Pion Total Cross Section in Nucleon - Nucleon Collisions

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Nomenclature

Throughout this paper, natural units are used, where $\hbar \equiv c \equiv 1$.

$E_{\text{lab}}$ Total energy of the projectile in the lab frame, in units of MeV or GeV.

$T_{\text{lab}}$ Kinetic energy of the projectile in the lab frame, in units of MeV or GeV.

$p_{\text{lab}}$ Momentum of the projectile in the lab frame, in units of MeV or GeV.

$m$ Projectile mass, in units of MeV or GeV.

$\sqrt{s}$ Center of momentum total energy, in units of MeV or GeV.

$p$ Symbol for proton.

$n$ Symbol for neutron.

d Symbol for deuteron (bound state of neutron and proton).

$X$ Generic symbol for unspecified particles.

$\pi$ Symbol for pion.

$\pi^+$ Symbol for positively charged pion.

$\pi^-$ Symbol for negatively charged pion.

$\pi^\pm$ Symbol for charged pions.

$\pi^0$ Symbol for neutral pion.

$\sigma$ Pion production total cross section, in units of mb.

$\sigma_{pp \rightarrow \pi X}$ Total cross section for the reaction $p + p \rightarrow \pi + X$. 
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Abstract

Total cross section parameterizations for neutral and charged pion production in nucleon - nucleon collisions are compared to experimental data over the projectile momentum range from threshold to 300 GeV. Both proton - proton and proton - neutron reactions are considered. Overall excellent agreement between parameterizations and experiment is found, except for notable disagreements near threshold. In addition, the hypothesis that the neutral pion production cross section can be obtained from the average charged pion cross section is checked. The theoretical formulas presented in the paper obey this hypothesis for projectile momenta below 500 GeV. The results presented provide a test of engineering tools used to calculate the pion component of space radiation.

1 Introduction

Hadron production in nuclear and nucleon collisions can be a significant source of space radiation, especially for planetary atmospheres. The most important hadron is the pion, because it has the lowest threshold for production. While total production cross sections are not used in transport equations, it is nevertheless worthwhile to understand them thoroughly because integrated differential cross sections must always equal total cross sections. The aim of this research is to obtain an excellent parameterization of pion production cross sections from nucleon - nucleon and nucleus - nucleus collisions. A good understanding of the energy dependence and nuclear mass dependence will then provide a further test of the accuracy of differential cross sections, when they are integrated. In addition, total cross sections may be used to generate differential cross sections. The method for generating these differential cross sections is outlined in reference [1]. The idea is to multiply the total cross section by functions that give the shape of the differential cross section. One then multiplies by a normalizing factor, so that the result equals the total cross section when integrated. This method is useful if the shape of the differential cross section is accurately known, because the integrated differential cross section is guaranteed to give the correct total cross section. The total cross section parameterizations developed previously [2], were compared to only nine experimental data points all above 10 GeV. The cosmic ray spectrum peaks near the pion threshold at 280 MeV, and therefore it is worthwhile to compare to more low energy data. The present work compares parameterizations to over thirty experimental data points ranging from threshold to about 300 GeV. Finally, an approximation that one often sees in the literature is that the neutral pion cross section can be obtained from the average of the charged pion cross sections. This hypothesis will tested.
2 Proton - proton reactions

The total inclusive cross section for pion production in proton - proton collisions has been parameterized as [2]

\[
\sigma_{pp \rightarrow \pi^{+}X} = \left(0.00717 + 0.0652 \frac{\log T_{\text{lab}}}{T_{\text{lab}}} + 0.162 \frac{T_{\text{lab}}}{T_{\text{lab}}^{2}}\right)^{-1},
\]

(1)

\[
\sigma_{pp \rightarrow \pi^{-}X} = \left(0.00456 + 0.0846 \frac{T_{\text{lab}}^{0.5}}{T_{\text{lab}}^{1.5}} + 0.577 \right)^{-1},
\]

(2)

\[
\sigma_{pp \rightarrow \pi^{0}X} = \left(0.007 + 0.1 \frac{\log T_{\text{lab}}}{T_{\text{lab}}} + 0.3 \frac{T_{\text{lab}}}{T_{\text{lab}}^{2}}\right)^{-1},
\]

(3)

where \(T_{\text{lab}}\) is the kinetic energy of the projectile, which is given in units of GeV to give \(\sigma\) in units of mb. These cross sections are plotted in Figures 1 - 4 as a function of \(p_{\text{lab}}\), which is the momentum of the projectile. These quantities are related via

\[
T_{\text{lab}} = E_{\text{lab}} - m = \sqrt{p_{\text{lab}}^{2} + m^{2}} - m,
\]

(4)

where \(m\) is the projectile mass. In equations (1) - (3), the cross sections are set equal to zero for energies below threshold. The thresholds are shown in Table 1.

An interesting aspect of pion production is that the following relation holds.

\[
\sigma_{pp \rightarrow \pi^{0}X} \approx \frac{1}{2} \left(\sigma_{pp \rightarrow \pi^{-}X} + \sigma_{pp \rightarrow \pi^{+}X}\right),
\]

(5)

which is shown as the black dotted curve in Figures 1 - 4, which closely follows the red solid curve (cross section for \(\pi^{0}\)), and shows that equation (5) works very well for all energies. Figure 4 shows that nonphysical behavior sets in at about 500 GeV, where the curves intersect and cross over each other. This phenomenon is nonphysical because the \(\pi^{-}\) cross section should not be larger than the \(\pi^{+}\) cross section. The parameterizations should not be used above this energy.

The parameterizations in equations (1) - (3) are compared to experimental data in Figures 5 - 10, which are shown as both linear and logarithmic plots. The reason for this is seen clearly, for example, in Figures 5 and 6. Figure 6 seems to show overall very good agreement between the parameterization and experiment. However, when plotted on a logarithmic scale, as in Figure 5 one can now see poor agreement for the data point at 1.38 GeV, which was not apparent on the linear plot.

In reference [2], the parameterizations of equations (1) - (3) were compared only to three data points from Whitmore [3] at \(p_{\text{lab}} = 12, 19\) and 24 GeV. In the present work, parameterizations are also compared to the Whitmore data at 69, 102, 205 and 303 GeV, and data from other authors [4, 5]. More importantly, parameterizations are also compared to some other low energy data [6, 7, 8, 9, 10]. Comparison is made to the data point at \(p_{\text{lab}} = 954\) MeV, corresponding to \(T_{\text{lab}} = 400\) MeV, of Abd El-Samad et al. [11]. The cross section for the reaction \(pp \rightarrow pp\pi^{0}\) is 0.1 mb. Even though this is an exclusive final state, the exclusive cross section is equal to the
inclusive cross section because the energy is below the two pion threshold. See Table 1. The complete set of experimental data is listed in Tables 2, 3, and 4. Note that Booth et al. [4] only report the cross section for \( \pi^+ \) plus \( \pi^- \) production, defined as \( \sigma_{\pi^\pm} \) at \( p_{\text{lab}} = 8.8 \) GeV. Therefore, the individual \( \pi^+ \) and \( \pi^- \) cross sections listed in Table 3 represent the experimental cross section of Booth et al. [4], but with the theoretically calculated individual pion cross section subtracted off using equations 1 and 2, i.e.

\[
\sigma_{\pi^+} (\text{Table 3}) \equiv \sigma_{\pi^\pm} (\text{Booth}) - \sigma_{\pi^-} (\text{equation 2}), \tag{6}
\]

\[
\sigma_{\pi^-} (\text{Table 3}) \equiv \sigma_{\pi^\pm} (\text{Booth}) - \sigma_{\pi^+} (\text{equation 1}). \tag{7}
\]

Figures 5 and 6 show the comparison of equation (1) to the experimental data. The \( p + p \to \pi^+ + X \) threshold occurs at \( T_{\text{lab}} = 289 \) MeV, or \( p_{\text{lab}} = 791 \) MeV, because this is the lowest energy reaction for which a \( \pi^+ \) particle is produced. See Table 1. Comparison between the parameterization and experiment is excellent for high energies up to 300 GeV and down to 3.67 GeV. However, at the lower energy of 1.38 GeV, which is closer to threshold, there is significant disagreement between the parameterization and experiment, with with the experimentally obtained cross sections being much larger than those obtained from the parameterization. Norbury et al. [12] have recently shown that cross sections near threshold are important because they are enhanced due to larger cosmic ray flux. Figures 7 and 8 show the comparison of equation (2) to the experimental data. The \( p + p \to \pi^- + X \) threshold occurs at \( T_{\text{lab}} = 600 \) MeV, or \( p_{\text{lab}} = 1219 \) MeV (see Table 1). These thresholds are much larger than for \( \pi^+ \) production and explains why the \( \pi^+ \) cross section is always larger than the \( \pi^- \) cross section, as shown in Figure 1. The \( \pi^- \) cross section should never be larger than the \( \pi^+ \) cross section because more energy is required to produce \( \pi^- \) particles. Comparison between the parameterization and experiment is excellent for high energies up to 300 GeV and down to 3.67 GeV. However, at the lower energy of 1.38 GeV, there is significant disagreement between the parameterization and experiment, this time with the experimentally obtained cross sections being much smaller than those obtained from the parameterization. Note that now the momentum of 1.38 GeV is very close to the \( \pi^- \) threshold at 1.219 GeV, and is considered a near threshold cross section [12]. Figures 9 and 10 show the comparison of equation (3) to the experimental data. The \( p + p \to \pi^0 + X \) threshold occurs at \( T_{\text{lab}} = 280 \) MeV, or \( p_{\text{lab}} = 777 \) MeV. See Table 1. Comparison between the parameterization and experiment is excellent, but there is significant disagreement at the high momentum of 300 GeV. The near threshold cross section parameterization is significantly larger than experiment.

## 3 Neutron - neutron reactions

Examining the reactions shown in Table 1, reveals that the neutron - neutron (nn) cross section for pion production should be the same as for the proton - proton (pp) reaction, if one ignores isospin. While Coulomb effects may be important at low energies, the reactions should be equal at higher energies. Therefore,

\[
\sigma_{\text{nn} \to \pi X} \approx \sigma_{\text{pp} \to \pi X} \tag{8}
\]

for all pion species. Unfortunately, there is no data to test this.
4 Neutron - proton reactions

Consideration of the reactions shown in Table 1, plus consideration of multiple pion reactions [13] suggests that, at low energy, the neutron - proton (np) cross section might be a simple multiple of the pp cross section. The reaction, \( n + p \rightarrow \pi^- + X \), has been studied by Abdivaliev [10] and the data are shown in Figure 7, compared to the following simple parameterization (dashed line) of the np cross section,

\[
\sigma_{\text{np} \rightarrow \pi^- X} = 2.5 \sigma_{\text{pp} \rightarrow \pi^- X},
\]

where \( \sigma_{\text{pp} \rightarrow \pi^- X} \) is given in equation (2). The np experimental cross sections for other pion species (\( \pi^+ \) and \( \pi^0 \)) are not available, and parameterizations cannot be developed.

At higher energies, one may expect pion production cross sections for all pp, np and nn reactions to become equal given the proliferation of multiple pion production. This expectation seems to be verified by the two data points of Azimov [8] at 300 GeV shown in Figure 10. The open triangle is for the reaction \( p + p \rightarrow \pi^0 + X \), and the solid square is for \( p + n \rightarrow \pi^0 + X \). The experimental cross section in this high energy region, near 300 GeV, seems to obey the relation

\[
\sigma_{\text{pn} \rightarrow \pi^0 X} \approx \sigma_{\text{pp} \rightarrow \pi^0 X}.
\]

Finally, one would expect

\[
\sigma_{\text{np} \rightarrow \pi X} = \sigma_{\text{pn} \rightarrow \pi X}
\]

to hold at all energies for all pion species.

5 Conclusions

Total cross section parameterizations for neutral and charged pion production in nucleon - nucleon collisions have been compared to an extensive set of experimental data over the projectile momentum range from threshold to 300 GeV. For \( \pi^\pm \) production, the disagreement between the parameterization and experiment near threshold at \( p_{\text{lab}} = 1.38 \) GeV needs to be resolved. This disagreement is not obvious in the linear plots, but shows up clearly in the logarithmic plots. For \( \pi^0 \) production, the disagreement between the parameterization and experiment at \( p_{\text{lab}} = 300 \) GeV needs to be resolved. There is a need for more experimental measurements of pn cross sections for all pion species at low and high energy. In spite of these disagreements, there is excellent overall agreement between parameterizations and experiment. Above the energy region near threshold, the parameterizations provide accurate cross section predictions. The results can be used to validate engineering tools used to calculate the pion component of space radiation.
References


[8] See data listing for the reaction $p + p \rightarrow \pi^0 + X$, for author named Azimov at http://durpdg.dur.ac.uk/HEPDATA/


Figure 1: Parameterized inclusive cross sections for pion production in proton - proton collisions given by equations (1) - (3). The green dotted line is $\sigma_{pp \rightarrow \pi^+ X}$. The red solid line is $\sigma_{pp \rightarrow \pi^0 X}$. The blue long dashed line is $\sigma_{pp \rightarrow \pi^- X}$. The black dotted curve is calculated from $\frac{1}{2}(\sigma_{pp \rightarrow \pi^+ X} + \sigma_{pp \rightarrow \pi^- X})$. Cross sections are set to zero for energies below threshold.

Figure 2: Same as Figure 1, except only the low energy region is shown.
Figure 3: Same as Figure 1, except only the low energy region is shown and the vertical scale is logarithmic.

Figure 4: Same as Figure 1, except a higher energy region is shown and the vertical and horizontal scales are logarithmic. Nonphysical behavior sets in at about 500 GeV.
Figure 5: Log-log plot of the $\pi^+$ total inclusive cross section parameterization (green, solid line) of equation (1) versus experimental data (triangle symbols) of references [3, 4, 6, 7, 9, 14].

Figure 6: Same as figure 5, except for use of linear axes.
Figure 7: Log-log plot of total inclusive cross section parameterization (blue, solid line) of equation (2) for the reaction $p + p \rightarrow \pi^- + X$ versus experimental data (triangle symbols) of references [3, 4, 7, 9, 14, ?, 10]. Also shown is the cross section data (solid squares) from Abdvaliev [10], for the reaction $n + p \rightarrow \pi^- + X$, compared to the parameterization (blue, dashed line) of equation (9).

Figure 8: Same as Figure 7, except for use of linear axes and n+p reaction is not shown.
Figure 9: Log-log plot of total inclusive cross section parameterization (red, solid line) of equation (3) for the reaction $p + p \rightarrow \pi^0 + X$ versus experimental data (triangle symbols) of references [3, 5, 8, 11, 14, 15, 17, 18, 19, 20].

Figure 10: Same as Figure 9, except for use of linear axes, and the $p + n \rightarrow \pi^0 + X$ data point of Azimov [8] (solid square at 300 GeV) is now shown. The data points for $p + p \rightarrow \pi^0 + X$ of Whitmore [3] (solid triangle at 303 GeV) and Azimov [8] (open diamond at 300 GeV) have been slightly shifted horizontally so that they do not overlap.
Table 1: Reaction thresholds. $T_{\text{lab}}$ is the kinetic energy threshold, $p_{\text{lab}}$ is the momentum threshold, and $\sqrt{s}$ is the center of momentum total energy at threshold. The mass of the deuteron is 1876.125 MeV. Particle $C$ is the produced particle of interest and is the first particle listed on the right hand side of each reaction. $X$ are the remaining particles. Note that cross sections for $n + p$ are the same as for $p + n$.

<table>
<thead>
<tr>
<th>A + B → C + X</th>
<th>$T_{\text{lab}}$ (MeV)</th>
<th>$p_{\text{lab}}$ (MeV)</th>
<th>$\sqrt{s}$ (MeV)</th>
</tr>
</thead>
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<tr>
<td>$p + p \rightarrow \pi^0 + p + p$</td>
<td>279.7</td>
<td>776.6</td>
<td>2011.5</td>
</tr>
<tr>
<td>$\pi^0 + \pi^0 + p + p$</td>
<td>578.7</td>
<td>1192.1</td>
<td>2146.5</td>
</tr>
<tr>
<td>$\pi^+ + p + n$</td>
<td>292.3</td>
<td>796.2</td>
<td>2017.4</td>
</tr>
<tr>
<td>$\pi^+ + d$</td>
<td>288.6</td>
<td>790.5</td>
<td>2015.7</td>
</tr>
<tr>
<td>$\pi^- + \pi^+ + p + p$</td>
<td>599.8</td>
<td>1218.7</td>
<td>2155.7</td>
</tr>
<tr>
<td>$n + n \rightarrow \pi^0 + n + n$</td>
<td>279.7</td>
<td>777.0</td>
<td>2014.1</td>
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<tr>
<td>$\pi^+ + \pi^- + n + n$</td>
<td>599.8</td>
<td>1219.3</td>
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Table 2: Cross section data for pion production in proton - neutron reactions.

<table>
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<th>Reaction</th>
<th>(T_{lab}) (GeV)</th>
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<th>(\sqrt{s}) (GeV)</th>
<th>(\sigma) (mb)</th>
<th>Reference</th>
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<td>(p + n \rightarrow \pi^0 + X)</td>
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<td>300</td>
<td>23.8</td>
<td>136 ± 10</td>
<td>[8]</td>
</tr>
<tr>
<td>(n + p \rightarrow \pi^- + X)</td>
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<td>1.25</td>
<td>2.17</td>
<td>1.45 ± 0.06</td>
<td>[10]</td>
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<tr>
<td></td>
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<td>1.73</td>
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<td>4.1 ± 0.1</td>
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<td>[10]</td>
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<tr>
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<td>21.0 ± 0.53</td>
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Table 3: Cross section data for $\pi^\pm$ production in proton-proton reactions.

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<th>$\sqrt{s}$ (GeV)</th>
<th>$\sigma$ (mb)</th>
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<td>1.38</td>
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<td>2.98</td>
<td>$6.0 \pm 1.2$</td>
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<td>8.8</td>
<td>4.3</td>
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<td>$30 \pm 0.6$</td>
<td>[3, ?]</td>
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<tr>
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<td>205</td>
<td>19.7</td>
<td>$86 \pm 2$</td>
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<tr>
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<td>303</td>
<td>23.9</td>
<td>$99.5 \pm 3.0$</td>
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</table>
Table 4: Cross section data for $\pi^0$ production in proton - proton reactions.

<table>
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<tr>
<th>Reaction</th>
<th>$T_{\text{lab}}$ (GeV)</th>
<th>$p_{\text{lab}}$ (GeV)</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\sigma$ (mb)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p + p \rightarrow \pi^0 + X$</td>
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<td>0.954</td>
<td>2.07</td>
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<td>19</td>
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<td>24</td>
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<td>$53.5 \pm 3.1$</td>
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<tr>
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<td>69</td>
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<td>400</td>
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</table>
Total cross section parameterizations for neutral and charged pion production in nucleon - nucleon collisions are compared to experimental data over the projectile momentum range from threshold to 300 GeV. Both proton - proton and proton - neutron reactions are considered. Overall excellent agreement between parameterizations and experiment is found, except for notable disagreements near threshold. In addition, the hypothesis that the neutral pion production cross section can be obtained from the average charged pion cross section is checked. The theoretical formulas presented in the paper obey this hypothesis for projectile momenta below 500 GeV. The results presented provide a test of engineering tools used to calculate the pion component of space radiation.