Quantum coherence effects in atomic media such as electromagnetically-induced transparency and absorption, lasing without inversion, super-radiance and gain-assisted superluminality have become well-known in atomic physics. But these effects are not unique to atoms, nor are they uniquely quantum in nature, but rather are fundamental to systems of coherently coupled oscillators. In this talk I will review a variety of analogous photonic coherence phenomena that can occur in passive and active coupled optical resonators. Specifically, I will examine the evolution of the response that can occur upon the addition of a second resonator, to a single resonator that is side-coupled to a waveguide, as the coupling is increased, and discuss the conditions for slow and fast light propagation, coupled-resonator-induced transparency and absorption, lasing without gain, and gain-assisted superluminal pulse propagation. Finally, I will discuss the application of these systems to laser stabilization and gyroscopy.
Coherence Phenomena in Coupled Optical Resonators

Dave Smith
NASA
Marshall Space Flight Center
Outline

- WGM Splitting
- Coherence Effects in Passive Systems
  - Coupled-Resonator-Induced Transparency and Absorption (CRIT and CRIA)
  - Gain-Assisted Superluminality (GAS)
- Coherence Effects in Active Systems
  - Lasing Without Gain (LWG)
  - Reduced lasing thresholds
- Application: laser gyroscopy
Varieties of Coupled Optical Resonators

- Mutually-coupled
- Intra-resonator Coupling

(a) Waveguide-coupled

(b) Inter-resonator coupling

(c) (d)
Whispering-Gallery-Mode Splitting


Mode splitting in ring resonators

Mode splitting in \( \mu \)-particles

- Consequence of constructive (N odd) or destructive (N even) interference
- \( N \) resonators yield
Tight-Binding PBGs in Photonic Molecules

Transmittance

Reflectance

Transmittance, $T$

Reflectance, $R$

Single-pass phase shift, $\phi_f (1/\lambda)$
Coupled Resonators = Two Level Atom

Coupled Modes = Schrod. Eqn. in RWA:

\[ \dot{\alpha}(t) = -\frac{\hbar}{2} \begin{pmatrix} -\Delta & \kappa^* \\ \kappa & -\Delta \end{pmatrix} \alpha(t) = \hat{H} \alpha(t) \]

\[ \alpha(t) = \begin{bmatrix} a_1(t) \\ a_2(t) \end{bmatrix} \]

Rotating State

\[ \Omega_R \equiv \tilde{\kappa} \quad \Delta \equiv \tilde{\omega}_1 - \tilde{\omega}_2 = \Delta + i \frac{\gamma_{12}}{2} \]

Non-Hermitian Hamiltonian

Complex NL Coupling and Detuning

\[ \tilde{\chi}_r = \cos \left( \frac{\tilde{\Omega} t}{2} \right) + i \left( \frac{\Delta}{\tilde{\Omega}} \right) \sin \left( \frac{\tilde{\Omega} t}{2} \right) \]

\[ \tilde{\chi}_i = i \left( \frac{\Omega_R^*}{\tilde{\Omega}} \right) \sin \left( \frac{\tilde{\Omega} t}{2} \right) \]

Complex Generalized Rabi Frequency

Damped Rabi Oscillations!

If \( \gamma_{12} \neq 0 \rightarrow H \) is non-Hermitian, dressed states couple
Rabi Flopping

**Coupled Resonator Density Matrix:**

\[
\rho = \begin{pmatrix}
E_1 E_1^* & E_1 E_2^* \\
E_2 E_1^* & E_2 E_2^*
\end{pmatrix}
\]
Coherent Photon Transfer in Coupled Resonators

- Only coherent excitation yields full transfer
- Sensitive to pulse area and frequency
- Adiabatic transfer independent of pulse area but requires long slow chirp
Stimulated Raman Adiabatic Passage (STIRAP)

- Counterintuitive pulse sequence
- Insensitive to pulse area – rapid transfer
- Requires 2 but not 1 photon resonance
Coupled-Resonator-Induced Transparency (CRIT)

- Photon Trapping!
- Slow light with no absorption!

Coupled-Resonator-Induced Absorption (CRIA)

- Analogous to Electromagnetically-Induced Absorption (EIA)
- Typically results from *Constructive* Interference!
- Requires the second resonator to be over-coupled \( r_2 < a_2 \)

![Graph showing absorptance and phase shift](image)
$N$ Coupled Ring Resonators

\[
\begin{pmatrix}
E_{4(N-1)} \\
E_{4(N-1)+2}
\end{pmatrix}
= \begin{pmatrix}
r_j & it_j \\
it_j & r_j
\end{pmatrix}
\begin{pmatrix}
E_{4(N-1)} \\
E_{4(N-1)+1}
\end{pmatrix}
\]

Feedback
\[
E_{4(N-1)+1} = a_j e^{i\phi_j} E_{4(N-1)+3}
\]

\[\pi \text{ phase shift results after 2 passes across coupler!}\]
Frequency Response

\[ \tau_1(\phi_1) = \frac{r_1 - a_1 e^{i\phi}}{1 - r_1 a_1 e^{i\phi}} = |\tau_1| \exp\left[ i\phi_1^{\text{(eff)}} \right] \]

\[ \tau_2(\phi_1, \phi_2) = \frac{r_2 - a_2 \tau_1 e^{i\phi_2}}{1 - r_2 a_2 \tau_1 e^{i\phi_2}} = |\tau_2| \exp\left[ i\phi_2^{\text{(eff)}} \right] \]

\[ = \exp\left[ i\tilde{\phi}_2^{\text{(eff)}} \right] \quad \tilde{\phi}_2^{\text{(eff)}} = \phi_2^{\text{(eff)}} - i \ln |\tau_2| \]

Dispersive Response

\[ \phi_2^{(\text{eff})}(\phi_1, \phi_2) = \arg(\tau_2) \]

Absorptive Response

\[ T_2 = |\tau_2|^2 \quad A_2 = 1 - T_2 \]

- **Slow or Fast Light**
- **CRIT or CRIA**
Transmission Argand Diagrams

**Single Resonator**

- Over-coupled
- Under-coupled
- Critically coupled

**Two Resonators**

- CRIA (fast light)
- CRIA (slow light)

➤ Slow light on the left!

➤ Slow light on the right!
Impulse Response

**Single Resonator:**

\[ \tau_1 (0) = \int_{-\infty}^{\infty} G_1 (t) \, dt \]

- Under-coupled
- Critically coupled
- Over-coupled

**Coupled Resonators:**

- **Distinguishable CRIT**
- **Indistinguishable CRIT**
- **CRIA**

> Single - Exponential Decay

> Coupled - Non-exponential Decay
FDTD of CRIA and CRIT
Critical Coupling of CRIT and CRIA

\[ r_2 = a_2 a_1 \]

\[ r_2 = a_2 \left| \tau_1 \right| \]
Slow and Fast Pulse Propagation

- Slow light with no absorption
- But fast light requires loss to obey causality / Relativity
Gain-Assisted Superluminality

Fast light in a transparent medium!
Gain-assist reverses dispersion / CRIA boosted to transparency
Still does not violate causality


Lasing in Coupled Resonators

- Coherence affects lasing threshold!
- LWI and Super-radiance analogs

**Distinguishable**

![Graph showing distinguishable lasing in coupled resonators](image)

**Indistinguishable**

![Graph showing indistinguishable lasing in coupled resonators](image)
Lasing Without Gain in Coupled Resonators

- Analogous to LWI ($N_e < N_g$): Gain in resonator 1 < Loss in resonator 2
- Lasing occurs by photon trapping – indistinguishability required.

![Graph showing transmittance vs. net gain, $T_2(\phi = 0)$ and lasing threshold.](Note: $G_1G_2 < 1$)
Reduced Thresholds in Coupled Resonators

- Threshold for coupled resonators < threshold for single resonator
- Resonators must be properly phased for constructive interference

Upper resonator twice OPL of lower

\[ T_2 \left( \phi_2 = \pi \right) \left( \phi_1 = 0 \right) \]

\[ T_1 \left( \phi_1 = 0 \right) \]

Transmittance vs. gain factor, \( g \)
Beat Frequency and Relative Modulation

\[ Q = 1.5 \times 10^8 \]
\[ N_g^{(1)} = -3.3 \]

\[ Q = 0.8 \times 10^8 \]
\[ N_g^{(1)} = 0.2 \]
Laser Gyro Enhancement

- Scale-Factor increased by $1/N$
- Elimination of Gyro Dead-band
- Single Resonators: under-coupled (Anomalous) or over-coupled (Normal), but no gain.
- Coupled Resonators: CRIA or GAS but not CRIT.

$$Q = 0.8 \times 10^8$$

$$N_g^{(1)} = 0.2$$
Summary and Conclusions

- Coupled resonators are analogous to multilevel atoms and are described approximately by the damped Rabi problem.

- Photons can be shuffled from one resonator to another using coherent and adiabatic photon transfer techniques. Coupled resonators can store light.

- Coherence phenomena such as EIT, EIA, GAS, and LWI are fundamental to systems of coherently coupled oscillators. They are not unique to atoms nor are they uniquely quantum phenomena.

- The dispersion in optical micro-resonators has application to the improvement of laser gyroscopes.
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