HIGHLY EFFICIENT DESIGN-OF-EXPERIMENTS METHODS FOR COMBINING CFD ANALYSIS AND EXPERIMENTAL DATA

Bernhard H. Anderson
NASA John Glenn Research Center
Cleveland, Ohio, 44135

Harold S. Haller
Real World Quality Systems
Cleveland, Ohio, 44116

ABSTRACT

It is the purpose of this study to examine the impact of “highly efficient” Design-of-Experiments (DOE) methods for combining sets of CFD generated analysis data with smaller sets of Experimental test data in order to accurately predict performance results where experimental test data were not obtained. The study examines the impact of micro-ramp flow control on the shock wave boundary layer (SWBL) interaction where a complete paired set of data exist from both CFD analysis and Experimental measurements By combining the complete set of CFD analysis data composed of fifteen (15) cases with a smaller subset of experimental test data containing four/five (4/5) cases, compound data sets (CFD/EXP) were generated which allows the prediction of the complete set of Experimental results No statistical difference were found to exist between the combined (CFD/EXP) generated data sets and the complete Experimental data set composed of fifteen (15) cases. The same optimal micro-ramp configuration was obtained using the (CFD/EXP) generated data as obtained with the complete set of Experimental data, and the DOE response surfaces generated by the two data sets were also not statistically different.
Highly Efficient Design-of-Experiments Methods for Combining CFD Analysis and Experimental data

Bernhard H. Anderson  
NASA Glenn Research Center  
Cleveland, Ohio 44135

Harold H. Haller  
Real World Quality Systems  
Cleveland, Ohio 44116
Micro-Ramp Oblique SWBL Flow Control Research Goals

• Demonstrate the applicability of combining CFD analysis data and experimental test data to gain an efficiency and cost effectiveness advantage.

• Statistically evaluate the combined CFD/Exp. Data set in comparison to the complete Experimental Data set.
Micro-Ramp Oblique SWBL Flow Control

Nomenclature, Geometry and Grid

Flow

$X_p$

$c/2$

$s = h y / n$

$y$

$Flow$

$Ap$

$Geometry$

$Composition$

$Nomenclature$
Micro-Ramp Oblique SWBL Flow Control
AIP Reference Plane Location
Mach Number Contours

Transverse Mach Number Contours
AIP Reference Plane

X (cm)
Z (cm)
M

AIP Reference Plane

2.10
2.00
1.90
1.80
1.70
1.60
1.50
1.40
1.30
1.20
1.10
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
Micro-Ramp Oblique SWBL Flow Control
Variables Held Constant

<table>
<thead>
<tr>
<th>Factor Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free Stream Mach Number, $Mo$</td>
<td>2.0</td>
</tr>
<tr>
<td>Free Stream Total Pressure (lbs/ft$^2$), $Po$</td>
<td>2112.0</td>
</tr>
<tr>
<td>Free Stream Total Temperature (°R), $To$</td>
<td>517.0</td>
</tr>
<tr>
<td>Shock Wave Generator Angle (degs.), $\delta$</td>
<td>$8.461 \pm 0.064$</td>
</tr>
<tr>
<td>Micro-Ramp Wedge Angle (degs.), $Ap$</td>
<td>24.0</td>
</tr>
</tbody>
</table>
Micro-Ramp Oblique SWBL Flow Control
DOE Factor Variables

<table>
<thead>
<tr>
<th>Factor Variable</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transverse Spacing (mm), s</td>
<td>25.0 – 35.0</td>
</tr>
<tr>
<td>Micro-Ramp Height (mm), h</td>
<td>3.0 – 5.0</td>
</tr>
<tr>
<td>Micro-Ramp Chord Length (mm), c</td>
<td>12.0 – 24.0</td>
</tr>
</tbody>
</table>
## Micro-Ramp Oblique SWBL Flow Control
### DOE Response Variables

<table>
<thead>
<tr>
<th>Response Variable</th>
<th>Nomenclature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary Layer Pitot Pressure Recovery</td>
<td>PTAVE</td>
</tr>
<tr>
<td>Boundary Layer Total Pressure Recovery</td>
<td>PFAVE</td>
</tr>
<tr>
<td>Compressible Displacement Thickness, (cm)</td>
<td>( \delta^* )</td>
</tr>
<tr>
<td>Compressible Momentum Thickness, (cm)</td>
<td>( \theta )</td>
</tr>
<tr>
<td>Transformed Form Factor</td>
<td>( H_{tr} )</td>
</tr>
</tbody>
</table>
## Micro-Ramp Oblique SWBL Flow Control
### Central Composite Face Center (CCF) Design
#### CFD Analysis Data Set

<table>
<thead>
<tr>
<th>Config.</th>
<th>s (mm)</th>
<th>h (mm)</th>
<th>c (mm)</th>
<th>PTAVE</th>
<th>PFAVE</th>
<th>δ* (cm)</th>
<th>θ (cm)</th>
<th>Htr</th>
</tr>
</thead>
<tbody>
<tr>
<td>rvg300</td>
<td>____</td>
<td>____</td>
<td>____</td>
<td>0.55746</td>
<td>0.67420</td>
<td>0.31805</td>
<td>0.10940</td>
<td>1.30205</td>
</tr>
<tr>
<td>rvg400</td>
<td>____</td>
<td>____</td>
<td>____</td>
<td>0.63962</td>
<td>0.65320</td>
<td>0.54031</td>
<td>0.18391</td>
<td>1.89615</td>
</tr>
<tr>
<td>rvg401</td>
<td>25.0</td>
<td>3.0</td>
<td>12.0</td>
<td>0.63219</td>
<td>0.64284</td>
<td>0.54972</td>
<td>0.19921</td>
<td>1.80391</td>
</tr>
<tr>
<td>rvg402</td>
<td>35.0</td>
<td>3.0</td>
<td>12.0</td>
<td>0.62456</td>
<td>0.63632</td>
<td>0.60199</td>
<td>0.20737</td>
<td>1.83447</td>
</tr>
<tr>
<td>rvg403</td>
<td>25.0</td>
<td>5.0</td>
<td>12.0</td>
<td>0.64658</td>
<td>0.65530</td>
<td>0.61546</td>
<td>0.21740</td>
<td>1.75787</td>
</tr>
<tr>
<td>rvg404</td>
<td>35.0</td>
<td>5.0</td>
<td>12.0</td>
<td>0.64087</td>
<td>0.64907</td>
<td>0.58951</td>
<td>0.21108</td>
<td>1.75554</td>
</tr>
<tr>
<td>rvg405</td>
<td>25.0</td>
<td>3.0</td>
<td>24.0</td>
<td>0.62279</td>
<td>0.63195</td>
<td>0.64810</td>
<td>0.21870</td>
<td>1.83855</td>
</tr>
<tr>
<td>rvg406</td>
<td>35.0</td>
<td>3.0</td>
<td>24.0</td>
<td>0.62399</td>
<td>0.63431</td>
<td>0.65956</td>
<td>0.22384</td>
<td>1.83640</td>
</tr>
<tr>
<td>rvg407</td>
<td>25.0</td>
<td>5.0</td>
<td>24.0</td>
<td>0.65325</td>
<td>0.65770</td>
<td>0.65920</td>
<td>0.22850</td>
<td>1.72099</td>
</tr>
<tr>
<td>rvg408</td>
<td>35.0</td>
<td>5.0</td>
<td>24.0</td>
<td>0.65805</td>
<td>0.66585</td>
<td>0.62662</td>
<td>0.21618</td>
<td>1.74131</td>
</tr>
<tr>
<td>rvg409</td>
<td>25.0</td>
<td>4.0</td>
<td>18.0</td>
<td>0.62845</td>
<td>0.63684</td>
<td>0.64084</td>
<td>0.21828</td>
<td>1.81235</td>
</tr>
<tr>
<td>rvg410</td>
<td>35.0</td>
<td>4.0</td>
<td>18.0</td>
<td>0.63326</td>
<td>0.64324</td>
<td>0.61887</td>
<td>0.21210</td>
<td>1.80763</td>
</tr>
<tr>
<td>rvg411</td>
<td>30.0</td>
<td>3.0</td>
<td>18.0</td>
<td>0.63266</td>
<td>0.64054</td>
<td>0.67356</td>
<td>0.22964</td>
<td>1.79437</td>
</tr>
<tr>
<td>rvg412</td>
<td>30.0</td>
<td>5.0</td>
<td>18.0</td>
<td>0.64613</td>
<td>0.65443</td>
<td>0.61295</td>
<td>0.21409</td>
<td>1.73988</td>
</tr>
<tr>
<td>rvg413</td>
<td>30.0</td>
<td>4.0</td>
<td>12.0</td>
<td>0.63150</td>
<td>0.64190</td>
<td>0.60965</td>
<td>0.21110</td>
<td>1.81368</td>
</tr>
<tr>
<td>rvg414</td>
<td>30.0</td>
<td>4.0</td>
<td>24.0</td>
<td>0.63490</td>
<td>0.64341</td>
<td>0.64770</td>
<td>0.21940</td>
<td>1.80623</td>
</tr>
<tr>
<td>rvg415</td>
<td>30.0</td>
<td>4.0</td>
<td>18.0</td>
<td>0.63535</td>
<td>0.64491</td>
<td>0.60281</td>
<td>0.20941</td>
<td>1.80081</td>
</tr>
</tbody>
</table>
### Micro-Ramp Oblique SWBL Flow Control
Central Composite Face Center (CCF) Design
15 x 15 cm. SWT Experimental Data Set

<table>
<thead>
<tr>
<th>Config.</th>
<th>s (mm)</th>
<th>h (mm)</th>
<th>c (mm)</th>
<th>PTAVE</th>
<th>PFAVE</th>
<th>δ*(cm)</th>
<th>θ (cm)</th>
<th>Htr</th>
</tr>
</thead>
<tbody>
<tr>
<td>rvg300</td>
<td>____</td>
<td>____</td>
<td>____</td>
<td>0.5675</td>
<td>0.6647</td>
<td>0.3264</td>
<td>0.1180</td>
<td>1.3219</td>
</tr>
<tr>
<td>rvg400</td>
<td>____</td>
<td>____</td>
<td>____</td>
<td>0.6571</td>
<td>0.6761</td>
<td>0.5158</td>
<td>0.1903</td>
<td>1.7460</td>
</tr>
<tr>
<td>rvg401</td>
<td>25.0</td>
<td>3.0</td>
<td>12.0</td>
<td>0.6421</td>
<td>0.6605</td>
<td>0.5265</td>
<td>0.1969</td>
<td>1.7239</td>
</tr>
<tr>
<td>rvg402</td>
<td>35.0</td>
<td>3.0</td>
<td>12.0</td>
<td>0.6558</td>
<td>0.6758</td>
<td>0.5296</td>
<td>0.1963</td>
<td>1.7308</td>
</tr>
<tr>
<td>rvg403</td>
<td>25.0</td>
<td>5.0</td>
<td>12.0</td>
<td>0.6952</td>
<td>0.7180</td>
<td>0.5523</td>
<td>0.2133</td>
<td>1.6657</td>
</tr>
<tr>
<td>rvg404</td>
<td>35.0</td>
<td>5.0</td>
<td>12.0</td>
<td>0.6838</td>
<td>0.7055</td>
<td>0.5382</td>
<td>0.2062</td>
<td>1.6788</td>
</tr>
<tr>
<td>rvg405</td>
<td>25.0</td>
<td>3.0</td>
<td>24.0</td>
<td>0.6734</td>
<td>0.6939</td>
<td>0.5373</td>
<td>0.2045</td>
<td>1.6901</td>
</tr>
<tr>
<td>rvg406</td>
<td>35.0</td>
<td>3.0</td>
<td>24.0</td>
<td>0.6667</td>
<td>0.6867</td>
<td>0.5312</td>
<td>0.2008</td>
<td>1.7008</td>
</tr>
<tr>
<td>rvg407</td>
<td>25.0</td>
<td>5.0</td>
<td>24.0</td>
<td>0.7053</td>
<td>0.7226</td>
<td>0.5556</td>
<td>0.2256</td>
<td>1.6170</td>
</tr>
<tr>
<td>rvg408</td>
<td>35.0</td>
<td>5.0</td>
<td>24.0</td>
<td>0.6966</td>
<td>0.7153</td>
<td>0.5446</td>
<td>0.2165</td>
<td>1.6314</td>
</tr>
<tr>
<td>rvg409</td>
<td>25.0</td>
<td>4.0</td>
<td>18.0</td>
<td>0.7037</td>
<td>0.7267</td>
<td>0.5377</td>
<td>0.2101</td>
<td>1.6391</td>
</tr>
<tr>
<td>rvg410</td>
<td>35.0</td>
<td>4.0</td>
<td>18.0</td>
<td>0.6882</td>
<td>0.7107</td>
<td>0.5263</td>
<td>0.2019</td>
<td>1.6632</td>
</tr>
<tr>
<td>rvg411</td>
<td>30.0</td>
<td>3.0</td>
<td>18.0</td>
<td>0.6427</td>
<td>0.6659</td>
<td>0.5296</td>
<td>0.1967</td>
<td>1.7202</td>
</tr>
<tr>
<td>rvg412</td>
<td>30.0</td>
<td>5.0</td>
<td>18.0</td>
<td>0.6664</td>
<td>0.6841</td>
<td>0.5089</td>
<td>0.2044</td>
<td>1.6266</td>
</tr>
<tr>
<td>rvg413</td>
<td>30.0</td>
<td>4.0</td>
<td>12.0</td>
<td>0.6808</td>
<td>0.7035</td>
<td>0.5269</td>
<td>0.2002</td>
<td>1.6783</td>
</tr>
<tr>
<td>rvg414</td>
<td>30.0</td>
<td>4.0</td>
<td>24.0</td>
<td>0.7015</td>
<td>0.7239</td>
<td>0.5350</td>
<td>0.2090</td>
<td>1.6438</td>
</tr>
<tr>
<td>rvg415</td>
<td>30.0</td>
<td>4.0</td>
<td>18.0</td>
<td>0.6900</td>
<td>0.7131</td>
<td>0.5282</td>
<td>0.2037</td>
<td>1.6532</td>
</tr>
</tbody>
</table>
Combined Analysis/Experimental Data
D-Optimal Multi Level Design (1)

\[ Y = \sum_{i=1}^{N} A_i X_i + \sum_{i=1}^{N} \sum_{j=1}^{N} A_{ij} X_i X_j + \sum_{i=1}^{N} A_{ii} X_i^2 + Rn(df, S_{y.x}, X_1, X_2, \ldots, X_N) \]

Source (1) = CFD Analysis
15 DOE Experiments

Source (2) = Experimental Data
4 DOE Experiments
Micro-Ramp Oblique SWBL Flow Control
Combined Analysis/Experimental Data
Significant Regression Terms (1)

Main Effect:

$AAX_A$

First Order Interactions:

$ASAXsX_A$

$AhAXhX_A$

$AcAXcX_A$
## Combined Analysis/Experimental Data

### D-Optimal Multi Level Design (1)

<table>
<thead>
<tr>
<th>Config.</th>
<th>s (mm)</th>
<th>h (mm)</th>
<th>c (mm)</th>
<th>$X_A$</th>
<th>PTAVE</th>
<th>PFAVE</th>
<th>$\delta^*$ (cm)</th>
<th>$\theta$ (cm)</th>
<th>Htr</th>
</tr>
</thead>
<tbody>
<tr>
<td>rvg300</td>
<td>____</td>
<td>____</td>
<td>____</td>
<td>Analysis</td>
<td>0.55746</td>
<td>0.67420</td>
<td>0.31805</td>
<td>0.10940</td>
<td>1.30205</td>
</tr>
<tr>
<td>rvg400</td>
<td>____</td>
<td>____</td>
<td>____</td>
<td>Analysis</td>
<td>0.63962</td>
<td>0.65320</td>
<td>0.54031</td>
<td>0.18391</td>
<td>1.89615</td>
</tr>
<tr>
<td>rvg401</td>
<td>25.0</td>
<td>3.0</td>
<td>12.0</td>
<td>Analysis</td>
<td>0.63219</td>
<td>0.64284</td>
<td>0.54972</td>
<td>0.19921</td>
<td>1.80391</td>
</tr>
<tr>
<td>rvg402</td>
<td>35.0</td>
<td>3.0</td>
<td>12.0</td>
<td>Analysis</td>
<td>0.62456</td>
<td>0.63632</td>
<td>0.60199</td>
<td>0.20737</td>
<td>1.83447</td>
</tr>
<tr>
<td>rvg403</td>
<td>25.0</td>
<td>5.0</td>
<td>12.0</td>
<td>Analysis</td>
<td>0.64658</td>
<td>0.65530</td>
<td>0.61546</td>
<td>0.21740</td>
<td>1.75787</td>
</tr>
<tr>
<td>rvg404</td>
<td>35.0</td>
<td>5.0</td>
<td>12.0</td>
<td>Analysis</td>
<td>0.64087</td>
<td>0.64907</td>
<td>0.58951</td>
<td>0.21108</td>
<td>1.75554</td>
</tr>
<tr>
<td>rvg405</td>
<td>25.0</td>
<td>3.0</td>
<td>24.0</td>
<td>Analysis</td>
<td>0.62279</td>
<td>0.63195</td>
<td>0.64810</td>
<td>0.21870</td>
<td>1.83855</td>
</tr>
<tr>
<td>rvg406</td>
<td>35.0</td>
<td>3.0</td>
<td>24.0</td>
<td>Analysis</td>
<td>0.62399</td>
<td>0.63431</td>
<td>0.65956</td>
<td>0.22384</td>
<td>1.83640</td>
</tr>
<tr>
<td>rvg407</td>
<td>25.0</td>
<td>5.0</td>
<td>24.0</td>
<td>Analysis</td>
<td>0.65325</td>
<td>0.65770</td>
<td>0.65920</td>
<td>0.22850</td>
<td>1.72099</td>
</tr>
<tr>
<td>rvg408</td>
<td>35.0</td>
<td>5.0</td>
<td>24.0</td>
<td>Analysis</td>
<td>0.65805</td>
<td>0.66585</td>
<td>0.62662</td>
<td>0.21618</td>
<td>1.74131</td>
</tr>
<tr>
<td>rvg409</td>
<td>25.0</td>
<td>4.0</td>
<td>18.0</td>
<td>Analysis</td>
<td>0.62845</td>
<td>0.63684</td>
<td>0.64084</td>
<td>0.21828</td>
<td>1.81235</td>
</tr>
<tr>
<td>rvg410</td>
<td>35.0</td>
<td>4.0</td>
<td>18.0</td>
<td>Analysis</td>
<td>0.63326</td>
<td>0.64324</td>
<td>0.61887</td>
<td>0.21210</td>
<td>1.80763</td>
</tr>
<tr>
<td>rvg411</td>
<td>30.0</td>
<td>3.0</td>
<td>18.0</td>
<td>Analysis</td>
<td>0.63266</td>
<td>0.64054</td>
<td>0.67356</td>
<td>0.22964</td>
<td>1.79437</td>
</tr>
<tr>
<td>rvg412</td>
<td>30.0</td>
<td>5.0</td>
<td>18.0</td>
<td>Analysis</td>
<td>0.64613</td>
<td>0.65443</td>
<td>0.61295</td>
<td>0.21409</td>
<td>1.73988</td>
</tr>
<tr>
<td>rvg413</td>
<td>30.0</td>
<td>4.0</td>
<td>12.0</td>
<td>Analysis</td>
<td>0.63150</td>
<td>0.64190</td>
<td>0.60965</td>
<td>0.21110</td>
<td>1.81368</td>
</tr>
<tr>
<td>rvg414</td>
<td>30.0</td>
<td>4.0</td>
<td>24.0</td>
<td>Analysis</td>
<td>0.63490</td>
<td>0.64341</td>
<td>0.64770</td>
<td>0.21940</td>
<td>1.80623</td>
</tr>
<tr>
<td>rvg415</td>
<td>30.0</td>
<td>4.0</td>
<td>18.0</td>
<td>Analysis</td>
<td>0.63535</td>
<td>0.64491</td>
<td>0.60281</td>
<td>0.20941</td>
<td>1.80081</td>
</tr>
<tr>
<td>rvg416</td>
<td>25.0</td>
<td>3.0</td>
<td>12.0</td>
<td>Exp. Data</td>
<td>0.6421</td>
<td>0.6605</td>
<td>0.5265</td>
<td>0.1969</td>
<td>1.7239</td>
</tr>
<tr>
<td>rvg417</td>
<td>25.0</td>
<td>5.0</td>
<td>24.0</td>
<td>Exp. Data</td>
<td>0.7053</td>
<td>0.7226</td>
<td>0.5556</td>
<td>0.2256</td>
<td>1.6170</td>
</tr>
<tr>
<td>rvg418</td>
<td>35.0</td>
<td>3.0</td>
<td>24.0</td>
<td>Exp. Data</td>
<td>0.6667</td>
<td>0.6867</td>
<td>0.5312</td>
<td>0.2008</td>
<td>1.7008</td>
</tr>
<tr>
<td>rvg419</td>
<td>35.0</td>
<td>5.0</td>
<td>12.0</td>
<td>Exp. Data</td>
<td>0.6838</td>
<td>0.7055</td>
<td>0.5382</td>
<td>0.2062</td>
<td>1.6788</td>
</tr>
</tbody>
</table>
# Micro-Ramp Oblique SWBL Flow Control

Comparison of CFD/Exp. and Experiment

Paired t-Test Results From Set (1)

<table>
<thead>
<tr>
<th>Response</th>
<th>MEAN</th>
<th>STDEV</th>
<th>t*</th>
<th>t(0.95,df)</th>
<th>LOW</th>
<th>HIGH</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTAVE</td>
<td>-0.0068</td>
<td>0.0187</td>
<td>0.3649</td>
<td>2.145</td>
<td>-0.0466</td>
<td>0.0332</td>
<td>Not Diff.</td>
</tr>
<tr>
<td>PFAVE</td>
<td>-0.0081</td>
<td>0.0238</td>
<td>0.4012</td>
<td>2.145</td>
<td>-0.0519</td>
<td>0.0356</td>
<td>Not Diff.</td>
</tr>
<tr>
<td>δ* (cm)</td>
<td>-0.0002</td>
<td>0.0252</td>
<td>0.0073</td>
<td>2.145</td>
<td>0.0542</td>
<td>0.0538</td>
<td>Not Diff.</td>
</tr>
<tr>
<td>θ (cm)</td>
<td>-0.0016</td>
<td>0.0045</td>
<td>0.3650</td>
<td>2.145</td>
<td>-0.0080</td>
<td>0.0112</td>
<td>Not Diff.</td>
</tr>
<tr>
<td>Htr</td>
<td>0.0179</td>
<td>0.0261</td>
<td>0.6861</td>
<td>2.145</td>
<td>-0.0384</td>
<td>0.0737</td>
<td>Not Diff.</td>
</tr>
</tbody>
</table>

Sample Difference,

\[ \Delta_j = (Y_{CFD} - Y_{Exp})_j \]

Mean of the Sample Difference,

\[ MEAN = \frac{1}{N} \sum_{j=1}^{n} \Delta_j \]

Standard Deviation of the Sample Difference,

\[ STDEV = \sqrt{\frac{(\Delta_j - \bar{\Delta})^2}{N-1}} \]
Combined Analysis/Experimental Data
D-Optimal Multi Level Design (2)

\[ Y = \sum_{i=1}^{N} A_i X_i + \sum_{i=1}^{N} \sum_{j=1}^{N} A_{ij} X_i X_j + \sum_{i=1}^{N} A_{ii} X_i^2 + Rn(df, S_{y.x}, X_1, X_2, ..., X_N) \]

Source (1) = CFD Analysis
15 DOE Experiments

Source (2) = Experimental Data
5 DOE Experiments
Micro-Ramp Oblique SWBL Flow Control
Combined Analysis/Experimental Data
Significant Regression Terms (2)

Main Effect:
$$A_{AA}X_A$$

First Order Interactions:
$$A_{SA}X_sX_A$$
$$A_{hA}X_hX_A$$
$$A_{cA}X_cX_A$$

Two Possible Second Order Interactions:
$$A_{chA}X_cX_hX_A$$
$$A_{hhA}(X_h)^2 X_A$$
## Combined Analysis/Experimental Data

#### D-Optimal Multi Level Design (2)

<table>
<thead>
<tr>
<th>Config.</th>
<th>s (mm)</th>
<th>h (mm)</th>
<th>c (mm)</th>
<th>$X_A$</th>
<th>PTAVE</th>
<th>PFAVE</th>
<th>$\delta^*$ (cm)</th>
<th>$\theta$ (cm)</th>
<th>Htr</th>
</tr>
</thead>
<tbody>
<tr>
<td>rvg300</td>
<td>______</td>
<td>______</td>
<td>______</td>
<td>Analysis</td>
<td>0.55746</td>
<td>0.67420</td>
<td>0.31805</td>
<td>0.10940</td>
<td>1.30205</td>
</tr>
<tr>
<td>rvg400</td>
<td>______</td>
<td>______</td>
<td>______</td>
<td>Analysis</td>
<td>0.63962</td>
<td>0.65320</td>
<td>0.54031</td>
<td>0.18391</td>
<td>1.89615</td>
</tr>
<tr>
<td>rvg401</td>
<td>25.0</td>
<td>3.0</td>
<td>12.0</td>
<td>Analysis</td>
<td>0.63219</td>
<td>0.64284</td>
<td>0.54972</td>
<td>0.19921</td>
<td>1.80391</td>
</tr>
<tr>
<td>rvg402</td>
<td>35.0</td>
<td>3.0</td>
<td>12.0</td>
<td>Analysis</td>
<td>0.62456</td>
<td>0.63632</td>
<td>0.60199</td>
<td>0.20737</td>
<td>1.83447</td>
</tr>
<tr>
<td>rvg403</td>
<td>25.0</td>
<td>5.0</td>
<td>12.0</td>
<td>Analysis</td>
<td>0.64658</td>
<td>0.65530</td>
<td>0.61546</td>
<td>0.21740</td>
<td>1.75787</td>
</tr>
<tr>
<td>rvg404</td>
<td>35.0</td>
<td>5.0</td>
<td>12.0</td>
<td>Analysis</td>
<td>0.64087</td>
<td>0.64907</td>
<td>0.58951</td>
<td>0.21108</td>
<td>1.75554</td>
</tr>
<tr>
<td>rvg405</td>
<td>25.0</td>
<td>3.0</td>
<td>24.0</td>
<td>Analysis</td>
<td>0.62279</td>
<td>0.63195</td>
<td>0.64810</td>
<td>0.21870</td>
<td>1.83855</td>
</tr>
<tr>
<td>rvg406</td>
<td>35.0</td>
<td>3.0</td>
<td>24.0</td>
<td>Analysis</td>
<td>0.62399</td>
<td>0.63431</td>
<td>0.65956</td>
<td>0.22384</td>
<td>1.83640</td>
</tr>
<tr>
<td>rvg407</td>
<td>25.0</td>
<td>5.0</td>
<td>24.0</td>
<td>Analysis</td>
<td>0.65325</td>
<td>0.65770</td>
<td>0.65920</td>
<td>0.22850</td>
<td>1.72099</td>
</tr>
<tr>
<td>rvg408</td>
<td>35.0</td>
<td>5.0</td>
<td>24.0</td>
<td>Analysis</td>
<td>0.65805</td>
<td>0.66585</td>
<td>0.62662</td>
<td>0.21618</td>
<td>1.74131</td>
</tr>
<tr>
<td>rvg409</td>
<td>25.0</td>
<td>4.0</td>
<td>18.0</td>
<td>Analysis</td>
<td>0.62845</td>
<td>0.63684</td>
<td>0.64084</td>
<td>0.21828</td>
<td>1.81235</td>
</tr>
<tr>
<td>rvg410</td>
<td>35.0</td>
<td>4.0</td>
<td>18.0</td>
<td>Analysis</td>
<td>0.63326</td>
<td>0.64324</td>
<td>0.61887</td>
<td>0.21210</td>
<td>1.80763</td>
</tr>
<tr>
<td>rvg411</td>
<td>30.0</td>
<td>3.0</td>
<td>18.0</td>
<td>Analysis</td>
<td>0.63266</td>
<td>0.64054</td>
<td>0.67356</td>
<td>0.22964</td>
<td>1.79437</td>
</tr>
<tr>
<td>rvg412</td>
<td>30.0</td>
<td>5.0</td>
<td>18.0</td>
<td>Analysis</td>
<td>0.64613</td>
<td>0.65443</td>
<td>0.61295</td>
<td>0.21409</td>
<td>1.73988</td>
</tr>
<tr>
<td>rvg413</td>
<td>30.0</td>
<td>4.0</td>
<td>12.0</td>
<td>Analysis</td>
<td>0.63150</td>
<td>0.64190</td>
<td>0.60965</td>
<td>0.21110</td>
<td>1.81368</td>
</tr>
<tr>
<td>rvg414</td>
<td>30.0</td>
<td>4.0</td>
<td>24.0</td>
<td>Analysis</td>
<td>0.63490</td>
<td>0.64341</td>
<td>0.64770</td>
<td>0.21940</td>
<td>1.80623</td>
</tr>
<tr>
<td>rvg415</td>
<td>30.0</td>
<td>4.0</td>
<td>18.0</td>
<td>Analysis</td>
<td>0.63535</td>
<td>0.64491</td>
<td>0.60281</td>
<td>0.20941</td>
<td>1.80081</td>
</tr>
<tr>
<td>rvg416</td>
<td>25.0</td>
<td>3.0</td>
<td>12.0</td>
<td>Exp. Data</td>
<td>0.6421</td>
<td>0.6605</td>
<td>0.5265</td>
<td>0.1969</td>
<td>1.7239</td>
</tr>
<tr>
<td>rvg417</td>
<td>25.0</td>
<td>5.0</td>
<td>24.0</td>
<td>Exp. Data</td>
<td>0.7053</td>
<td>0.7226</td>
<td>0.5556</td>
<td>0.2256</td>
<td>1.6170</td>
</tr>
<tr>
<td>rvg418</td>
<td>35.0</td>
<td>3.0</td>
<td>24.0</td>
<td>Exp. Data</td>
<td>0.6667</td>
<td>0.6867</td>
<td>0.5312</td>
<td>0.2008</td>
<td>1.7008</td>
</tr>
<tr>
<td>rvg419</td>
<td>35.0</td>
<td>5.0</td>
<td>12.0</td>
<td>Exp. Data</td>
<td>0.6838</td>
<td>0.7055</td>
<td>0.5382</td>
<td>0.2062</td>
<td>1.6788</td>
</tr>
<tr>
<td>rvg420</td>
<td>30.0</td>
<td>4.0</td>
<td>18.0</td>
<td>Exp. Data</td>
<td>0.6900</td>
<td>0.7131</td>
<td>0.5282</td>
<td>0.2037</td>
<td>1.6532</td>
</tr>
</tbody>
</table>
### Micro-Ramp Oblique SWBL Flow Control CFD/Exp. DOE Prediction Data Set (2) Statistically Significant Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Coeff.</th>
<th>p-Value</th>
<th>% Signif.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.642</td>
<td>0.0001</td>
<td>99.99</td>
</tr>
<tr>
<td>h</td>
<td>0.009639</td>
<td>0.0001</td>
<td>99.99</td>
</tr>
<tr>
<td>Xa</td>
<td>0.07104</td>
<td>0.0001</td>
<td>99.99</td>
</tr>
<tr>
<td>s*c</td>
<td>0.002908</td>
<td>0.0119</td>
<td>98.81</td>
</tr>
<tr>
<td>s*Xa</td>
<td>0.006285</td>
<td>0.0033</td>
<td>99.67</td>
</tr>
<tr>
<td>h*c</td>
<td>0.00401</td>
<td>0.0017</td>
<td>99.83</td>
</tr>
<tr>
<td>h*Xa</td>
<td>0.01349</td>
<td>0.0001</td>
<td>99.99</td>
</tr>
<tr>
<td>c*Xa</td>
<td>0.0183</td>
<td>0.0001</td>
<td>99.99</td>
</tr>
<tr>
<td>s²</td>
<td>______</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>h²</td>
<td>0.004711</td>
<td>0.0087</td>
<td>99.13</td>
</tr>
<tr>
<td>h²*Xa</td>
<td>-0.02405</td>
<td>0.0001</td>
<td>99.99</td>
</tr>
</tbody>
</table>

#### Total Pressure Recovery, PFAVE

<table>
<thead>
<tr>
<th>Term</th>
<th>Coeff.</th>
<th>p-Value</th>
<th>% Signif.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>1.803</td>
<td>0.0001</td>
<td>99.99</td>
</tr>
<tr>
<td>h</td>
<td>-0.03722</td>
<td>0.0001</td>
<td>99.99</td>
</tr>
<tr>
<td>Xa</td>
<td>-0.1501</td>
<td>0.0001</td>
<td>99.99</td>
</tr>
<tr>
<td>s*c</td>
<td>______</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>s*Xa</td>
<td>______</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>h*c</td>
<td>-0.01053</td>
<td>0.0065</td>
<td>99.35</td>
</tr>
<tr>
<td>h*Xa</td>
<td>______</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>c*Xa</td>
<td>-0.02122</td>
<td>0.0024</td>
<td>94.76</td>
</tr>
<tr>
<td>s²</td>
<td>0.01219</td>
<td>0.0907</td>
<td>90.93</td>
</tr>
<tr>
<td>h²</td>
<td>-0.03069</td>
<td>0.0006</td>
<td>99.94</td>
</tr>
<tr>
<td>h²*Xa</td>
<td>0.04543</td>
<td>0.0082</td>
<td>99.18</td>
</tr>
</tbody>
</table>

#### Transformed Shape Factor, Htr
Micro-Ramp Oblique SWBL Flow Control
Comparison of CFD/Exp. and Experiment
Paired t-Test Results From Set (2)

<table>
<thead>
<tr>
<th>Response</th>
<th>MEAN</th>
<th>STDEV</th>
<th>t*</th>
<th>t(0.95,df)</th>
<th>LOW</th>
<th>HIGH</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTAVE</td>
<td>0.0002</td>
<td>0.0142</td>
<td>0.0123</td>
<td>2.145</td>
<td>-0.0301</td>
<td>0.0304</td>
<td>Not Diff.</td>
</tr>
<tr>
<td>PFAVE</td>
<td>0.0001</td>
<td>0.0155</td>
<td>0.0052</td>
<td>2.145</td>
<td>-0.0334</td>
<td>0.0332</td>
<td>Not Diff.</td>
</tr>
<tr>
<td>δ* (cm)</td>
<td>0.0021</td>
<td>0.0181</td>
<td>0.1152</td>
<td>2.145</td>
<td>-0.0367</td>
<td>0.0408</td>
<td>Not Diff.</td>
</tr>
<tr>
<td>θ (cm)</td>
<td>0.0010</td>
<td>0.0044</td>
<td>0.2192</td>
<td>2.145</td>
<td>-0.0085</td>
<td>0.0104</td>
<td>Not Diff.</td>
</tr>
<tr>
<td>Htr</td>
<td>0.0003</td>
<td>0.0119</td>
<td>0.0241</td>
<td>2.145</td>
<td>-0.0252</td>
<td>0.0257</td>
<td>Not Diff.</td>
</tr>
</tbody>
</table>

Sample Difference,

$$\Delta_j = (Y_{CFD} - Y_{Exp})_j$$

Mean of the Sample Difference,

$$\text{MEAN} = \frac{1}{N} \sum_{j=1}^{n} \Delta_j$$

Standard Deviation of the Sample Difference,

$$\text{STDEV} = \sqrt{\frac{(\Delta_j - \bar{\Delta})^2}{(N-1)}}$$
Micro-Ramp Oblique SWBL Flow Control
Comparison of CFD/Exp. and Experiment Total/Pitot Pressure Correlation

Correlation Coefficient, $r = 0.998$

- Least Square Regression of Analyses Data
- 95% Confidence Interval on Analyses Data
- DOE Analyses Data

CFD/Exp Data Set (2)
Correlation Coefficient, $r = 0.998$

Experimental Data Set
Correlation Coefficient, $r = 0.995$
Micro-Ramp Oblique SWBL Flow Control
Comparison of CFD/Exp. and Experiment
Shape Factor/Total Pressure Correlation

Correlation Coefficient, $r = 0.938$
- Least Square Regression of Analyses Data
- 95% Confidence Interval on Analyses Data

Correlation Coefficient, $r = 0.738$
- Least Square Regression of Analyses Data
- 95% Confidence Interval on Analyses Data

CFD/Exp Data Set (2)
Correlation Coefficient, $r = 0.938$

Experimental Data Set
Correlation Coefficient, $r = 0.738$
Micro-Ramp Oblique SWBL Flow Control

$s_{opt} = 25.0 \, \text{mm}$, $h_{opt} = 5.0 \, \text{mm}$, $c_{opt} = 24.0 \, \text{mm}$

Optimal Design Point

<table>
<thead>
<tr>
<th>Source</th>
<th>Response</th>
<th>Y</th>
<th>-95.0%Y</th>
<th>+95.0%Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFD/Exp. (2)</td>
<td>PTAVE</td>
<td>0.70530</td>
<td>0.69468</td>
<td>0.71593</td>
</tr>
<tr>
<td></td>
<td>PFAVE</td>
<td>0.72269</td>
<td>0.71458</td>
<td>0.73104</td>
</tr>
<tr>
<td></td>
<td>$\delta^* (\text{cm})$</td>
<td>0.54165</td>
<td>0.49242</td>
<td>0.59051</td>
</tr>
<tr>
<td></td>
<td>$\theta (\text{cm})$</td>
<td>0.22115</td>
<td>0.20512</td>
<td>0.23718</td>
</tr>
<tr>
<td></td>
<td>Htr</td>
<td>1.61115</td>
<td>1.58008</td>
<td>1.64223</td>
</tr>
<tr>
<td>Experiment</td>
<td>PTAVE</td>
<td>0.70110</td>
<td>0.68023</td>
<td>0.72198</td>
</tr>
<tr>
<td></td>
<td>PFAVE</td>
<td>0.72003</td>
<td>0.69674</td>
<td>0.74327</td>
</tr>
<tr>
<td></td>
<td>$\delta^* (\text{cm})$</td>
<td>0.53766</td>
<td>0.51496</td>
<td>0.56090</td>
</tr>
<tr>
<td></td>
<td>$\theta (\text{cm})$</td>
<td>0.22151</td>
<td>0.21379</td>
<td>0.22923</td>
</tr>
<tr>
<td></td>
<td>Htr</td>
<td>1.61754</td>
<td>1.59448</td>
<td>1.64069</td>
</tr>
</tbody>
</table>
Micro-Ramp Oblique SWBL Flow Control
Comparison of Two DOE Models
CFD/Exp. and Experiment

\[
t^* = \frac{|Y_{\text{CFD}/\text{Exp}} - Y_{\text{Exp}}|}{\sqrt{\left(\frac{Y_A - Y_{\text{CFD}/\text{Exp}}}{t_{\text{CFD}/\text{Exp}}(0.975, df_{\text{CFD}/\text{Exp}})}\right)^2 + \left(\frac{Y_B - Y_{\text{Exp}}}{t_{\text{Exp}}(0.975, df_{\text{Exp}})}\right)^2}}
\]

\[Y_{\text{CFD}/\text{Exp}} = \text{Response from model (CFD/Exp)}\]
\[Y_{\text{Exp}} = \text{Response from model (Exp)}\]
\[Y_A = +95.0\% \text{ interval from response model (CFD/Exp)}\]
\[Y_B = +95.0\% \text{ interval from response model (Exp)}\]
Micro-Ramp Oblique SWBL Flow Control
Comparison of Two DOE Models
CFD/Exp. and Experiment

If $t^* > t(0.975, df)$, DOE models are different
If $t^* < t(0.975, df)$, DOE models are not different
Where $t(0.975, df)$ is based on the pooled degrees of freedom:

$$
df = \frac{\left(\frac{S_{CFD/Exp}^2}{df_{CFD/Exp}} + \frac{S_{Exp}^2}{df_{Exp}}\right)^2}{\frac{S_{CFD/Exp}^4}{df_{CFD/Exp}^3} + \frac{S_{Exp}^4}{df_{Exp}^3}}
$$
## Micro-Ramp Oblique SWBL Flow Control
Comparison of Two DOE Models
Optimal Design Point

<table>
<thead>
<tr>
<th>Response</th>
<th>$Y_{CFD/EXP}$</th>
<th>$Y_{Exp}$</th>
<th>$S_{y.x}$</th>
<th>$t^*$</th>
<th>df</th>
<th>t(0.975,df)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTAVE</td>
<td>0.7053</td>
<td>0.7011</td>
<td>0.0109</td>
<td>0.3854</td>
<td>23</td>
<td>2.069</td>
<td>Not Diff.</td>
</tr>
<tr>
<td>PFAVE</td>
<td>0.7227</td>
<td>0.7200</td>
<td>0.0112</td>
<td>0.2307</td>
<td>20</td>
<td>2.088</td>
<td>Not Diff.</td>
</tr>
<tr>
<td>$\delta^*$ (cm)</td>
<td>0.5416</td>
<td>0.5377</td>
<td>0.0249</td>
<td>0.1600</td>
<td>22</td>
<td>2.074</td>
<td>Not Diff.</td>
</tr>
<tr>
<td>$\theta$ (cm)</td>
<td>0.2212</td>
<td>0.2215</td>
<td>0.0083</td>
<td>0.0432</td>
<td>23</td>
<td>2.069</td>
<td>Not Diff.</td>
</tr>
<tr>
<td>Htr</td>
<td>1.6112</td>
<td>1.6175</td>
<td>0.0201</td>
<td>0.3185</td>
<td>22</td>
<td>2.074</td>
<td>Not Diff.</td>
</tr>
</tbody>
</table>
Micro-Ramp Oblique SWBL Flow Control
Total Pressure Response Surface
hxc Statistical Interaction

CFD/Exp. DOE Data Set (2)
Experimental DOE Data Set
Micro-Ramp Oblique SWBL Flow Control
Displacement Thickness Response Surface
hxc Statistical Interaction

CFD/Exp. DOE Data Set (2)  Experimental DOE Data Set
Micro-Ramp Oblique SWBL Flow Control
Momentum Thickness Response Surface
hxc Statistical Interaction

CFD/Exp. DOE Data Set (2)

Experimental DOE Data Set
Micro-Ramp Oblique SWBL Flow Control
Shape Factor Response Surface
hxc Statistical Interaction

CFD/Exp. DOE Data Set (2)  Experimental DOE Data Set
Advanced Flow Control for Supersonic Inlets

Conclusions

• A CFD analysis data set and a smaller set of experimental data was combined into a single data set and analyzed to gain an efficiency and cost effectiveness advantage.

• There was no statistical difference between the CFD/Exp. data set and the complete experimental data set.

• There was no statistical difference between the DOE model generated by the CFD/Exp. data set and the DOE model generated by the complete experimental data set.
Advanced Flow Control for Supersonic Inlets
Concluding Advice on Data Scaling

• Care must be exercised in choosing both the CFD data set and the smaller experimental data set in order to maintain an average error of prediction ratio (EOPR) close to 1.0 for the combined data set.

• Choose only the statistically significant terms in the regression model during the DOE analysis.

• Choose a linear approach to data scaling by starting from the main and first order interaction effects and adding higher order interaction terms as statistically necessary.