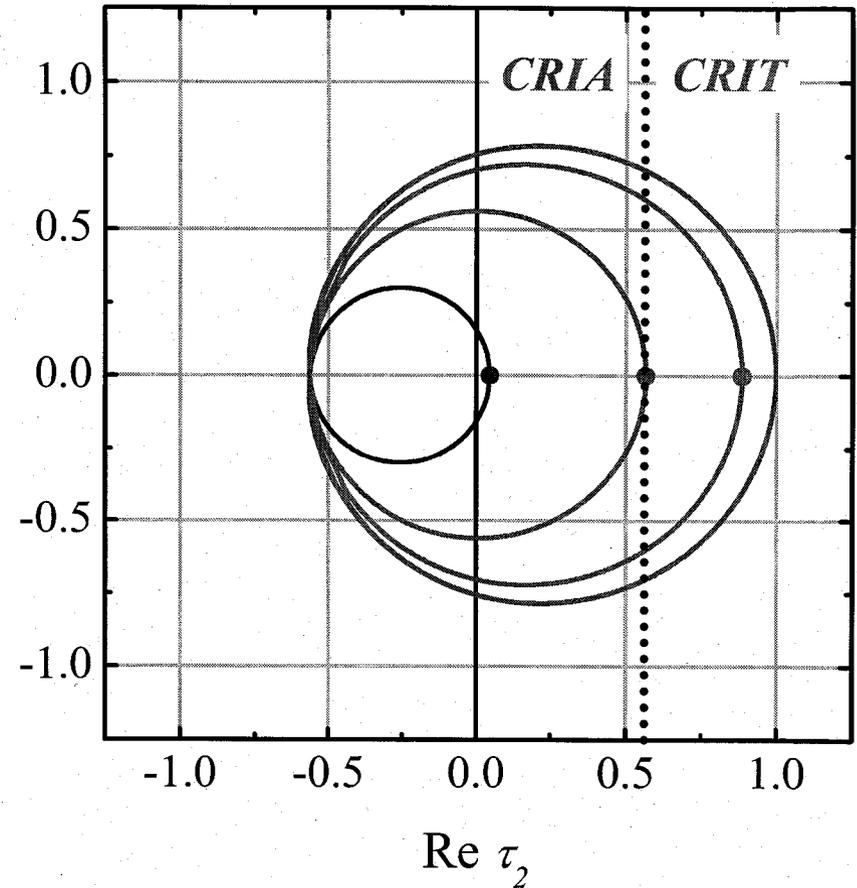
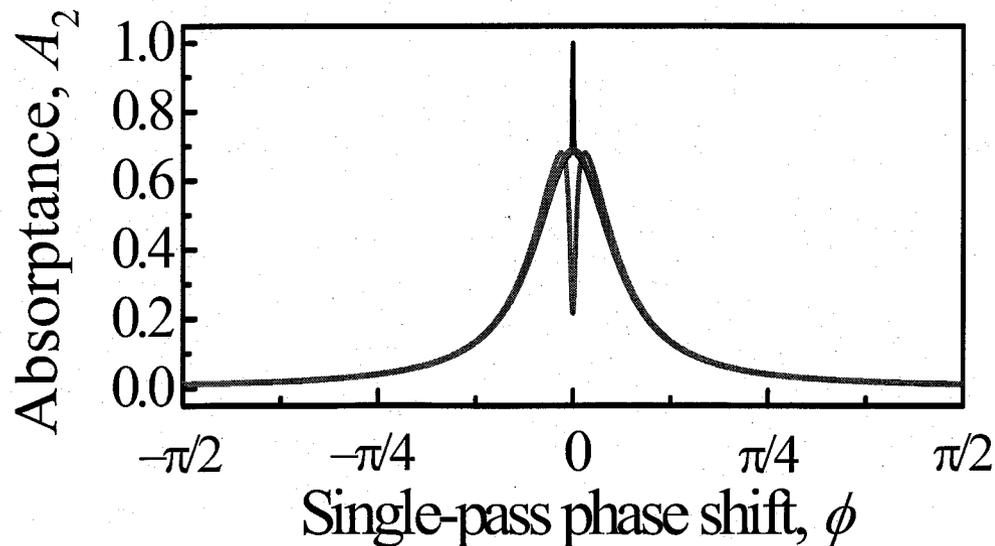
Coupling



CRIT / CRIA Transition

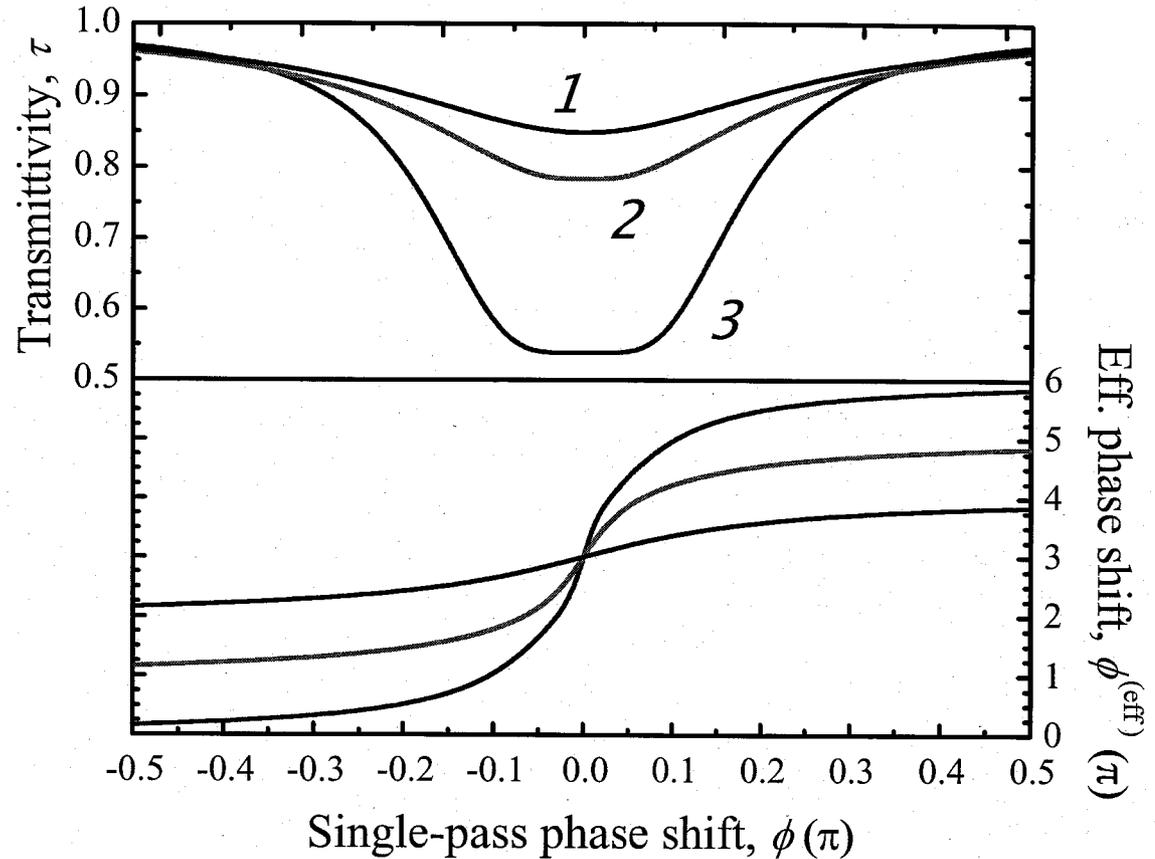
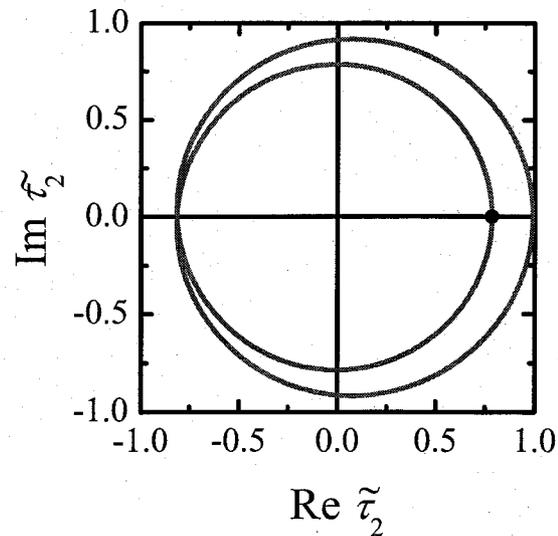
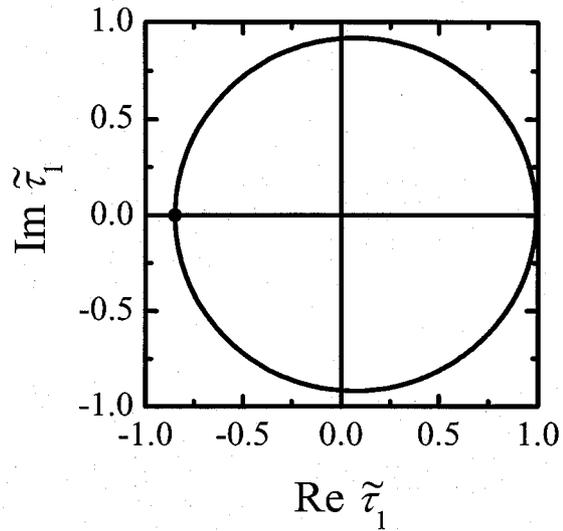
- Resonator #2 strongly over-coupled for CRIA to occur
- CRIT / CRIA transition at

$$T_2(0) - T_2(\phi_{sp}) = 0$$





Expanding the Slow Light Bandwidth



➤ *Bandwidth AND delay both increase!*

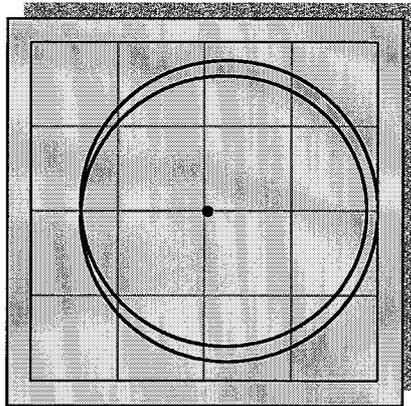
➤ *FSR-Limited to THz modulations*



Pulse Compression / Expansion

$$\beta = \frac{d}{d\delta} \left[\vec{\tau} \cdot \frac{d\vec{\tau}}{d\delta} \right] \Big|_{\delta=0} = 0 \Rightarrow \text{Perfect Concentric Circle} = \text{No Pulse Expansion or Compression over a Broad-band}$$

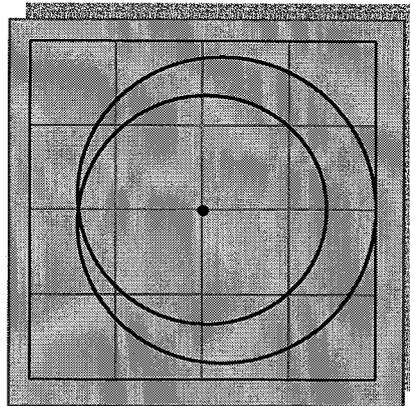
CRIT



Expansion

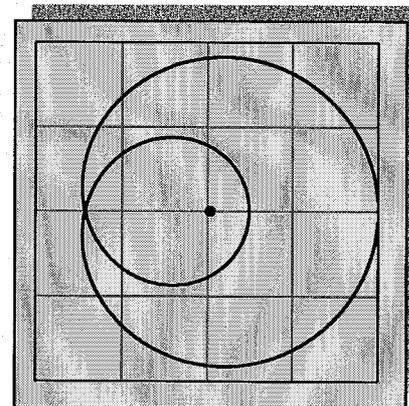
$$\beta < 0$$

CRIT / CRIA



$$\beta \approx 0$$

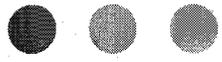
CRIA



Compression

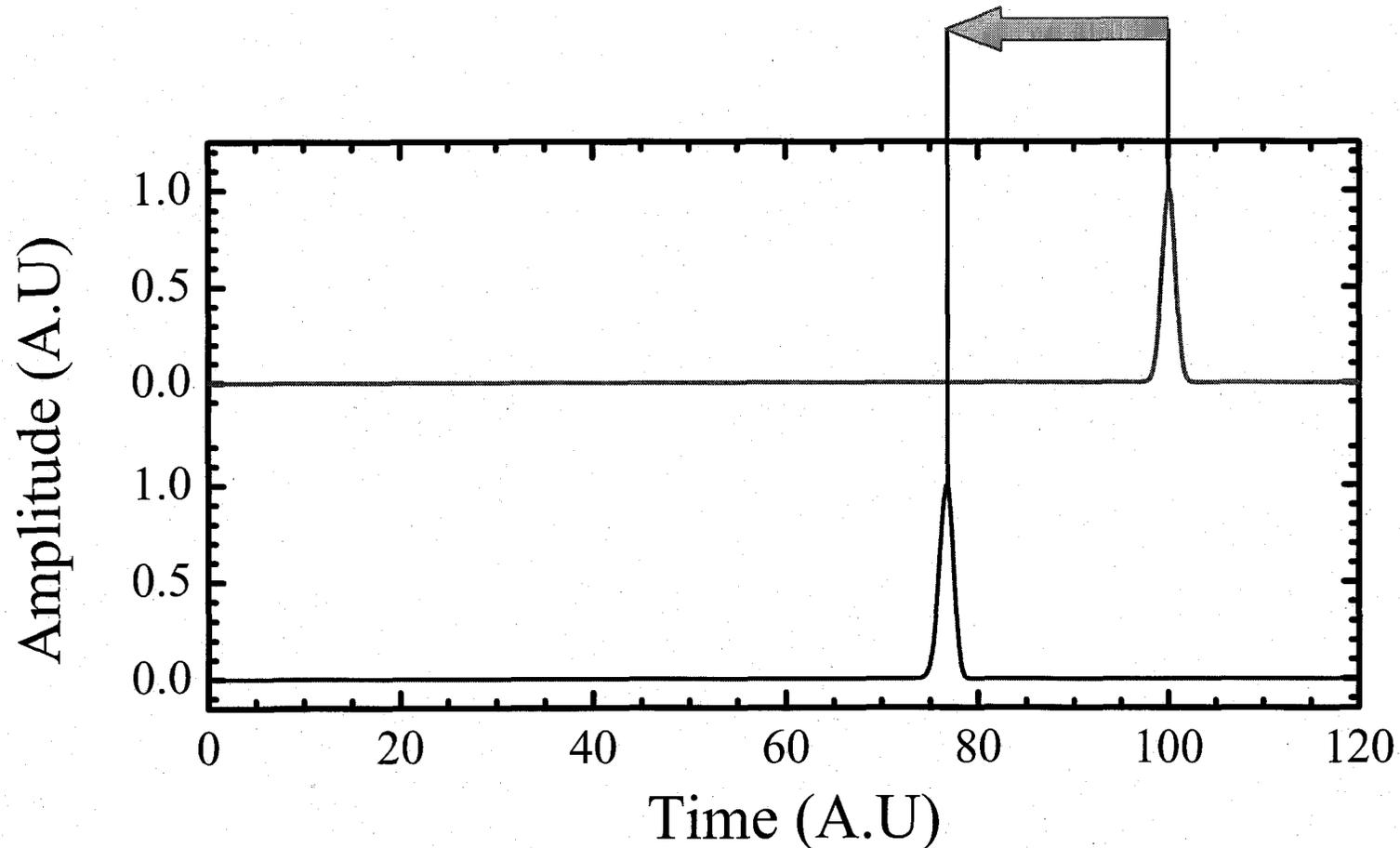
$$\beta > 0$$

➤ *CRIT / CRIA transition* \Rightarrow *Minimum Pulse Distortion!*



SCISSORS SLOB

*Near distortion-free delay
of 14 pulse lengths by 100
CRIT/CRIA structures*



➤ *Pulse delay increased by using sequence of resonators*

Summary and Conclusions

- Bandwidth AND Pulse Delay simultaneously increased while pulse distortion minimized by operating at CRIT / CRIA transition.
- Can use multiple coupled resonators to further increase bandwidth, but pulse delay still limited to a few pulse lengths.
- A resonator chain or SCISSORS structure can be used to further increase pulse delay.

Collaborators:

- *K. A. Fuller* / University of Alabama in Huntsville
- *A. Odutola* / Alabama A&M University
- *R. W. Boyd, A. Schweinsberg, G. Gehring* / University of Rochester