begins with the concept of an anisoplanatic function, defined as a phase function representative of the degree to which imaging performance varies over the field of view. The wavefront phase at a given field point is assumed to be given by the sum of the isoplanatic and anisoplanatic contributions. It is further assumed that an estimate of an isoplanatic phase function at a given field point can be modeled as a sum, over all other field points, of the convolutions of the filter function with the wavefront phase. Then the filter-function problem is formulated as one of choosing the filter coefficients to minimize the sum, over all field points, of the squares of the differences between the estimated and exact phase values of the isoplanatic phase function. To minimize this sum, one sets the partial derivatives of this sum with respect to the filter coefficients equal to zero. After some further algebraic manipulations, one obtains equations for the filter coefficients and an equation for the corrected wavefront generated by use of the filter function.

The filter function was tested in a computational simulation based on the optical design of the James Webb Space Telescope. Among the results, the variation of phase error over the field of view was 83 percent less in the case of a multiple-field-point/filter-function control law than in the case of a single-field-point control law (see figure).

This work was done by Bruce H. Dean of Goddard Space Flight Center. Further information is contained in a TSP (see page 1).

GSC-14900-1

### Mean Values of Corrected Root-Mean-Square Wavefront Error

Mean Values of Corrected Root-Mean-Square Wavefront Error were computed for several field points and fitted with straight lines to show that errors can be reduced and/or distributed more evenly when multiple field points and the filter function are used.

#### Iterative-Transform Phase Retrieval Using Adaptive Diversity

High- and low-spatial-frequency contents are recovered with high dynamic range.

**Goddard Space Flight Center, Greenbelt, Maryland**

A phase-diverse iterative-transform phase-retrieval algorithm enables high-spatial-frequency, high-dynamic-range, image-based wavefront sensing. [The terms “phase-diverse,” “phase retrieval,” “image-based,” and “wavefront sensing” are defined in the first of the two immediately preceding articles, “Broadband Phase Retrieval for Image-Based Wavefront Sensing” (GSC-14899-1).] As described below, no prior phase-retrieval algorithm has offered both high dynamic range and the capability to recover high-spatial-frequency components.

Each of the previously developed image-based phase-retrieval techniques can be classified into one of two categories: iterative transform or parametric. Among the modifications of the original iterative-transform approach has been the introduction of a defocus diversity function (also defined in the cited companion article). Modifications of the original parametric approach have included minimizing alternative objective functions as well as implementing a variety of nonlinear optimization methods. The iterative-transform approach offers the advantage of ability to recover low, middle, and high spatial frequencies, but has disadvantage of having a limited dynamic range to one wavelength or less. In contrast, parametric phase retrieval offers the advantage of high dynamic range, but is poorly suited for recovering higher spatial frequency aberrations.

The present phase-diverse iterative-transform phase-retrieval algorithm offers both the high-spatial-frequency capability of the iterative-transform approach and the high dynamic range of parametric phase-recovery techniques. In implementation, this is a focus-diverse iterative-transform phase-retrieval algorithm that incorporates an adaptive diversity function, which makes it possible to avoid phase unwrapping while preserving high-spatial-frequency recovery.

The algorithm includes an inner and an outer loop (see figure). An initial estimate of phase is used to start the algorithm on the inner loop, wherein multiple intensity images are processed, each using a different defocus value. The processing is done by an iterative-transform method, yielding individual phase estimates corresponding to each image of the defocus-diversity data set. These individual phase estimates are combined in a weighted average to form a new phase estimate, which serves as the initial phase estimate for either the next iteration of the iterative-transform method or, if the maximum number of iterations has been reached, for the next...
several steps, which constitute the outer-loop portion of the algorithm.

The details of the next several steps must be omitted here for the sake of brevity. The overall effect of these steps is to adaptively update the diversity defocus values according to recovery of global defocus in the phase estimate. Aberration recovery varies with differing amounts as the amount of diversity defocus is updated in each image; thus, feedback is incorporated into the recovery process. This process is iterated until the global defocus error is driven to zero during the recovery process.

The amplitude of aberration may far exceed one wavelength after completion of the inner-loop portion of the algorithm, and the classical iterative transform method does not, by itself, enable recovery of multi-wavelength aberrations. Hence, in the absence of a means of “off-loading” the multi-wavelength portion of the aberration, the algorithm would produce a wrapped phase map. However, a special aberration-fitting procedure can be applied to the wrapped phase data to transfer at least some portion of the multi-wavelength aberration to the diversity function, wherein the data are treated as known phase values. In this way, a multi-wavelength aberration can be recovered incrementally by successively applying the aberration-fitting procedure to intermediate wrapped phase maps. During recovery, as more of the aberration is transferred to the diversity function following successive iterations around the outer loop, the estimated phase ceases to wrap in places where the aberration values become incorporated as part of the diversity function. As a result, as the aberration content is transferred to the diversity function, the phase estimate resembles that of a reference flat.

This work was done by Bruce H. Dean of Goddard Space Flight Center. Further information is contained in a TSP (see page 1). GSC-14879-1

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**Wavefront Sensing With Switched Lenses for Defocus Diversity**

*It is no longer necessary to translate a camera to precisely controlled defocus positions.*

*Goddard Space Flight Center, Greenbelt, Maryland*

In an alternative hardware design for an apparatus used in image-based wavefront sensing, defocus diversity is introduced by means of fixed lenses that are mounted in a filter wheel (see figure) so that they can be alternately switched into a position in front of the focal plane of an electronic camera recording the image formed by the optical system under test. [The terms “image-based”, “wavefront sensing”, and “defocus diversity” are defined in the first of the three immediately