Status on Iterative Transform Phase Retrieval applied to the GBT Data

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Image-Based Wavefront Sensing and Control of the NRAO Green Bank Radio Telescope

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Overview

Introduction
- Phase Retrieval / NASA Projects
- JWST TRL-6

GBT Data / Notes:
- Data Format and Sampling
- Ray Trace Model & Wavefront
- Symmetry of GBT Data
- Pupil and Fourier Model
- Pupil Amplitude

PR Simulations
- Wavefront derived from GBT Data symmetry
- Wavefront Sensing accuracy and Coherent / Incoherent Assumptions

GBT Results
Applications and Technology Development

- NASA Investments in Image-Based WFSC
  - Developments through JWST Pre Phase-A and Phase-B,
  - WFSC Demonstrated to TRL-6 using the Ball Aerospace TBT,
  - Have investigated a number of performance and implementation details, e.g., optimal diversity defocus, bandpass, phase wrapping, Branch Points,
  - Compact Supercomputing Architecture utilizing DSPs

<table>
<thead>
<tr>
<th>Date</th>
<th>Projects</th>
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<tbody>
<tr>
<td>1990</td>
<td>Hubble Primary Mirror Aberration Determination</td>
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<td>1994</td>
<td>Mars Observer Camera In-flight Diagnosis</td>
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<td>1996</td>
<td>Cassini ISS Narrow Angle Camera Verification Testing</td>
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<td>09/1998</td>
<td>NASA Developmental Comparative Active Telescope Testbed (DCATT)</td>
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<td>NASA Wavefront Control Testbed (WCT)</td>
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<td>IRAC Testing (Spitzer Space Telescope)</td>
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<td>Phase Retrieval Camera</td>
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<td>04/2002</td>
<td>RIVMOS Testing</td>
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<td>07/2002</td>
<td>NIRSpec Microshutter (MSA) Testbed</td>
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<td>09/2002</td>
<td>HUBBLE Simulator Hardware (CASTLE)</td>
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<td>04/2003</td>
<td>TPF’s High Contrast Imaging Testbed (HCIT)</td>
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<td>04/2003</td>
<td>Mercury Laser Altimeter (MLA)</td>
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<td>06/2003</td>
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<td>JWST AMSD Mirror Testing with a Phase Retrieval Camera</td>
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<td>Ball Aerospace RA-6 (Boulder, CO)</td>
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<td>HST Wide-Field Camera III</td>
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<td>07/2005</td>
<td>Palomar 200” Telescope Adaptive Optics System (PALAO) Calibration</td>
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<td>10/2005</td>
<td>JWST Testbed Telescope (TBT; Ball Aerospace, Boulder, CO)</td>
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James Webb Space Telescope (JWST)

- Successor to the Hubble Space Telescope
- Current Launch Date is 2013

- 18 Segment PM
- 6.5 meter aperture
- Orbit at L2
Testbed Telescope (TBT)
Flight traceable, 1/6 scale, 18 segment design

Algorithm Performance requirements dictated by NASA’s TRL-6
- Testbed provides functionally accurate simulation platform for developing deliverable WFSC algorithms and software,
- Used to perform TRL-6 end to end testing,
- a solution is a fine-phasing algorithm that incorporates feedback,
- an adaptive diversity function, eliminates Branch Points, and Wrapping
TRL-6 Comparison with Interferometer

Phase Retrieval:  Interferometer:

![Image of Phase Retrieval and Interferometer comparison](image_url)
Phase Retrieval Approaches

- Two main approaches commonly used:
  - Iterative Transform (ITA)
    - Gerchberg-Saxton
    - Misell-Gerchberg-Saxton
    - HDA (extends dynamic range)
  - Parametric (non-linear least squares model fitting)
    - Solve for aberration coefficients
    - Solve for point-point phase in the pupil

\[
\min [\text{Objective Function}]: \\
\sum_{\forall \text{m}, \text{n}} \left\| \text{PSF}_{\text{data}, \text{m}, \text{n}} - \text{PSF}_{\text{model}, \text{m}, \text{n}} \right\|^2 = 0
\]

For JWST - adopted a hybrid approach that incorporates features of both types of algorithms.

Concept:

- phase from intensity data? \( z = x + iy = re^{i\theta} \)
- complex numbers:
  \[ |z|^2 = r^2e^{i\theta}e^{-i\theta} = r^2 \neq r^2(\theta) \]
- phase part is decoupled from intensity
- phase-recovery fact - optical aperture scatters phase information into the intensity data
- star image – normally like an airy disk for a circular aperture:

- intensity is now a function of the phase:
  \[ r^2 = r^2(\theta) \]
- algorithm: indirectly recover phase from intensity.
Earlier Work using ITA with Radio Antennas

Notes / Understanding of GBT Data

- Consists of two feeds (pixels), two polarizations,
- Separated by 58 arc-seconds,
- Output of receivers is differenced to minimize the effect of sky-brightness variations.
- Effective response of the telescope is modeled as the real beam convolved by two delta functions separated by 58” in the azimuth direction
  - aberrations due to both of the feeds being off (and on opposite sides of) the optical axis are negligible?
  - if this is not negligible, then a “single beam convolved by two delta functions” assumption may not be valid.
Raw Data Contributed by the NRAO

- Data Filename: s114-l-db.fits, April 2005
- Read: dx, dy, fnu, ufnu, ttime ([5806×1 double])

Scan Pattern: (plot dx, dy):

Signal vs time (plot fnu, ttime):
NRAO Data: Non-uniform data samples are interpolated:

- Data values: dx (azm), dy (elv) are used to form a rectangular coordinate array.

Two Options:

- **down-sample in x:**

- **up-sample in y:**

- First interpolated to a uniform rectangular grid (azm-elev),

- A rectangular coordinate grid of 17 by 68 is formed and then the 5806 fnu data values are interpolated to this grid using cubic interpolation.
NRAO Data & Sampling

Azimuth direction (x), approximately 350 samples/per scan line.
Sampling in Azimuth = 3600*(180/pi)*(dx(251)-dx(250)) = 2.42 Arcsec / pixel = px = 1.1732e-05 radians
Qx = λ/(D*px) = 6.96e-3 / (100*1.1732e-05) = 5.9325

Elevation direction (y), 17 scan lines
Sampling in Elevation = 3600*(180/pi)*(dy(5600)-dy(250))/17 = 7.5 Arcsec / pixel = py = 3.6361e-05 radians
Qy = λ/(D*px) = 6.96e-3 / (100*3.6361e-05) = 1.9141

Nyquist sampling is 7.2 arcsec / sample, Q = 2; Under-sampled by 0.96 in Elevation; over-sampled by 2.97 in Azimuth

v = 43.1 GHz;  λ = 6.96 mm
NRAO GBT Aperture

- Panels are arranged in such a way that rings are concentric with a parent parabola.
- Zemax design: GBT is setup as a single off-axis section of the parent parabola.
Note that Translation Shift of Fourier Transform produces a phase factor:

\[ \Im[g(x - x_0)] = G(\omega)e^{-i2\pi\omega x_0} \]
Pupil Illumination - I


\[ A(r) = 1, \quad (\text{uniform}) \]
\[ = \exp[-\alpha(r/r_0)^2], \quad (\text{tapered Gaussian}) \]
\[ \alpha = (T_e/20)\ln 10, \quad (\text{edge taper factor}) \]

Using the formula for edge taper in dB:

\[ \Delta A_{dB} = 10\log[(A_1/A_2)^2] = 20\log(1/0.18) \approx 15 \, \text{dB} \]
Pupil Illumination - II

\[ A(r) = 1, \quad \text{(uniform)} \]
\[ = \exp[-\frac{r^2}{2\sigma^2}], \quad \text{(tapered Gaussian)} \]
with \( \sigma = 0.3 \),

from PTCSPN47.pdf

... the aperture plane amplitude distribution, that is, the illumination of the primary surface. This was approximated as a well-centered and circular Gaussian with a width (in radius-normalized units) defined by \( \sigma = 0.3 \), which corresponds to 14.5 dB of illumination taper at the edge of the dish ...

\[ \sigma \approx 0.55 \]

amplitude variation:

\[ \Delta A_{db} = 10 \log[(A_1 / A_2)^2] = 20 \log(1 / 0.18) \approx 15 \text{ dB} \]
Challenge for ITA Phase Retrieval

- Two incoherently subtracted irradiance values appear in the GBT data.
- Data collection process, $I = I_1 - I_2$
- For the ITA approach to work, these irradiance values should be the result of one FFT.
- So make the approximation that:

Coherent Approximation for Incoherent Data:

\[ |\Imag{A_L(-\theta_t) + A_R(+\theta_t)}|^2 \approx |\Imag{A_L(-\theta_t)}|^2 + |\Imag{A_R(+\theta_t)}|^2 \]

or simply \[ I \approx I_L + L_R \]
Coherent Approximation for Incoherent Data

\[ |\mathcal{I}\{A_L(-\theta_t) + A_R(+\theta_t)\}|^2 \approx |\mathcal{I}\{A_L(-\theta_t)\}|^2 + |\mathcal{I}\{A_R(+\theta_t)\}|^2 \]

beam tilt:
Validity of Approximation?

- Good approximation for large tilt (i.e., there is little interference)
- Plot of squared error as a function of tilt:

\[
error^2 = \sum_k \sum_j |I_{\text{coherent}}(j,k) - I_{\text{incoherent}}(j,k)|^2
\]

How does error propagate to Phase Retrieval?
Proof of Concept: PR Simulation

Input Wavefront:

RMS = 557 μ; λ/13; PV = 0.4561 λ

Malacara Basis Set:

<table>
<thead>
<tr>
<th>#</th>
<th>radial</th>
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<th>term</th>
<th>aberration</th>
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<tr>
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<td>30° trefoil</td>
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<tr>
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<td>3</td>
<td>1</td>
<td>$r(3r^2 - 2) \sin \alpha$</td>
<td>$y$-coma</td>
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<tr>
<td>9</td>
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<td>2</td>
<td>$r(3r^2 - 2) \cos \alpha$</td>
<td>$x$-coma</td>
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<td>$r^3 \cos 3\alpha$</td>
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<td>$r^4 \sin 4\alpha$</td>
<td>22.5° tetrafoil</td>
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<td>12</td>
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<td>3</td>
<td>$(4r^4 - 3r^2) \sin 2\alpha$</td>
<td>45° astig (2nd order)</td>
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<td>3</td>
<td>$r^4 \cos 4\alpha$</td>
<td>0° tetrafoil</td>
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Check: Coherent PR Simulation

Image on 1-side of focus:  Dual aperture model:  Pupil Amplitude:

Results:

Decomposition

<table>
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<tr>
<th>Term</th>
<th>Input</th>
<th>Left Beam</th>
<th>Right Beam</th>
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Recovered:
Comment on GBT Beam Symmetry

Model  Data  Incoherent Model

Let PR solve for the wavefront that is consistent with the data:
Incoherent PR Results (simulation)

Incoherent Data:

abs()

dual wavefront:

Results:

Pupil Amplitude:

Recovered:
Incoherent PR Results - worst case
-- Simulation: 2 waves beam tilt

Incoherent Data: Model: dual wavefront:

Results:

Pupil Amplitude: Recovered:
Estimate Initial Sampling Parameters from focused GBT Data

- $Q \approx 4.5$
- Beam tilt $\approx 1.5 \lambda$

Can also tune parameters by matching the FFT of the data
Wavefront Sensing Results applied to GBT Data - 3

GBT Data: 

![GBT Data Image]

Model: 

![Model Image]

wavefront: 

![Wavefront Image]

Results: 

![Results Image]

Pupil Amplitude: 

![Pupil Amplitude Image]

Recovered: 

![Recovered Image]
Wavefront Sensing Results applied to GBT Data - 2

GBT Data: Model: wavefront:

Results:

Pupil Amplitude: Recovered:
Summary

- In principle, coherent ITA PR may work on incoherent GBT data,
- Errors increases as beam tilt decreases