A Systematic Approach to Sensor Selection for Aircraft Engine Health Estimation

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Abstract

A systematic approach for selecting an optimal suite of sensors for on-board aircraft gas turbine engine health estimation is presented. The methodology optimally chooses the engine sensor suite and the model tuning parameter vector to minimize the Kalman filter mean squared estimation error in the engine’s health parameters or other unmeasured engine outputs. This technique specifically addresses the underdetermined estimation problem where there are more unknown system health parameters representing degradation than available sensor measurements. This paper presents the theoretical estimation error equations, and describes the optimization approach that is applied to select the sensors and model tuning parameters to minimize these errors. Two different model tuning parameter vector selection approaches are evaluated: the conventional approach of selecting a subset of health parameters to serve as the tuning parameters, and an alternative approach that selects tuning parameters as a linear combination of all health parameters. Results from the application of the technique to an aircraft engine simulation are presented, and compared to those from an alternative sensor selection strategy.

Nomenclature

\( A, A_{xh}, A_{xq} \) system matrices

\( B, B_{xh}, B_{xq} \)

\( C, C_{xh}, C_{xq} \)

\( D, F, F_{xh}, F_{xq} \)

\( G, L, M, N \)

C-MAPSS Commercial Modular Aero-Propulsion System Simulation

\( G_{xh}, G_{x}, G_{h}, G_{z} \) estimation bias matrices

HPC high pressure compressor

HPT high pressure turbine

\( I \) identity matrix

\( K_{x} \) Kalman filter gain

LPC low pressure compressor

LPT low pressure turbine

\( P_{x}, P_{z} \) health and auxiliary parameter covariance matrices

\( P_{xh,k}, P_{xq,k}, P_{zh,k}, P_{zq,k} \) covariance matrices of estimated parameters

\( Q, Q_{xh}, Q_{xq} \) process noise covariance matrices

\( R \) measurement noise covariance matrix

\( V^{*} \) transformation matrix relating \( h_{k} \) to \( q_{k} \)

\( W_{x} \) auxiliary parameter weighting matrix

\( h_{k} \) health parameter vector

\( m \) dimension of tuning parameter vector

\( p \) dimension of health parameter vector

\( q_{k} \) Kalman filter tuning parameter vector

\( r \) number of sensors to choose from

\( s \) number of additional sensors to add

\( u_{k} \) actuator command vector

\( v_{k} \) measurement noise vector

\( w_{k}, w_{h,k}, w_{xh,k} \) process noise vectors

\( x_{k} \) state vector

\( x_{xh,k} \) augmented state vector (\( x_{k} \) and \( h_{k} \))

\( x_{xq,k} \) reduced order augmented state vector (\( x_{k} \) and \( q_{k} \))

\( y_{k} \) vector of measured outputs

\( z_{k} \) vector of unmeasured (auxiliary) outputs

\( e_{xq,k} \) residual vector (estimate minus its expected value)

Subscripts

\( k \) discrete time step index

\( ss \) steady-state

\( xh \) augmented state vector (\( x \) and \( h \))

\( xq \) reduced order augmented state vector (\( x \) and \( q \))

Superscripts

\( \dagger \) pseudo-inverse

\( ^{\wedge} \) estimated value

\( \sim \) error value

\( \bar{\cdot} \) mean value

\( T \) transpose

Operators

\( E[\cdot] \) expected value of argument

\( tr[\cdot] \) trace of matrix

\( SSEE(\cdot) \) sum of squared estimation errors

Introduction

An emerging approach in the field of aircraft engine controls and health management is the inclusion of real-time
on-board models for the in-flight estimation of engine performance variables (Refs. 1, 2, and 3) that can be directly utilized by controls, prognostics and health management applications. This includes the estimation of engine health parameters, such as efficiencies and flow capacities, that reflect the level of degradation in each major engine module, and might indicate the existence of faults.

Aircraft operators conventionally apply a ground-based engine condition monitoring process known as gas path analysis to estimate and trend the health parameters of individual engines (Refs. 4 and 5). This analysis is typically conducted based on steady-state snapshot measurement data collected each flight, and uses linear estimation techniques such as weighted least squares (Ref. 4) or maximum a posteriori estimation (Ref. 5). With the emergence of on-board model-based technology, the real-time continuous estimation of engine health parameters is becoming possible. The conventional approach used for real-time estimation is based on Kalman filter concepts. As will be explained, a requirement for Kalman filter estimation is that there be at least as many sensors as health parameters. However, in an aircraft engine this is typically not the case, presenting an underdetermined estimation problem. The steady-state gas path analysis approaches discussed in References 4 and 5 are capable of estimating more health parameters than sensed measurements due to the inclusion of a priori knowledge regarding health parameter variations. This enables estimation, albeit biased estimation, when faced with an underdetermined estimation problem. For Kalman filter estimation applications, a common approach to address the underdetermined estimation shortcoming is to only estimate a subset of the health parameters, referred to as model tuning parameters, and to assume that other health parameters remain constant. While this approach enables on-line Kalman filter-based estimation, it also results in biased estimation, as the effects of the unestimated health parameters will be reflected in those that are estimated.

Two steps that can be taken to improve the accuracy of on-board models are to optimize the selection of the model tuning parameter vector, and to add additional gas path measurement sensors. In order to optimize overall performance estimation accuracy, it is desirable to develop and apply a systematic approach for combined model tuning parameter and sensor selection.

In a departure from the conventional technique of selecting a subset of health parameters to serve as the model tuning parameter vector, Litt (Ref. 6) and the current authors (Refs. 7 and 8) have presented methodologies that create a tuning parameter vector as a linear combination of all health parameters that is of appropriate dimension to enable Kalman filter estimation. These studies have shown that an optimally selected model tuning parameter vector can significantly reduce the estimation error in the engine performance parameters of interest (Ref. 8).

Several sensor selection approaches have been investigated within the engine health monitoring community. In Reference 9, an information theoretic approach to aircraft engine sensor selection is performed. This method selects sensor suites to optimize a metric defined from the Fisher information matrix. Reference 10 presents a sensor selection approach that seeks to optimize the diagnostic information contained in the Jacobian matrix. The Jacobian matrix, sometimes referred to as the influence coefficient matrix, relates the effects of health parameter deviations to corresponding deviations in sensed engine outputs. In Reference 11 a sensor selection performance metric is defined as a function of the steady-state covariance of Kalman filter estimates, which also includes sensor cost. In References 12 and 13 a performance metric is defined to maximize the detectability and discriminability of user-specified fault types. Sensor cost and fault criticality are also considered in this approach.

This work extends previous research by applying the dual approach of tuning parameter and sensor selection for minimizing on-board Kalman filter estimation errors. The remaining sections of this paper are organized as follows. First, the mathematical formulation of the linear Kalman estimator is presented, and theoretical estimation error equations are introduced assuming open-loop, steady-state operating conditions. Next, the estimation error information is utilized within the methodology for tuner parameter and sensor selection. The methodology is then applied to an aircraft turbofan engine simulation. Theoretical and experimental results, including comparisons with another sensor selection method, are presented to illustrate the advantages of utilizing this methodology for underdetermined health parameter estimation problems. Finally, conclusions are presented.

### Problem Formulation

The discrete linear time-invariant engine state space equations about an operating point are given as

\[
\begin{align*}
    x_{k+1} &= Ax_k + Bu_k + Lh_k + w_k \\
    y_k &= Cx_k + Du_k + Mh_k + v_k 
\end{align*}
\]

where \(k\) is the time index, \(x\) is the vector of state variables, \(u\) is the vector of control inputs, and \(y\) is the vector of measured outputs. The vector \(h\) represents the engine health parameters. As the health parameters deviate from their nominal values, they induce shifts in other variables. The vectors \(w\) and \(v\) are uncorrelated zero-mean white noise input sequences. The matrix \(Q\) will be used to denote the covariance of \(w\), and \(R\) to denote the covariance of \(v\). The matrices \(A, B, C, D, L, M\) are of appropriate dimension. The health parameters, represented by the vector \(h\), are unknown inputs to the system. They may be treated as a set of biases, and are thus modeled without dynamics. With this interpretation Equation (1) can be written as:
The vector $w_{sh}$ is zero-mean white noise associated with the augmented state vector, $[x^T \ h^T]^T$, with a covariance of $Q_{sh}$. The vector $w_{sh}$ consists of the original state process noise, $w$, concatenated with the process noise associated with the health parameter vector, $w_h$.

Once the $h$ vector is appended to the state vector, it may be directly estimated, provided that the realization in Equation (2) is observable. Using this formulation, the number of health parameters that can be estimated is limited to the number of sensors, the dimension of $y$ (Ref. 14). An aircraft gas turbine engine typically has fewer sensors than health parameters, thus presenting an underdetermined estimation problem.

This paper presents a systematic methodology for the selection of additional engine sensors and model tuning parameters. These selections are performed to minimize the estimation error in engine health parameters. An approximation of the health parameter can be directly estimated, provided that the realization in Equation (2) is observable. Using this formulation, the number of health parameters that can be estimated is limited to the number of sensors. This means that while the Kalman filter is a dynamic system, the state estimation error covariance matrix and the Kalman gain matrix are invariant—instead of these matrices being updated each time step, they are pre-converged and held constant.

Kalman Filter Formulation

In this study, steady-state Kalman filtering is applied. This means that while the Kalman filter is a dynamic system, the state estimation error covariance matrix and the Kalman gain matrix are invariant—instead of these matrices being updated each time step, they are pre-converged and held constant. Given the reduced order linear state space equations shown in Equation (5), and assuming steady-state, open-loop operation ($u = 0$), the Kalman filter estimator becomes (Ref. 15)

$$\dot{\hat{x}}_{sh,k} = A_{sq} \hat{x}_{sq,k-1} + K_{e} \left( y_{k} - C_{sq} A_{sq} \hat{x}_{sq,k-1} \right)$$

where $K_{e}$ is the steady-state Kalman filter gain. The reduced order augmented state vector estimate, $\hat{x}_{sq}$, produced by Equation (7) can be used to obtain an estimate of the augmented state vector as

$$\hat{x}_{sh,k} = \begin{bmatrix} \hat{x}_{q,k} \\ \hat{h}_{k} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & V^* \end{bmatrix} \hat{x}_{sq,k}$$

where $V^*$ is the pseudo-inverse of $V$.

Substituting Equation (4) into Equation (2) yields the following reduced order state space equations that will be used to formulate the Kalman filter

$$\begin{bmatrix} x_{q,k+1} \\ q_{k+1} \end{bmatrix} = \begin{bmatrix} A & LV^* \\ 0 & I \end{bmatrix} \begin{bmatrix} x_{q,k} \\ q_{k} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} w_{k} \\ w_{q,k} \end{bmatrix}$$

$$y_k = C_{sq} \begin{bmatrix} x_{q,k} \\ q_{k} \end{bmatrix} + Du_k + v_k$$

$$\begin{bmatrix} x_{q,k+1} \\ q_{k+1} \end{bmatrix} = A_{sq} x_{sq,k} + B_{sq} u_k + w_{sq,k}$$

$$y_k = C_{sq} x_{sq,k} + Du_k + v_k$$

The state process noise, $w_{sq}$, and its associated covariance, $Q_{sq}$, for the reduced order system are calculated as

$$w_{sq,k} = \begin{bmatrix} I & 0 \\ 0 & V^* \end{bmatrix} w_{sh,k}$$

$$Q_{sq} = \begin{bmatrix} I & 0 \\ 0 & V^* \end{bmatrix} Q_{sh} \begin{bmatrix} I & 0 \\ 0 & V^* \end{bmatrix}^T$$

Reduced-Order State Space Model

To enable Kalman filter formulation when faced with an underdetermined estimation problem, a reduced-order system model is applied. First a reduced model tuning parameter vector, $q$, is constructed from the health parameter vector, $h$,

$$q = V^* h$$

where $q \in \mathbb{R}^m$, $h \in \mathbb{R}^p$, $m < p$, and $V^*$ is an $m \times p$ transformation matrix of rank $m$, relating $h$ to $q$. In this formulation $q$ and the sensor measurement vector, $y$, are defined to be of equivalent dimension (i.e., $y \in \mathbb{R}^m$). The transformation matrix $V^*$ can be defined to construct $q$ as a subset of $h$ (the conventional approach to tuner selection), or $V^*$ can be defined to construct $q$ as a linear combination of all health parameters. An approximation of the health parameter vector, $\hat{h}$, can be obtained as

$$\hat{h} = V^* q$$
Using Equation (8), an estimate of the entire health parameter vector can be obtained when faced with the underdetermined estimation problem.

**Kalman Filter Estimation Error**

The estimation errors in \( \hat{x}_{sh,k} \) are defined as the difference between estimated and actual values

\[
\tilde{x}_{sh,k} = \hat{x}_{sh,k} - x_{sh,k}
\]

When facing an underdetermined estimation problem, it will be impossible for the Kalman filter estimator to completely restore all information when transforming \( \hat{q} \) into \( h \). Therefore, the Kalman filter will be a biased estimator (i.e., the expected values of \( \tilde{x}_{sh,k} \) will be non-zero). The estimation errors can be considered to consist of two components: an estimation error bias, and an estimation variance. Complete derivations of the Kalman filter estimation error bias and variance under steady-state, open-loop operating conditions are provided in Reference 8. Abbreviated derivations are provided in the following subsections.

**Estimation Error Bias**

As shown in Reference 8 the steady-state augmented state estimation error bias for an arbitrary health parameter vector, \( h \), is given as

\[
\bar{x}_{sh,\text{av}} = E\left[ \bar{x}_{sh,k} - x_{sh,k} \right]
\]

\[
= \begin{bmatrix}
0 \\
I
\end{bmatrix} (I - A_{sq} + K_{c}C_{sq}A_{sq})^{-1} \ldots \\
- \begin{bmatrix}
(I - A)^{-1}L \\
I
\end{bmatrix}
\]

\[
\times K_{c} \left[ C(I - A)^{-1}L + M \right] \ldots h
\]

\[
= \begin{bmatrix}
\bar{x}_{sh,\text{av}} \\
\bar{h}_{sh,av}
\end{bmatrix} = \begin{bmatrix}
G_{s} \\
G_{h}
\end{bmatrix} h = \bar{G}_{sh} h
\]

where the operator \( E[\bullet] \) represents the expected value of the argument. The estimation error bias equation, Equation (10), is a function of an arbitrary \( h \). As such it is representative of the parameter estimation error biases in a single engine, at a given point in its lifetime of use where its deterioration is represented by \( h \). The average sum of squared estimation error biases across a fleet of engines can be calculated as

\[
\bar{x}_{sh,\text{av, fleet}} = E\left[ \bar{x}_{sh,\text{av}}^T \bar{x}_{sh,\text{av}} \right]
\]

\[
= \text{tr}\left( \bar{G}_{sh} P_{h} \bar{G}_{sh}^T \right)
\]

where the matrix \( P_{h} \), defined as \( E[hh^T] \), reflects a priori or historical knowledge of the covariance in the health parameters across all engines. If available, it can be used to predict the sum of squared estimation errors biases as shown in (11).

**Estimation Variance**

Reference 8 also presents the derivation of the augmented state estimate covariance matrix, \( P_{sq,k} \). This matrix can be calculated as a function of the reduced-order state vector estimation covariance matrix, \( P_{sq,k} \), defined as

\[
P_{sq,k} = E\left[ \left( \hat{x}_{sq,k} - E[\hat{x}_{sq,k}] \right) \left( \hat{x}_{sq,k} - E[\hat{x}_{sq,k}] \right)^T \right]
\]

\[
= \begin{bmatrix}
A_{xq} - K_{c}C_{xq}A_{xq} \\
A_{xq} - K_{c}C_{xq}A_{xq}
\end{bmatrix} \ldots \\
+ K_{c} \left[ C(I - A)^{-1}L + M \right] \ldots h
\]

\[
\times \begin{bmatrix}
A_{xq} - K_{c}C_{xq}A_{xq} \\
A_{xq} - K_{c}C_{xq}A_{xq}
\end{bmatrix}^T
\]

\[
+ K_{c} \left[ C(I - A)^{-1}L + M \right] \ldots h
\]

Once \( P_{sq,k} \) is obtained, it can be used to calculate \( P_{sh,k} \), the covariance of \( \hat{x}_{sh,k} \), defined as

\[
P_{sh,k} = E\left[ \left( \hat{x}_{sh,k} - E[\hat{x}_{sh,k}] \right) \left( \hat{x}_{sh,k} - E[\hat{x}_{sh,k}] \right)^T \right]
\]

\[
P_{sh,k} = \begin{bmatrix}
I & 0 \\
0 & V^T
\end{bmatrix} \begin{bmatrix}
P_{sq,k} & I \\
0 & V^T
\end{bmatrix}^T
\]

The augmented state vector estimation covariance given in Equation (14) can be partitioned into covariance information for the original state vector, \( P_{x,k} \) (upper left corner of \( P_{sh,k} \)), and the health parameter vector, \( P_{h,k} \) (lower right corner of \( P_{sh,k} \))

\[
P_{sh,k} = \begin{bmatrix}
P_{x,k} & \cdots \\
\cdots & P_{h,k}
\end{bmatrix}
\]

The variance in \( \hat{x}_{sh,k} \) can be obtained from the diagonal of the covariance matrix produced by (14).
Sum of Squared Estimation Errors

Once Eqs. (11) and (14) are obtained, they may be used to analytically calculate the mean sum of squared estimation errors over all engines by combining the respective estimation error bias and estimation variance. The mean augmented state vector sum of squared estimation errors, \( SSEE(\tilde{x}_{sh,fleet}) \), becomes

\[
SSEE(\tilde{x}_{sh,fleet}) = \tilde{x}_{sh,fleet}^2 + tr\left\{ P_{l,h,k} \right\}
\]

(16)

Similarly, the mean health parameter sum of squared estimation errors is

\[
SSEE(\tilde{h}_{fleet}) = tr\left\{ G_{h,k} P_{h,k} G_{h,k}^T + P_{h,k} \right\}
\]

(17)

From the previous equations it can be observed that both estimation bias and variance are affected by the selection of the sensor suite and the model tuning parameter vector, \( q \), since both contain \( C \) and \( V^r \). The sum of squared estimation errors presented in this section gives rise to an optimization problem: selecting the sensor suite and the tuner vector to minimize the SSEE in the Kalman filter estimates. The approach applied for selecting these parameters will be described in the next section.

Optimal Sensor and Tuner Selection

Prior to initiating the search for an optimal sensor suite, specific system design information must be defined or obtained. This includes:

- Specifying the number of sensors to be added to the baseline sensor suite, and the candidate list of sensors to choose from.
- Generating system state space equations at a fleet average (50 percent deteriorated) engine trim point. Thus at the trim point, the mean value of \( h \) is zero, and the variations in \( h \) for an individual engine are equally likely to be positive or negative.
- Defining measurement noise covariance matrix, \( R \).
- Defining augmented state process noise covariance matrix, \( Q_{sh} \).
- Defining average health parameter covariance, \( P_h \), for a fleet of engines.

After the necessary system information has been obtained, the search for the sensor suite and/or tuner vector to minimize the Kalman filter sum of squared estimation errors can commence. This study assumes that a baseline sensor suite is given, for example a suite of engine control sensors, and it is desired to assess the estimation accuracy improvement that can be gained by adding additional sensors. Other factors such as sensor cost, weight, or reliability are not considered, but could be included by applying a similar strategy to that described in References 12 and 13, which combines the effects of sensor cost and diagnostic effectiveness into a single merit value.

If the target number of sensors (baseline + additional) is greater than or equal to the number of health parameters, and the system is fully observable, the tuner vector will consist of all health parameters, and \( V^r \) simply becomes the identity matrix. Conversely, in the case where there are fewer sensors than health parameters a reduced order tuner vector will be necessary, and consequently \( V^r \) will not equal the identity matrix. The following subsections will first describe sensor selection for a fully observable system, followed by a description of combined sensor and tuner selection for rank deficient cases. In each case the selections will be performed to minimize the Kalman filter SSEE.

Optimal Sensor Selection (Fully Observable System With \( V^r = I \))

For the fully observable estimation case, the relative merits of different candidate sensors suites are assessed by applying an exhaustive brute force search. Given a set of \( r \) additional sensors to choose from, and a target number, \( s \), of additional sensors, the total number of sensor suite combinations will be:

\[
\binom{r}{s} = \frac{r!}{s!(r-s)!}
\]

(18)

One requirement that must be satisfied in this search is that the dimension of the sensor suite (baseline + additional sensors) cannot be less than the dimension of \( h \). The matrix \( V^r \) is set equal to the identity matrix, and the exhaustive search evaluates the SSEE (Equation (17)) for each candidate sensor suite. The sensor suite that produces the minimum SSEE is identified and returned as the optimal choice.

Combined Sensor and Tuner Selection (\( q \) Defined as a Subset of \( h \))

For rank deficient systems, combined sensor and tuner selection will be required. In this study two different tuner selection approaches are considered. These are the conventional approach of selecting \( q \) vectors that are subsets of \( h \), and the new approach described in Reference 8 that selects \( q \) vectors that are combinations of all elements of \( h \). For the case where \( q \) is defined as a subset of \( h \), an exhaustive search of all sensor and tuner vector combinations is applied. Given the following information:

- a set of \( r \) additional sensors to choose from, and a target number, \( s \), of additional sensors, and
- a \( p \times 1 \) health parameter vector, and a sensor suite and consequent \( q \) vector of size \( m \times 1 \),

the total number of sensor and tuner combinations is
Combinations = \( \frac{r!}{s!(r-s)!} \times \frac{p!}{m!(p-m)!} \) \( \text{sensor combinations tuner combinations} \) \( \text{(19)} \)

For each sensor suite and tuner vector combination, the SSEE (Eq. (17)) is calculated. The sensor suite and tuner vector combination found to produce the lowest SSEE is selected as the optimal design.

**Combined Sensor and Tuner Selection (q Defined as a Combination of All Elements of h)**

The second tuner selection approach considered defines \( q \) as a combination of all elements of \( h \). This approach also applies an exhaustive search of all possible sensor combinations. However, the corresponding optimal search for the \( V^* \), that minimizes the SSEE for a given sensor suite is performed by applying a multiparameter optimal iterative search as described in Reference 8. This is done using the `lsqnonlin` function of the Matlab (The MathWorks, Inc.) Optimization Toolbox. The steps in the combined optimal sensor and tuner selection search are:

A. Given a set of \( r \) additional sensors to choose from, and a target number, \( s \), of additional sensors, the number of sensor suite combinations considered will be as shown in Equation (18).

B. For each sensor suite, the optimal iterative search is applied to find the corresponding tuning parameter vector that minimizes the SSEE. The steps in this process are:

1. Generate an initial random guess of \( V^* \).
2. Construct a reduced order state-space model based on \( V^* \), (Eq. (5)).
3. Formulate Kalman filter (Eq. (7)).
4. Calculate the SSEE (Eq. (17)).
5. On each iteration, the change in SSEE relative to the previous iteration is assessed to determine if convergence within a user specified tolerance has been achieved.
   a. If converged, skip step 6 and proceed to step 7.
   b. If not converged, proceed to step 6 to update \( V^* \).
6. \( V^* \) is updated via the Matlab `lsqnonlin` function, and the process returns to step 2.
7. Upon convergence, the routine returns the optimal value of \( V^* \), and ends.

Upon completion of the search, the sensor suite and tuner vector combination found to produce the lowest SSEE is selected as the optimal combination.

**Optimal Sensor and Tuner Selection for Auxiliary (Unmeasured) Output Estimation**

The previous section of this paper presented an approach for the selection of sensors and model tuning parameters for the minimization of health parameter estimation errors. If desired, the process can be reformulated for minimizing the estimation error of unmeasured, or auxiliary, engine outputs such as thrust or stall margin. Linear equations representing auxiliary outputs as functions of the state variables, actuator commands, and health parameters can be written as

\[ z_k = FV_k + GV_k + NH_k \] \( \text{(20)} \)

where \( z \) denotes the auxiliary output vector, and \( F, G, \) and \( N \) are appropriately sized matrices. Following a theoretical derivation similar to the one presented in this paper for \( h \) estimation errors, the Kalman filter auxiliary output sum of squared estimation errors (SSEE) can be obtained as

\[ \text{SSEE}(\hat{z}_{\text{next}}) = \text{tr} \left\{ P_{x,\text{next}} \right\} + \text{tr} \left\{ P_{x,k} \right\} + \text{tr} \left\{ P_{z,k} \right\} \]

\( \text{tr} \left\{ G_i P_{z,k} G_i^T + P_{z,k} \right\} \) \( \text{(21)} \)

The above equation can be used in place of Equation (17) within the methodology to find a design that is optimal for auxiliary parameter estimation. A complete derivation of Equation (21) is given in Reference 8.

**Turbofan Engine Example**

A linearized cruise operating point extracted from the NASA Commercial Modular Aero-Propulsion System Simulation (C-MAPSS) (Ref. 16) high-bypass turbofan engine model is used to evaluate the systematic sensor and tuner selection methodology. The linear model has two state variables and three control inputs (shown in Table 1), and 10 health parameters (shown in Table 2). The engine is assumed to have five baseline sensors, and six additional (optional) sensors are to be assessed for the estimation accuracy improvement they will provide if added individually, or in combination, to the baseline sensor suite. All sensors, along with their sensor noise standard deviations, are shown in Table 3. In this study the model is assumed to run open-loop, so all control inputs remain at 0, i.e., they do not deviate from their trim values for the linear model, and no actuator bias is present. Deviations in all ten health parameters are assumed to be uncorrelated, and randomly shifted from their trim conditions with a standard deviation of \( \pm 0.02 \) (\( \pm 2 \) percent). Since a parameter’s variance is equal to its standard deviation squared, the health parameter covariance matrix, \( P_{h} \), is defined as a diagonal matrix with all diagonal elements equal to 0.0004.

**TABLE 1.——STATE VARIABLES AND ACTUATORS**

<table>
<thead>
<tr>
<th>State variables</th>
<th>Actuators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nf—fan speed</td>
<td>Wf—fuel flow</td>
</tr>
<tr>
<td>Ne—core speed</td>
<td>VSV—variable stator vane</td>
</tr>
<tr>
<td></td>
<td>VBV—variable bleed valve</td>
</tr>
</tbody>
</table>
Theoretical Kalman Filter Estimation Errors

The methodology described in the previous section was applied to assess the theoretical Kalman filter estimation errors that can be obtained through optimal sensor and tuner selection for the given test case. This assessment was performed for the baseline sensor suite containing five sensors, and sensor suites of successively increasing size, ranging from 6 to 11 sensors. For sensor suites consisting of 10 or 11 sensors, the tuner vector was simply set to the entire 10 element health parameter vector (i.e., \( q = h \) and \( V^* = I \)). For rank deficient cases (i.e., fewer sensors than health parameters), combined sensor and tuner selection was performed. For these cases, theoretical mean SSEE’s were formulated by applying the following two tuner selection approaches:

- Selecting \( q \) as a subset of \( h \) to minimize the health parameter \( (q) \) SSEE.
- Selecting \( q \) as a combination of all elements of \( h \) to minimize the health parameter \( (h) \) SSEE.

<table>
<thead>
<tr>
<th>TABLE 2.—HEALTH PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
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<tr>
<td>1</td>
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<tr>
<td>2</td>
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<tr>
<td>3</td>
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<td>10</td>
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</table>

<table>
<thead>
<tr>
<th>TABLE 3.—SENSED OUTPUTS AND STANDARD DEVIATION AS PERCENT OF OPERATING POINT TRIM VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
</tr>
<tr>
<td>Baseline Sensors</td>
</tr>
<tr>
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<td>3</td>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
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<tr>
<td>Additional Sensors</td>
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<td>3</td>
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<tr>
<td>4</td>
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<td>5</td>
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<td>6</td>
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</tbody>
</table>

Theoretical estimation errors for the two cases are summarized in Table 4 and Table 5. The SSEE values were obtained by taking the trace of the health parameter estimation error covariance matrix as shown in Equation (17). Theoretical mean squared estimation errors for individual health parameters were obtained from the diagonal of that same matrix. Readers will notice that the bottom two rows of Table 4 and Table 5 are identical. These rows correspond to the 10 and 11 element sensor suite cases. In these cases the tuner vector is simply set equal to the health parameter vector.

<table>
<thead>
<tr>
<th>TABLE 4.—HEALTH PARAMETER THEORETICAL MEAN SQUARED ESTIMATION ERRORS (( q ) SELECTED AS A SUBSET OF ( h )^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
</tr>
<tr>
<td>P24</td>
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<tr>
<td>5</td>
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<td>6</td>
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<tr>
<td>10</td>
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<tr>
<td>11</td>
</tr>
</tbody>
</table>

*Bold italic font denotes health parameter used as a tuning parameter.

<table>
<thead>
<tr>
<th>TABLE 5.—HEALTH PARAMETER THEORETICAL MEAN SQUARED ESTIMATION ERRORS (( q ) IS A COMBINATION OF ALL ELEMENTS OF ( h ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
</tr>
<tr>
<td>P24</td>
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<tr>
<td>5</td>
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<tr>
<td>6</td>
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<td>7</td>
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<tr>
<td>10</td>
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<tr>
<td>11</td>
</tr>
</tbody>
</table>

The methodology described in the previous section was applied to assess the theoretical Kalman filter estimation errors that can be obtained through optimal sensor and tuner selection for the given test case. This assessment was performed for the baseline sensor suite containing five sensors, and sensor suites of successively increasing size, ranging from 6 to 11 sensors. For sensor suites consisting of 10 or 11 sensors, the tuner vector was simply set to the entire 10 element health parameter vector (i.e., \( q = h \) and \( V^* = I \)). For rank deficient cases (i.e., fewer sensors than health parameters), combined sensor and tuner selection was performed. For these cases, theoretical mean SSEE’s were formulated by applying the following two tuner selection approaches:

- Selecting \( q \) as a subset of \( h \) to minimize the health parameter \( (q) \) SSEE.
- Selecting \( q \) as a combination of all elements of \( h \) to minimize the health parameter \( (h) \) SSEE.

Theoretical estimation errors for the two cases are summarized in Table 4 and Table 5. The SSEE values were obtained by taking the trace of the health parameter estimation error covariance matrix as shown in Equation (17). Theoretical mean squared estimation errors for individual health parameters were obtained from the diagonal of that same matrix. Readers will notice that the bottom two rows of Table 4 and Table 5 are identical. These rows correspond to the 10 and 11 element sensor suite cases. In these cases the tuner vector is simply set equal to the health parameter vector.
are a subset of the health parameter vector (Table 4). The and Table 5 are not consistent—different sensors are selected depending upon the tuner selection approach applied.

Another observation drawn from these data is that for underdetermined estimation scenarios the approach that selects tuner parameters as a linear combination of all health parameters (Table 5) provides improved SSEE results compared to the conventional approach of selecting tuners that are a subset of the health parameter vector (Table 4). The improvement is more pronounced when fewer sensors are available. As more sensors are added, the benefit of the new tuner selection approach becomes progressively less.

Readers are reminded that the optimization strategy seeks to determine the sensor suite and tuner vector combination that minimizes the mean sum of squared estimation errors in the entire health parameter vector. The estimation accuracy of individual health parameters can vary widely. This is illustrated in Figure 1 for three of the cases shown in the Table 4 and Table 5 including:

- a) Five sensors with defined as a subset of ; first row of Table 4 (black bars)
- b) Seven sensors with defined as a subset of ; third row of Table 4 (grey bars)
- c) Seven sensors with defined as a combination of all elements of ; third row of Table 5 (white bars)

This figure shows the mean squared estimation error for each of the 10 health parameters as opposed to the SSEE for the entire health parameter vector.

For the two cases in which is selected as a subset of (cases and ), any instance where the mean squared estimation error is exactly equal to 4.0 (shown in squared percentage units) is indicative of a health parameter excluded from the tuner vector. This is expected given the defined for this example. In general, case c has the best estimation accuracy, but this is not true in all cases. For example, case b shows improved estimation accuracy for the and health parameters. However, the overall SSEE for case b (20.51) is much worse than that for case c (15.06).

Experimental Kalman Filter Estimation Errors

Simulation studies were conducted to experimentally validate the theoretically predicted Kalman filter health parameter estimation errors. This experimental validation was conducted for the same three cases described above (cases a, b, and c). A separate Kalman filter was designed for each case. The experimental results were obtained through a Monte Carlo simulation analysis where the health parameters were randomly distributed in accordance with the defined covariance matrix, . The test cases were concatenated to produce a single time history input that was provided to the C-MAPSS linear discrete state space model given in Equation (1), with an update rate of 15 ms. Each individual health parameter test case lasted 30 sec. A total of 375 30 sec test cases were evaluated, resulting in an 11,250 s input time history.

The theoretical and experimental estimation error results, shown in Table 6, exhibit good agreement. A visual illustration of the effect that sensor and tuner selection has on Kalman filter health parameter estimation accuracy can be seen in Figures 2 to 4, which show actual and estimated results for three of the 10 health parameters (, and ). The three subplots shown in each figure correspond to the three different Kalman filter designs (cases a, b, and c). Each plot shows a 300 sec segment of the evaluated test cases. The step changes that can be observed in each plot every 30 sec correspond to a transition to a different health parameter vector. True model health parameter inputs are shown in black, and Kalman filter estimates are shown in red. These plots corroborate the information shown in Table 6. Figure 2 shows that for (HPT efficiency) estimation, case c has the best estimation accuracy, followed by case b, and finally case a. Figure 3 shows that for (HPT flow capacity) all three Kalman filters are able to provide good estimates. Figure 4 shows that for (LPT efficiency), the estimates in case b are superior to those in case c. Case a excludes from , and thus its corresponding estimate in the top subplot of Figure 4 is invariant.
### TABLE 6.—COMPARISON OF THEORETICAL AND EXPERIMENTAL MEAN SQUARED ESTIMATION ERRORS

<table>
<thead>
<tr>
<th>Case</th>
<th>Sensors added to baseline</th>
<th>Mean squared error</th>
<th>Health parameter mean squared estimation errors (percent squared)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\eta_{\text{FAN}}$</td>
</tr>
<tr>
<td>a</td>
<td></td>
<td></td>
<td>4.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Theoretical</td>
<td>4.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Experimental</td>
<td>3.56</td>
</tr>
<tr>
<td>b</td>
<td>x</td>
<td></td>
<td>4.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Theoretical</td>
<td>3.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Experimental</td>
<td>2.57</td>
</tr>
</tbody>
</table>

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**Figure 2.** $\eta_{\text{HPT}}$ estimation (sensor and tuner selection comparison).

**Figure 3.** $\gamma_{\text{HPT}}$ estimation (sensor and tuner selection comparison).

**Figure 4.** $\eta_{\text{LPT}}$ estimation (sensor and tuner selection comparison).
These experimental results validate the theoretical developments in this paper for the sensor and tuner selection methodology. Furthermore the results show that although overall (global) health parameter vector estimation accuracy is optimized, the relative estimation accuracy of individual elements of the health parameter vector may vary.

Discussion

The results presented in this paper suggest that the sensor and tuner selection process cannot be performed devoid of corresponding knowledge regarding the intended end use or application. This was illustrated by the different sensor suites selected for health parameter estimation (see Tables 4 and 5) when different tuner vector formulations were considered. A further illustration of this fact can be shown by optimizing sensor suite and tuner vector selection for auxiliary (unmeasured) engine output estimation (Eq. (21)), and comparing the results to those obtained when optimizing for health parameter estimation (Eq. (17)). To conduct this comparison the following two sensor and tuner selection problems were established (similar to the one previously described for case e health parameter estimation) for auxiliary output estimation:

Case d:
- Assume that the five baseline sensors previously shown in Table 3 are available.
- Select the same two additional sensors optimal for $h$ estimation in case e (T30 and P45).
- Optimally select the tuner vector (where $q$ is defined as a combination of all elements of $h$) that minimizes the auxiliary parameter SSEE (Eq. (21)) in the following four outputs:
  - T40: combustor temperature
  - T50: LPT exit temperature
  - Fn: net thrust
  - SmLPC: LPC stall margin

Case e:
- Assume that the five baseline sensors previously shown in Table 3 are available.
- Consider all possible combinations of two additional sensors from the six optional sensors shown in Table 3.
- For each sensor combination, optimally select the tuner vector (where $q$ is defined as a combination of all $h$) that minimizes the SSEE in the auxiliary parameters of interest (same auxiliary parameters shown for case d).
- Choose the sensor suite and tuner vector combination that minimizes the SSEE as the optimal design.

The sensor selection and auxiliary output estimation results for case c, d, and e are shown in Table 7. In comparing case e and d, it can be seen that given the same sensor suite (baseline + T30 and P45), choosing a tuner vector optimal for $z$ estimation instead of $h$ estimation reduces the $z$ SSEE by approximately 30 percent. If combined sensor and tuner selection is applied (case e) an 87 percent reduction can be gained compared to case c.

<table>
<thead>
<tr>
<th>Case and optimization objective</th>
<th>Sensors added to baseline</th>
<th>SSEE (z) results</th>
</tr>
</thead>
<tbody>
<tr>
<td>case c: $h$ estimation</td>
<td>P24 T30 T45</td>
<td>188.1</td>
</tr>
<tr>
<td>case d: $z$ estimation</td>
<td>P24</td>
<td>132.8</td>
</tr>
<tr>
<td>case e: $z$ estimation</td>
<td>P24 T30</td>
<td>24.9</td>
</tr>
</tbody>
</table>

The sensors selected through the approach presented in this paper can also be compared to those selected through the Systematic Sensor Selection Strategy (S4) presented in Reference 13. Reference 13 also used C-MAPSS to perform sensor selection. One example shown in Reference 13 assumed a baseline suite of commanded fuel flow and six sensed measurements, and the merits of adding additional sensors were considered. The six baseline and five optional sensors used in that study are shown in Table 8. In order to compare results, the approach presented in this paper (where $q$ is a combination of all elements of $h$ for rank deficient problems) was applied to this same test case.

The results obtained by applying the two strategies are shown in Table 9. Both approaches produced identical selections for the 8 and 10 element sensor suite cases, and both select T21 as the last sensor added (11 element sensor suite case). However, they produce different selections for the seven and nine element sensor suite cases. The fact that the two approaches do not produce identical results in all cases is not surprising as there are fundamental differences in the objectives of the two approaches. The work in Reference 13 focused on the diagnosis of five fault types assumed to occur in isolation, whereas this paper focuses on the estimation...
accuracy of 10 health parameters that are uncorrelated and can vary simultaneously. Also, the focus of Reference 13 was on the fault detection and isolation capabilities offered by the selected sensor suite. Estimating or quantifying the fault magnitude was not considered. Furthermore, in Reference 13 sensor selection was based on two operating points—takeoff and cruise. The different sensor selection results, while perhaps not unexpected, do illustrate the importance of performing sensor selection with the intended application in mind.

Table 9.—Comparison of sensor suites selected applying S4 versus this paper’s approach

<table>
<thead>
<tr>
<th>No.</th>
<th>Sensors added to baseline (S4)</th>
<th>Sensors added to baseline (this paper’s approach)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>P15 T21 P24 T50</td>
<td>P15 T21 P24 T50</td>
</tr>
<tr>
<td>7</td>
<td>x</td>
<td>x</td>
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<tr>
<td>8</td>
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<td>9</td>
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<td>10</td>
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<tr>
<td>11</td>
<td>x x x x</td>
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</table>

Conclusions

A systematic approach to combined sensor suite and model tuning parameter selection for on-line Kalman filter-based health parameter estimation has been presented. The Kalman filter estimation accuracy improvement that could be gained by adding additional sensors was theoretically predicted and experimentally validated through simulation studies. Of the two model-tuning parameter selection approaches considered, the new approach of selecting the model tuner vector as a linear combination of all health parameters was found to provide improved estimation accuracy over the conventional approach of selecting a subset of health parameters to serve as model tuner vector. The new technique was demonstrated to consistently produce smaller sum of squared health parameter estimation errors, and in most cases produce smaller errors in individual health parameters estimates as well. The methodology presented in this paper is general, and can facilitate combined sensor and tuner parameter selection for different objective functions such as health parameter estimation or auxiliary output estimation. The systematic sensor and tuning parameter selection methodology is envisioned to be a valuable tool for system designers to assess the estimation accuracy enabled by different design choices. Areas for future work include extending the technique to select sensors and tuning parameters optimal over a range of operating conditions, and evaluating the technique on a non-linear engine model, under both steady-state and transient operating conditions.

References

A systematic approach for selecting an optimal suite of sensors for on-board aircraft gas turbine engine health estimation is presented. The methodology optimally chooses the engine sensor suite and the model tuning parameter vector to minimize the Kalman filter mean squared estimation error in the engine’s health parameters or other unmeasured engine outputs. This technique specifically addresses the underdetermined estimation problem where there are more unknown system health parameters representing degradation than available sensor measurements. This paper presents the theoretical estimation error equations, and describes the optimization approach that is applied to select the sensors and model tuning parameters to minimize these errors. Two different model tuning parameter vector selection approaches are evaluated: the conventional approach of selecting a subset of health parameters to serve as the tuning parameters, and an alternative approach that selects tuning parameters as a linear combination of all health parameters. Results from the application of the technique to an aircraft engine simulation are presented, and compared to those from an alternative sensor selection strategy.

Aircraft engines; Systems health monitoring; Gas turbine engines; Kalman filtering; State estimation