Method and system for monitoring and analyzing, in real time, variation with time of an aircraft flight parameter. A time-dependent recovery band, defined by first and second recovery band boundaries that are spaced apart at at least one time point, is constructed for a selected flight parameter and for a selected time recovery time interval length $\Delta t(FP;rec)$. A flight parameter, having a value $FP(t_0)$ at a time $t_0$, is likely to be able to recover to a reference flight parameter value $FP(t;ref)$, lying in a band of reference flight parameter values $FP(t;ref;CB)$, within a time interval given by $t_0 - t_0 \pm \Delta t(FP;rec)$, if (or only if) the flight parameter value lies between the first and second recovery band boundary traces.

20 Claims, 8 Drawing Sheets
Fig. 1A

Fig. 1B
Drag Appliance Activation

FIG. 2

FIG. 3
Fig. 4

Fig. 5A

Fig. 5B
Provide sequence of at least two FP reference trace values \( \{FP(t=t_{ref})\}_n \) for time values \( t=t_n \) and a recovery time interval length \( \Delta t(FP;\text{rec}) \).

Provide a constrained solution \( u(t;t_{ref}) \) for the FP for initial time values \( t=t_n \) with initial values \( u(t=t_n,t) \) displaced from the corresponding values \( FP(t=t_n,\text{ref}) \).

For each of at least two constrained solutions \( u(t;t_{ref}) \), provide a reference curve-crossing time value, \( t = t_{zc}(n) \), where \( u(t;t_{ref}) \) crosses the reference trace \( FP(t;t_{ref}) \).

Compute coordinate pair \( (t_{zc}(n)-\Delta t(FP;\text{rec}), u(t_{zc}(n)-\Delta t(FP;\text{rec});t_n) \) for at least two time values \( t=t_n \), and identify each of these coordinate pairs with at least one point on a recovery band boundary.

Connect at least two points with coordinates \( (t_{zc}(n)-\Delta t(FP;\text{rec}), u(t_{zc}(n)-\Delta t(FP;\text{rec});t_n) \) to provide a portion of a recovery band boundary trace (optional).

FIG. 6
Provide a recovery time interval length $\Delta t_{(FP;rec)}$ and corresponding recovery band boundary traces $FP(t_{RB;1})$ and $FP(t_{RB;2})$ ($> FP(t_{RB;1})$) for a selected flight parameter $FP$ in a selected phase of flight.

Provide a selected present value $FP(t_m)$ of the flight parameter $FP$.

Is $FP(t_m)$ within, or on a boundary of, the recovery band?

- **No**: Flight parameter $FP$ is unlikely to recover to the reference curve or reference band within a recovery time interval length, beginning at the present time $t \approx t_m$.
  - Recommend at least one action to subsequently bring the flight parameter value within the recovery band (optional).

- **Yes**: Flight parameter $FP$ can recover to the reference curve or reference band within a recovery time interval length, beginning at the present time $t \approx t_m$.
  - Display recovery band for $FP$ and one or more contemporaneous value(s) $FP(t_m)$ with reference to the recovery band (optional).

**FIG. 7**
Fig. 14

Fig. 15

Fig. 16
REAL TIME CORRECTION OF AIRCRAFT FLIGHT CONFIGURATION

ORIGIN OF THE INVENTION

This invention was made by an employee of the U.S. government. The U.S. government has the right to make, use and/or sell the invention described herein without payment of compensation therefor, including but not limited to payment of royalties.

FIELD OF THE INVENTION

This invention relates to real time monitoring, analysis and display of aircraft flight configuration variables.

BACKGROUND OF THE INVENTION

An aircraft that is ascending following takeoff or descending on approach will have measurable kinetic energy and potential energy components, and these components will change with time in measurable manners. Desirable energy states for both takeoff and landing can be determined from aircraft manufacturer guidance for these phases of flight. For example, where the approach occurs at an airport with an operable and reliable instrument landing system (ILS), the ILS may provide data recorded on the aircraft, to serve as a standard for comparing observed kinetic and potential energy components for an aircraft near the ground, below 2,500 feet altitude and for an assumed straight glide slope path to a touchdown site. If the airport has no operable and reliable ILS, or if the aircraft is not near the ground, another mechanism for providing a standard for measurements or estimates is needed. On takeoff, where no electronic guidance comparable to a guide slope sensor is available, the aircraft climb profile can be compared to manufacturer guidance or to observed performance for recorded aircraft departures from the particular airport.

The airline industry has become concerned with the problem of unstable aircraft approaches, because approach and landing accidents often begin as unstable approaches. An “unstable approach” is often defined as an approach where, below a threshold altitude (1,000 feet for IFR and 500 feet for VFR), the aircraft is not established on a proper glide slope path and with a proper air speed, with a stable descent rate and engine power setting, and with a proper landing configuration (landing gear and flaps extended). Airlines have developed approach procedures that call for abandonment of an approach that is determined to be unstable.

Development and testing of methods for detecting atypical flights by N.A.S.A. has revealed that high aircraft energy arrivals were detected (after touchdown); and the resulting unstable approaches were found to occur more frequently than the recoveries.

It may be possible to identify a first class of high energy arrivals where recovery and subsequent stabilization is possible and relatively easy, and to identify a second class of high energy arrivals in which recovery and subsequent stabilization is likely to be difficult or impossible. However, the present procedures for determining presence of a normal approach include an electronic glide slope that extends linearly from the end of a target runway to the aircraft, whereas a typical (normal) aircraft approach path is curved and follows the electronic glide slope only from about 1,800 feet above the field to the end of the runway.

A 3-to-1 glide path slope, corresponding to decrease of 1,000 feet in altitude for every 3 nautical miles horizontal travel, is often desirable during an arrival phase. The air speed is 250 knots or less by regulation below 10,000 feet, and the aircraft decelerates to a reference speed before joining the approach path. These parameters are directly available but are unlikely to prove to be the only relevant parameters in determining whether a flight arrival phase is normal or other than normal.

Presently, analysis of the possibility of a normal aircraft approach or other normal aircraft maneuver is provided after the fact, in an off-line comparison of the present aircraft flight parameter value(s) with a collection of corresponding reference flight parameter values that provide a “normal” aircraft maneuver. This off-line approach might be improved upon by reference to charts and tables, provided in the aircraft cabin, but the time required for such comparisons is realistically measured in minutes, and this does not allow for a real time response.

What is needed is a system for monitoring, analyzing and displaying, in real time, a present flight configuration, of an aircraft and comparison of this configuration with one or more reference flight configurations for the same or similar circumstances. Preferably, the system should include the effects of aircraft performance constraints and should indicate whether a reference flight configuration can be achieved within a selected time interval, which may vary from one flight parameter to another and from one flight phase to another.

SUMMARY OF THE INVENTION

These needs are met by the invention, which provides measurements or estimates of present flight parameter (FP) values and compares the present value(s) with corresponding reference flight parameter values for similar circumstances. For a given flight parameter and a reference (ideal) trace or reference band for that parameter, a recovery band is determined and displayed, in real time, indicating whether the present flight parameter value can recover to a reference trace or band value within a selected recovery time interval (normally 2-5 sec), including the effects of aircraft system performance constraints on the rate of recovery. The aircraft performance constraints are optionally expressed in terms of one or more constrained differential equations involving relevant time derivatives of flight parameter values. Where recovery to a reference flight parameter value is not possible within the recovery time interval, the system optionally provides one or more alternative responses for the pilot in command in order to bring subsequent flight parameter values within the recovery band RB. The recovery band can be
generated and provided before the RB is used or can be
generated at the time of its use.

BRIEF DESCRIPTION OF THE DRAWINGS

FIGS. 1A and 1B illustrates an environment in which the
invention can be practiced.

FIG. 2 graphically illustrates drag force developed versus
percentage activation of an aircraft drag appliance.

FIGS. 3 and 4 graphically illustrate a screen displaying
present FP values, a trace and/or band of reference FP values,
and corresponding recovery band traces determined according
to the invention.

FIGS. 5A/5B graphically illustrate recovery band boundary
trace determination for a reference trace and a reference band.

FIGS. 6 and 7 are flow charts of procedures for practicing
the invention.

FIGS. 8-16 graphically illustrate examples of constrained
FP values u and the applicable constraints for various con-
straint classes.

DESCRIPTION OF BEST MODES OF THE
INVENTION

FIGS. 1A and 1B illustrate environments for an ascending
aircraft (1A) and for a descending aircraft (1B) where the
invention can be practiced. In FIG. 1A, an aircraft 11 is
ascending, either after takeoff or in moving from a first flight
to a second flight altitude. In FIG. 1B, an aircraft 11 is
descending, either in preparation for a subsequent touch-
down or in moving from a first flight altitude to a second flight altitude. The aircraft 11 has at least one of an associated kinetic energy KE(t), potential energy PE(t), energy component E(t)=d1•KE(t)+d2•PE(t), where (d1,d2) are selected non-negative real numbers; energy component time derivative (d/dt)E(t), thrust power, vertical speed, ground air speed, aircraft mass, height above ground, flap position, speed brake position, landing gear position, other drag indices, roll, pitch and/or yaw angles, angle of attack, or another relevant flight parameter.

The aircraft kinetic energy and potential energy components are, respectively,

\[ KE(t) = \frac{1}{2} m(t) v(t)^2 \]

\[ PE(t) = \frac{1}{2} g h(t) \]

where \( m(t) \) is the instantaneous aircraft mass (taking account of fuel consumption), \( v(t) \) is an instantaneous moment of inertia tensor for the aircraft, \( a(t) \) is an aircraft rotation vector, computed with reference to a center of gravity or other selected location determined on the aircraft, \( g \) is local gravity coefficient, and \( h(t) \) is the instantaneous height of aircraft cg above a local reference height, such as local ground height. The rotational component of kinetic energy may be negligible or may be ignored for other reasons. The aircraft potential energy may be taken as in Eq. (2).

An aircraft system measures or estimates or otherwise provides a value (referred to as a “measured value” for convenience herein)

\[ E(t) = \frac{d}{dt} KE(t) + \frac{d}{dt} PE(t) \]  (3)

of an energy component of an aircraft during an ascent phase, a cruise phase or a descent phase of a flight, at each of a first sequence of times \( t_1, t_2, \ldots, t_N \), where \( d1 \) and \( d2 \) are selected real values, not both 0; for example, the pair \((d1,d2) = (1,0), (0,1), (1,1) \) or \((1,-1)\). Optionally, the system also provides an energy component time derivative

\[ \frac{d}{dt} E(t) = d3 + \frac{d}{dt} KE(t) + d4 \frac{d}{dt} PE(t) \]

(4)

where \( d3 \) and \( d4 \) are selected real values, not both 0. The system also provides a reference value \( E(t_0) \) of the energy component and a reference value time derivative \( \frac{d}{dt} E(t_0) \); ref for a time, \( t=t_0 \), determined with reference to the time \( t_0 \).

The screen in FIG. 3 displays first and second curves, \( T1 \) and \( T2 \) or recovery band boundary traces \( FP(t, RB) \), representing a boundary trace or traces of a recovery band (RB), as a function of time, where each trace can be a continuous curve or can be discrete. FIG. 3 also displays a third trace \( T3 \) (or group of traces) representing a sequence of reference values.
FP(tref) within the recovery band RB; and a fourth trace \( t_4 \) representing the measured present FP values \( FP(t) \), which may lie inside or outside the recovery band RB. The reference FP values trace \( t_3 \) is a sequence of FP values that correspond to a desirable aircraft flight configuration for an environment similar to the present flight. Alternatively, the trace \( t_3 \) can be replaced by a central band of reference FP values, denoted \( FP(t_{ref};CB) \), corresponding to a collection of desirable FP values for a given time, \( t=t_3 \), as illustrated in FIG. 4.

The trace \( t_3 \) in FIG. 3 refers to reference flight parameter values \( FP(t_{ref}) \) for a flight reference, against which the corresponding present flight parameter values (trace \( t_4 \)) is to be compared. It is preferable to present the RB traces, \( t_1 \) and \( t_2 \), in the physical units for that flight parameter. However, all traces can be presented in terms of dimensionless quantities with a common divisor.

The RB traces, \( t_1 \) and \( t_2 \), are defined with reference to the reference FP values \( FP(t_{ref}) \), as discussed in the following. The RB traces, \( t_1 \) and \( t_2 \), may be, but need not be, symmetrically arranged with respect to the reference FP values. Optionally, the reference FP value trace is replaced by the central band \( FP(t_{ref};CB) \) and has central band boundary traces \( t_3 \); and the RB boundary traces, \( t_1 \) and \( t_2 \), can be referenced to the respective central band boundary traces, \( t_3A \) and \( t_3B \), as illustrated in FIG. 4.

For purposes of illustration, each trace shown in FIGS. 3 and 4 is assumed to be discrete, for example, corresponding to the sequence of measurement times \( t=t_n \). Optionally, the present FP values \( FP(t) \) are provided (measured or estimated) in real time as the aircraft proceeds. The values \( FP(t_{ref}) \), \( FP(t_{ref};CB;3A \) and \( 3B) \), and \( FP(t;RB) \) are preferably referenced to the respective central band boundary traces, \( t_3A \) and \( t_3B \), as illustrated in FIG. 4.

As a first example, Eq. (5) with \( c_1<0 \) and \( c_2<0 \), which may correspond to a limitation on rate of change of a variable such as angle of attack. A third example is Eq. (5) with \( c_1=0 \) and \( c_2=0 \), which may correspond to a limitation on radial acceleration of \( u \), such as turn rate for the aircraft. Appendix 1 sets forth 13 classes of “constrained solutions” \( u(t_n) \) of the constraint relations (5) for \( t=t_n \), where the initial value \( u(t_{-n}t_n)-FP(t_n) \), the initial rate of change \( (dF/dt)(t_{-n}t_n) \), and/or the initial acceleration \( \{d^2F/dt^2\}_{t_n} \) is provided, where needed, in connection with Eq. (5).

Determination of the zero crossing time \( t^n(\bar{n}) \) is straightforward, because \( FP(t_{ref}) \) is known or specified and \( u(t_{-n}t_n) \) for \( t=t_n \) is determinable from the results in Appendix 1 or other relevant constraint expressions. The system then determines if a specified recovery time interval length \( \Delta(FP;rec) \) (dependent upon the present phase of the flight) satisfies

\[
\Delta(FP;rec)=\frac{u(t_{-n}t_n)}{FP(t_{ref})} < 0. \tag{6}
\]

The recovery time interval length may vary with the particular choice of FP and with the phase of the flight, but often lies in a range 2-15 sec. If the inequality in Eq. (8) is satisfied, the initial value \( u(t_{-n}t_n)-FP(t_n) \) lies within the recovery band RB, and the present FP value can recover to a subsequent reference FP value \( FP(t_{-n};ref) \) after the recovery time interval. If the inequality in Eq. (8) is not satisfied, the initial value \( u(t_{-n}t_n)-FP(t_n) \) does not lie within the recovery band RB, and additional time is likely required for recovery. The algebraically largest (or smallest) constraint equation solution initial value, \( u(t_{upper}) \), or \( u(t_{lower}) \) for \( t=t_n \), for which the inequality in Eq. (8) is satisfied determines one value \( u(t_{-n}t_n) \) on the recovery boundary RB (trace \( t_1 \) or \( t_2 \)). The locus of all points on one or the other of the boundary traces \( t_1 \) or \( t_2 \) defines that recovery band boundary (RBB) trace.

As noted in Appendix 1, to determine the coordinates of a point on the recovery band boundary (RBB), it is more convenient and straightforward to begin at the coordinates \( (u(t_{upper});FP(t_{ref};ref)) \) on the FP reference curve \( FP(t_{ref}) \) and to proceed backward in time, using a modified time variable \( \tau \), such as \( t_{-n}(\bar{n})=\Delta(FP;rec) \) to determine a boundary point on the RBB. Beginning at a point on a flight parameter graph with initial coordinates \( (u(t_{upper});FP(t_{ref};ref)) \) on the FP reference curve, a constrained solution is run in reverse to a point having final coordinates

\[
(t_{final})=(t_{upper}(\Delta(FP;rec));FP(t_{ref};ref)), \tag{11}
\]

which is identified as a point on the recovery band boundary RBB. This process is repeated for a sequence of initial coor-
may also be implemented during the flight, in real board or offboard) that is used on a flight. However, this procedure would be implemented din real time, or near-real time. This procedure is preferably implemented beforehand and provided as part of a software package in a computer (on-board or offboard) that is used on a flight. However, this procedure may also be implemented during the flight, in real time or near real time. This procedure would be implemented separately for each flight parameter FP of interest, and each FP will have its own characteristic constraint solutions, which may vary with the phase of the flight, with the initial value ut=(t0,t0), and with the local environment (origin airport, waypoint, destination airport, etc.).

Use of the Recovery Band

When the recovery band and its boundaries (for a given phase and a given flight parameter FP) have been determined or otherwise provided, an aircraft in flight or proceeding toward take off may use the RB, as indicated in a flow chart in FIG. 7, to determine if recovery to a reference curve or reference band is possible. In step 71, a recovery time interval length Δt(FP,rec) and corresponding recovery band boundary traces FP(t;RB;1) and/or FP(t;RB;2) are determined or otherwise provided, for a selected flight parameter FP and for an appropriate present phase of a flight. In step 72, a present flight parameter value FP((t0) of the flight parameter FP is provided. In step 73, the system determines if FP((t0) is within or on a boundary of the recovery band, for example,

\[ FP(t;RB;1) \leq FP(t) \leq FP(t;RB;2) \]  

If the answer to the query in step 73 is “yes,” the system interprets this condition as indicating that the flight parameter FP can (or is likely to be able to) recover to the reference curve or reference band within a recovery time interval length, beginning at the present time, t=t0, in step 74. If the answer to the query in step 73 is “no,” the system interprets this condition as indicating that the flight parameter FP is unlikely to recover to the reference curve or reference band within a recovery time interval length, beginning at the present time, t=t0, in step 74. The system then proceeds to step 77 (optional) and displays a selected section of the recovery band and one or more contemporaneous value(s) FP(t), with reference to the recovery band. The procedures in the flow charts in FIGS. 6 and 7 can be performed together.

This procedure is implemented din real time, or near-real time, as the flight progresses. Where recovery of K flight parameter values FP,(t)(k=1, ..., K,K≥2) is a concern, each flight parameter will have its own recovery time interval length Δt(FP;rec) and its own recovery band boundary traces, FP,(t;RB;1) and FP,(t;RB;2); and joint recovery of the K flight parameters should be examined with reference to the K recovery bands.

In order to generate a continuous trace of a reference curve or reference band boundary, such as FP(t;ref) or FP(t;ref; CB1 or 2), or a continuous trace of a recovery band boundary, such as FP(t;RB;1 or 2), one may need to extend the underlying definition of the reference or recovery function from discrete points to a continuum (here, of temporal values t). Given N function values f(t)(n=1, 2, ..., N; N≥2), with t0<t1<...<tn are defined, the estimation function N

\[ F(t) = \sum_{m=1}^{N} \frac{f(t) - f(t_m)}{t_m - t} \]  

is continuous in the range t0≤t≤tN and beyond and satisfies

\[ F(t=t_0) = f(t_0) \]  

Because the measurement times are spaced apart, none of the denominators in F(t) is 0, and the magnitude of F(t) is bounded. The definition for F(t) can be extended to any finite number N of time points.

More generally, one can define an estimation function F*(t; est) as a sum of two or more continuous characteristic func-
tions \( F^*(t;k) \) \((k=1, \ldots, N; N \geq 2)\), where the characteristic function component \( F^*(t;k) \) satisfies
\[
F^*(t^{-1},k)=f(t) (p-k-0 \neq 0).
\]

**APPENDIX I**

Some Constraint Equations for Aircraft Performance

Constraints on aircraft performance can arise from limitations in engine, airfoil and system performance, limitations imposed by an airline on its own flights, limitations imposed by a regulatory agency (such as the F.A.A. or the N.T.S.B.), and other limitations. For purposes of this analysis, a constraint on a flight parameter \( FP \) is assumed to be expressible in terms of, first and second order linear differential equations in a constrained solution \( u(t;\tau) \) for the flight parameter \( FP \):
\[
B^\prime \leq \text{coeff}(u(t;\tau),a) + c(t)(du/dt) + a0u \leq B^\prime,
\]
\[
 u(t;\tau_1,a) = 0(t;FP),
\]
\[
 (du/dt)(u(t;\tau_1,a) - a0u(t;FP)) \text{ (optional; dependent on } \tau_1),
\]
\[
 (d^2u/dt^2)(u(t;\tau_1,a) - a0u(t;FP)) \text{ (optional; dependent on } \tau_1),
\]
where \( c2, c1 \) and \( c0 \) are selected real numbers, at least one of which is non-zero, \( B0 \) and \( B1 \) are selected real numbers. 12 classes of constraint equation solutions are analyzed here, depending upon which of the coefficients \( c2, c1 \) and \( c0 \) are non-zero. For most relevant constraints (A-1), \( B0-B1 \).

Class 1: \( c1= c2=0; 0 < c0 \). This class includes limitations on an aircraft flap angle (or drag control) or an aileron angle (aircraft turn radius). Without loss of generality, the coefficient \( c0 \) is assumed to have the value +1. The quantity \( u \) is constrained by
\[
 B0 \leq u(0) \leq B1,
\]
\[
 B0 \leq u(t;\tau_1,a) = a0 \leq B1,
\]
is equal to \( u \) between \( B0 \) and \( B1 \), is limited at the lower end by the value \( B0 \) and is limited at the upper end by the value \( B1 \) (\( B0 \)), as illustrated in FIG. 8. The constraints in Eq. (A-3) are implemented analytically by the functional and constraint solution
\[
 u = a_{1}(a;u; a1; b1) = a1 + (u(t;\tau_1) + b1 - a0u(t;\tau_1) - b1)/2
\]
\[
 u = (B0 + B1)/2,
\]
\[
 b1 = (B1 - B0)/2,
\]
together with any other relevant initial conditions for \( u(t;\tau_1) \) and its derivatives. Optionally, one of the constraints can be deleted, by formally setting \( B0 = -\infty \) or \( B1 = +\infty \). The left hand constraint and right hand constraint in Eq. (A-3a) would be deleted, by formally setting \( a0 = -\infty \) or \( a0 = +\infty \). The left hand constraint on \( a0 \) is assumed to have the value +1. FIG. 8, with \( u \) replaced by \( du/dt \), illustrates the effect of this constraint on the variable \( du/dt \). The constrained solution \( u \) satisfies
\[
 u(t,lower) = B0 + a0u(t;\tau_1) + 0 \leq u(t;\tau_1) + a0u(t;\tau_1) \leq B1 + a0u(t;\tau_1),
\]
\[
 0 \leq u(0) \leq B1,
\]
and the function and constraint solution of this class are given by
\[
 u = a_{1}(a;u; a3; b3),
\]
\[
 a3(t) = (B1 - B0)/(\Delta t)^2,
\]
\[
 b3(t) = (B1 - B0)/(\Delta t)^2,
\]
\[
 u(\text{lower}) = B0 + a0u(t;\tau_1) + 0 \leq a0u(t;\tau_1) \leq B1 + a0u(t;\tau_1),
\]
\[
 u(\text{upper}) = B1 + a0u(t;\tau_1) + 0 \leq a0u(t;\tau_1) \leq B1 + a0u(t;\tau_1),
\]

The constraints set forth in Eq. (A-6d) correspond to Eq. (A-1).

Class 3: \( c0= c1= 0; c2=0 \). This class includes a limitation on second order rate of change of the flight parameter value, with \( u0 \) and \( u'0 \) unconstrained. Without loss of generality, the coefficient \( c2 \) is assumed to have the value +1. The functional and constraint solution for this class are given by
\[
 B0 \leq a0 \leq B1,
\]
\[
 a0 = a_{1}(a;u; a5; b5),
\]
\[
 a5(t) = (B1 - B0)/(\Delta t)^4,
\]
\[
 b5(t) = (B1 - B0)/(\Delta t)^4,
\]
\[
 u(\text{lower}) = B0 + a0u(t;\tau_1) + 0 \leq a0u(t;\tau_1) \leq B1 + a0u(t;\tau_1),
\]
\[
 u(\text{upper}) = B1 + a0u(t;\tau_1) + 0 \leq a0u(t;\tau_1) \leq B1 + a0u(t;\tau_1),
\]

Here, \( u0 \) and \( u'0 \) are constrained by Eq. (A-9). The upper branch and lower limit solutions and a permissible constrained solution \( u(t;\tau_1) \) are illustrated in FIG. 11.

Class 4: \( c0= 0; c1= 0; c2=0 \). This class includes a combined limitation of a sum of the zeroth and first order rate of change of the flight parameter value, with \( u0 \) and \( u'0 \) unconstrained. Without loss of generality, the coefficients \( c1 \) and \( c2 \) are assumed to have the value +1. The constrained solution \( u(t;\tau_1) \) for \( t \geq \tau_1 \) can be implemented and analytically using the functional
\[
 u = a_{1}(a;u; a7; b7),
\]
\[
 a7(t) = (B1 - B0)/(\Delta t)^6,
\]
\[
 b7(t) = (B1 - B0)/(\Delta t)^6,
\]
\[
 u(\text{lower}) = B0 + a0u(t;\tau_1) + 0 \leq a0u(t;\tau_1) \leq B1 + a0u(t;\tau_1),
\]
\[
 u(\text{upper}) = B1 + a0u(t;\tau_1) + 0 \leq a0u(t;\tau_1) \leq B1 + a0u(t;\tau_1),
\]

with \( u0 \) unconstrained. FIG. 9 illustrates the upper limit and lower limit solutions, \( u(t;\text{upper}) \) and \( u(t;\text{lower}) \), and a permissible constrained solution \( u(t) \), corresponding to Eqs. (A-5).

The constraints set forth in Eq. (A-6d) correspond to Eq. (A-1).
\[ b(t) = \begin{cases} a(t) & \text{for } t < 0 \\ b(t) & \text{for } t \geq 0 \end{cases} \]

The functional and constraint solution for this class is given by

\[ u(t) = \begin{cases} a(t) & \text{for } t < 0 \\ c(t) & \text{for } t \geq 0 \end{cases} \]

for some function \( c(t) \). The constrained solution \( u(t; a) \) is determined by replacing the parameter \( a \) by \( +ia \) or by \( -ia \) in Eqs. (15a) and (15b), respectively.

The left hand inequality in Eq. (11a) (B0 only) is re-expressed as

\[ \left\{ \begin{array}{l}
\left| u(t) \right| \leq \left| u(t; \text{upper}) \right|
\end{array} \right. \]

where the right hand inequality in Eq. (13a) is included by symmetry arguments. The constrained solution \( u(t; a) \) can be expressed as

\[ u(t) = \begin{cases} a(t) & \text{for } t < 0 \\ c(t) & \text{for } t \geq 0 \end{cases} \]

for some function \( c(t) \). The constrained solution \( u(t; a) \) is determined by replacing the parameter \( a \) by \( +ia \) or by \( -ia \) in Eqs. (15a) and (15b), respectively.

The left hand side of the defining equation (16) is re-expressed as

\[ \left\{ \begin{array}{l}
\left| \frac{\partial u(t)}{\partial t} \right| \leq \left| \frac{\partial u(t; \text{upper})}{\partial t} \right|
\end{array} \right. \]

where the right hand inequality in Eq. (11a) (B0 only) is included by symmetry arguments. The constrained solution \( u(t; a) \) can be expressed as

\[ u(t) = \begin{cases} a(t) & \text{for } t < 0 \\ c(t) & \text{for } t \geq 0 \end{cases} \]

for some function \( c(t) \). The constrained solution \( u(t; a) \) is determined by replacing the parameter \( a \) by \( +ia \) or by \( -ia \) in Eqs. (15a) and (15b), respectively.

The left hand side of the defining equation (16) is re-expressed as

\[ \left\{ \begin{array}{l}
\left| \frac{\partial^2 u(t)}{\partial t^2} \right| \leq \left| \frac{\partial^2 u(t; \text{upper})}{\partial t^2} \right|
\end{array} \right. \]

where the right hand inequality in Eq. (11a) (B0 only) is included by symmetry arguments. The constrained solution \( u(t; a) \) can be expressed as

\[ u(t) = \begin{cases} a(t) & \text{for } t < 0 \\ c(t) & \text{for } t \geq 0 \end{cases} \]

for some function \( c(t) \). The constrained solution \( u(t; a) \) is determined by replacing the parameter \( a \) by \( +ia \) or by \( -ia \) in Eqs. (15a) and (15b), respectively.

The left hand side of the defining equation (16) is re-expressed as

\[ \left\{ \begin{array}{l}
\left| \frac{\partial^3 u(t)}{\partial t^3} \right| \leq \left| \frac{\partial^3 u(t; \text{upper})}{\partial t^3} \right|
\end{array} \right. \]

where the right hand inequality in Eq. (11a) (B0 only) is included by symmetry arguments. The constrained solution \( u(t; a) \) can be expressed as

\[ u(t) = \begin{cases} a(t) & \text{for } t < 0 \\ c(t) & \text{for } t \geq 0 \end{cases} \]

for some function \( c(t) \). The constrained solution \( u(t; a) \) is determined by replacing the parameter \( a \) by \( +ia \) or by \( -ia \) in Eqs. (15a) and (15b), respectively.

The left hand side of the defining equation (16) is re-expressed as

\[ \left\{ \begin{array}{l}
\left| \frac{\partial^4 u(t)}{\partial t^4} \right| \leq \left| \frac{\partial^4 u(t; \text{upper})}{\partial t^4} \right|
\end{array} \right. \]

where the right hand inequality in Eq. (11a) (B0 only) is included by symmetry arguments. The constrained solution \( u(t; a) \) can be expressed as

\[ u(t) = \begin{cases} a(t) & \text{for } t < 0 \\ c(t) & \text{for } t \geq 0 \end{cases} \]

for some function \( c(t) \). The constrained solution \( u(t; a) \) is determined by replacing the parameter \( a \) by \( +ia \) or by \( -ia \) in Eqs. (15a) and (15b), respectively.
The solution component \( w(t) \) is found to satisfy
\[
dw/dt = 0,
\]
(A-26a)
\[
w(t) = w_0 + w_1 t.
\]
(A-26b)

The constraints in Eq. (24) can be re-expressed as
\[
B_1 u(t, -t'') - u_0  \leq B_3 t,
\]
(A-29a)
\[
B_2 t^2 u(t, -t'') - u_0  \leq B_3 t, (A-29b)
\]
where \( u(t, -t'') \) is the constrained solution computed in the preceding, \( t_0 \) is a selected “initial time value,” and \( u_1 \) and \( u_1' \) are selected initial values. Here, the approach (2) with the particular choices
\[
\tau = (t_n + At(FP; rec), t_{(n) - At(FP; rec)}),
\]
(A-38a)
\[
\tau(\text{initial}) = t_n, \tau(\text{initial}) = t_{(n)},
\]
(A-38b)
\[
\tau = \text{FP}(t_n; \text{ref}),
\]
(A-38c)
\[
u_1' = -d/dt \text{FP}(t_n; \text{ref}),
\]
(A-38d)
will be used, but the approach (1), mutatis mutandis, should yield substantially the same results. The constrained solution \( U(\tau) \) runs backward in time, beginning at the initial coordinates \( (\tau = t_n, U(\tau = t_n)) \), and proceeding to a coordinate location
\[
(\tau = t_n, \Delta t(\text{FP; rec}), U(\tau = t_n)) = \Delta t(\text{FP; rec}), U(\tau = t_n) = \text{FP}(t_n; \text{ref}),
\]
(A-39)
which is identified as a boundary point for the recovery band RB.

What is claimed is:

1. A method for monitoring and analyzing, in real time, variation with time of an aircraft flight parameter, the method comprising:

   measuring a value \( \text{FP}(t = t_n) \) of a selected flight parameter \( \text{FP} \) for an aircraft, for a time, \( t = t_n \) that is equal to or adjacent to a present time;

   providing a sequence \( \{\text{FP}(t_n; \text{ref})\}_{n} \) of FP reference values for at least two time values, \( t_n \) (\( n = n_1, n_2 \)), and an available recovery time interval length \( \Delta t(\text{FP; rec}) \);
providing at least one constrained solution value \( u(t_\text{ref}) \) for the FP on a graph \((t, u(t))\), with at least one initial constrained solution value \( u(t_1, t_\text{ref}) \) that is displaced from the corresponding value \( F(t_\text{ref}) \); for each constrained solution \( u(t_\text{ref}) \), providing a reference curve-crossing time value, \( t = t_\text{ref} \), at which \( u(t_\text{ref}) \) crosses at least one trace of the reference values \( F(t; \text{ref}) \) on the graph; computing a coordinate pair \((t, u) = (t_\text{ref} + \Delta t(FP; \text{rec}), u(t_\text{ref} + \Delta t(FP; \text{rec})) \) for the graph at the first two time values, \( t = t_\text{ref} \), \( n = n_1, n_2 \), and identifying this coordinate pair with at least one point on a recovery band boundary trace; connecting at least two of the points with the coordinates \( (t_\text{ref} + \Delta t(FP; \text{rec}), u(t_\text{ref} + \Delta t(FP; \text{rec})) \) to provide a portion of a recovery band boundary trace; providing at least first and second non-coincident recovery band boundary traces, \((t; F(t; \text{rec} = 1)) \) and \((t; F(t; \text{rec} = 2)) \), that are spaced apart for at least one time value \( t \) on the graph \((t, u(t))\); displaying the at least first and second recovery band boundary traces, \((t; F(t; \text{rec} = 1)) \) and \((t; F(t; \text{rec} = 2)) \), and the value \( F(t_\text{ref}) \) of the flight parameter at the time \( t = t_\text{ref} \) on the graph \((t, u(t))\); and providing an operator of the aircraft with information which may be used to bring the flight parameter \( FP \) within the recovery band defined by the first and second recovery band boundary traces.

2. The method of claim 1, further comprising:

providing a value \( F(t; \text{ref}) \) of said selected flight parameter in a selected phase \( \phi \) of a flight;

determining if the value \( F(t; \text{ref}) \) lies between, or lies on, said first and second recovery band boundary traces;

when the value \( F(t; \text{ref}) \) lies between said first and second recovery band boundary traces, or lies on at least one of said first and second recovery band boundary traces, interpreting this condition as indicating that said selected flight parameter is likely to be able to recover the value \( F(t; \text{ref}) \) to at least one of said FP reference values within a time interval of length no greater than \( \Delta t(FP; \text{rec}) \).

3. The method of claim 1, further comprising:

providing a value \( F(t; \text{ref}) \) of said selected flight parameter in a selected phase \( \phi \) of a flight;

determining if the value \( F(t; \text{ref}) \) lies between, or lies on, said first and second recovery band boundary traces;

when the value \( F(t; \text{ref}) \) does not lie between said first and second recovery band boundary traces, and does not lie on at least one of said first and second recovery band boundary traces, interpreting this condition as indicating that said selected flight parameter is unlikely to be able to recover from the value \( F(t; \text{ref}) \) to at least one of said FP reference values within a time interval of length no greater than \( \Delta t(FP; \text{rec}) \).

4. The method of claim 1, further comprising providing said constrained quantity \( u(t_\text{ref}) \) for said FP value as a dimensionless ratio \( F(t)/F_\text{PO} \), where \( F_\text{PO} \) is a selected measured or estimated value of said FP value.

5. The method of claim 1, further comprising providing said constrained quantity \( u(t_\text{ref}) \) for said flight parameter \( FP \) as a solution to a selected constrained differential equation

\[
B' = \frac{2}{3} \left[ B \left( \frac{d^2 u}{dt^2} \right) + c_1 \left( \frac{du}{dt} \right) + c_0 u \right] = B''
\]

where the coefficients \( c_0, c_1 \) and \( c_2 \) are first, second and third numerical constants, at least one being non-zero, and \( B' \) and \( B'' \) are fourth and fifth numerical constants, at least one being non-zero.

6. A method for monitoring and analyzing, in real time, variation with time of an aircraft flight parameter the method comprising:

providing a recovery band in a graph of values \((t, u(t))\) of a measured flight parameter \( FP \) for an aircraft versus time \( t \), the recovery band being defined by first and second non-coincident recovery band boundary traces that are spaced apart for at least one time value, and providing a recovery time interval length \( \Delta t(FP; \text{rec}) \), having a positive temporal value, where the flight parameter, having a value \( F(t; \text{ref}) \) at a time \( t = t_\text{ref} \), is likely to be able to recover to a reference flight parameter value \( F(t; \text{ref}) \), lying in a band of reference flight parameter values \( F(t; \text{ref}) \), and within a time interval given by \( t_\text{ref} \leq t \leq t_\text{ref} + \Delta t(FP; \text{rec}) \), if the flight parameter value lies between, or lies on, the first and second recovery band boundary traces; and

displaying at least one of the recovery band on the graph and the recovery time interval length \( \Delta t(FP; \text{rec}) \).

7. The method of claim 6, further comprising providing said reference flight parameter band as a curve having a width of substantially 0.

8. The method of claim 6, further comprising providing said quantity \( w(t) \) in said graph as a dimensionless ratio \( FP(t)/F_\text{PO} \), where \( F_\text{PO} \) is a selected measured or estimated value of said FP value.

9. The method of claim 6, further comprising providing said value \( w(t) \) for said flight parameter \( FP \) as a solution to a selected constrained differential equation

\[
B' = \frac{2}{3} \left[ B \left( \frac{d^2 w}{dt^2} \right) + c_1 \left( \frac{dw}{dt} \right) + c_0 w \right] = B''
\]

where the coefficients \( c_0, c_1 \) and \( c_2 \) are first, second and third numerical constants, at least one being non-zero, and \( B' \) and \( B'' \) are fourth and fifth numerical constants, at least one being non-zero.

10. A method for monitoring and analyzing, in real time, variation with time of an aircraft flight parameter the method comprising:

providing a recovery band in a graph of values \((t, u(t))\) of a measured flight parameter \( FP \) for an aircraft versus time \( t \), the recovery band being defined by first and second non-coincident recovery band boundary traces that are spaced apart for at least one time value, and providing a recovery time interval length \( \Delta t(FP; \text{rec}) \), having a positive temporal value, where the flight parameter, having a value \( F(t; \text{ref}) \) at a time \( t = t_\text{ref} \), is unlikely to be able to recover to a reference flight parameter value \( F(t; \text{ref}) \), lying in a band of reference flight parameter values \( F(t; \text{ref}) \), within a time interval given by \( t_\text{ref} \leq t \leq t_\text{ref} + \Delta t(FP; \text{rec}) \), if the flight parameter value does not lie between, and does not lie on, the first and second recovery band boundary traces; and

displaying at least one of the recovery band on the graph and the recovery time interval length \( \Delta t(FP; \text{rec}) \).

11. The method of claim 10, further comprising providing said reference flight parameter band as a curve having a width of substantially 0.

12. The method of claim 10, further comprising providing said reference flight parameter band as a curve having a width of substantially 0.
providing said value \( w(t) \) for said flight parameter \( FP \) as a solution to a selected constrained differential equation

\[
B'\dot{w} + B''w = \left[ c_0 + c_1 \frac{dw}{dt} + c_2 \right],
\]

where the coefficients \( c_0, c_1 \) and \( c_2 \) are first, second and third numerical constants, at least one being non-zero, and \( B' \) and \( B'' \) are fourth and fifth numerical constants, at least one being non-zero.

14. A system for monitoring and analyzing, in real time, variation with time of an aircraft flight parameter, the system comprising a computer that is programmed:

to measure a value \( FP(t_p) \) of a selected flight parameter \( FP \) for an aircraft, for a time, \( t = t_p \), that is equal to or adjacent to a present time;

to provide a sequence \( \{ FP(t_p; \text{ref}) \} \) of FP reference values for at least two time values, \( t = t_p \) \((n=n_1, n_2)\), and an available recovery time interval length \( \Delta t(FP; \text{rec}) \);

to provide at least one constrained solution \( u(t_1, \text{ref}) \) for the FP on a graph \( (t, u(t)) \), with at least one initial constrained solution value \( u(t_1, \text{ref}) \) that is displaced from the corresponding value \( FP(t_1; \text{ref}) \);

for each constrained solution \( u(t_1, \text{ref}) \), to provide a reference curve-crossing time value, \( t = t_1(n) \), at which \( u(t_1) \) crosses at least one trace of the reference values \( FP(t; \text{ref}) \) on the graph;

to compute a coordinate pair \( (t, u(t)) = (t_1(n) - \Delta t(FP; \text{rec}) u(t_1(n) - \Delta t(FP; \text{rec}) t_1) \) on the graph for the at least two time values, \( t = t_1(n=n_1, n_2) \), and to identify this coordinate pair at least one point on a recovery band boundary trace;

to connect at least two of the points with the coordinates \( (t_1(n) - \Delta t(FP; \text{rec}) u(t_1(n) - \Delta t(FP; \text{rec}) t_1) \)) to provide a portion of a recovery band boundary trace;

to provide at least first and second non-coincident recovery band boundary traces, \( (t, FP(t; \text{rec}; 1)) \) and \( (t, FP(t; \text{rec}; 2)) \), that are spaced apart from each other for at least one time value \( t \) on the graph \( (t, u(t)) \); and

to display at least first and second recovery band boundary traces, \( (t, FP(t; \text{rec}; 1)) \) and \( (t, FP(t; \text{rec}; 2)) \), and the value \( FP(t; \text{rec}) \) of the flight parameter at the time \( t = t_1 \), on the graph \( (t, u(t)) \); and

to provide an operative of the aircraft with information which may be used to bring the flight parameter within the recovery band defined by the first and second recovery band boundary traces.

15. The system of claim 14, wherein said computer is further programmed:

to provide a value \( FP(t = t_1; \phi) \) of said selected FP parameter in a selected phase \( \phi \) of a flight;

to determine if the value \( FP(t = t_1; \phi) \) lies between, or lies on, said first and second recovery band boundary traces; and

wherein the value \( FP(t = t_1; \phi) \) lies between said first and second recovery band boundary traces, to interpret this condition as indicating that said selected flight parameter is likely to be able to recover form the value \( FP(t = t_1; \phi) \) to at least one of said FP reference values within a time interval of length no greater than \( \Delta t(FP; \text{rec}) \).

16. The system of claim 14, wherein said computer is further reprogrammed:

to provide a value \( FP(t = t_1; \phi) \) of said selected FP parameter in a selected phase \( \phi \) of a flight;

to determine if the value \( FP(t = t_1; \phi) \) lies between, or on, said first and second recovery band boundary traces; and

when the value \( FP(t = t_1; \phi) \) does not lie between said first and second recovery band boundary traces, to interpret this condition as indicating that said selected flight parameter is unlikely to be able to recover from the value \( FP(t = t_1; \phi) \) to at least one of said FP reference values within a time interval of length no greater than \( \Delta t(FP; \text{rec}) \).

17. A system for monitoring and analyzing, in real time, variation with time of an aircraft flight parameter the system comprising a computer that is programmed:

to provide a recovery band as a graph of values \( (t, u(t)) \) of a measured flight parameter \( FP \) for an aircraft versus time \( t \), the recovery band being defined by first and second non-coincident recovery band boundary traces that are spaced apart at at least one time value, and to provide a recovery time interval length \( \Delta t(FP; \text{rec}) \), having a positive temporal value, where the flight parameter, having a value \( FP(t = t_p) \) at a time \( t = t_p \) is likely to be able to recover to a reference flight parameter value \( FP(t; \text{ref}) \), lying in a band of reference flight parameter values \( FP(t; \text{ref}; CB) \), within a time interval given by \( t_p \leq t \leq t_p + \Delta t(FP; \text{rec}) \), if the flight parameter value lies between, or on, the first and second recovery band boundary traces; and

to display at least one of the recovery band on the graph and the recovery time interval length \( \Delta t(FP; \text{rec}) \).

18. The system of claim 17, wherein said compute is further programmed to provide said reference flight parameter band as a curve having a width of substantially 0.

19. A system for monitoring and analyzing, in real time, variation with time of an aircraft flight parameter the system comprising a computer that is programmed:

to provide a recovery band as a graph of values \( (t, u(t)) \) of a measured flight parameter \( FP \) for an aircraft versus time \( t \), the recovery band being defined by first and second non-coincident recovery band boundary traces that are spaced apart at at least one time value, and to provide a recovery time interval length \( \Delta t(FP; \text{rec}) \), having a positive temporal value, where the flight parameter, having a value \( FP(t = t_p) \) at a time \( t = t_p \) is unlikely to be able to recover to a reference flight parameter value \( FP(t; \text{ref}) \), lying in a band of reference flight parameter values \( FP(t; \text{ref}; CB) \), within a time interval given by \( t_p \leq t \leq t_p + \Delta t(FP; \text{rec}) \), if the flight parameter value does not lie between the first and second recovery band boundary traces; and

to display at least one of the recovery band on the graph and the recovery time interval length \( \Delta t(FP; \text{rec}) \).

20. The system of claim 19, wherein said computer is further programmed to provide said reference flight parameter band as a curve having a width of substantially 0.

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