**COSMOLOGICAL EFFECTS IN PLANETARY SCIENCE.** H.-J. Blome[1] and T. L. Wilson[2], [1]University of Applied Sciences, Hohenstaufenallee 6, 52066 Aachen, Germany, [2]NASA, Johnson Space Center, Houston, TX 77058,

**Introduction:** In an earlier discussion of the planetary flyby anomaly [1], a preliminary assessment of cosmological effects upon planetary orbits exhibiting the flyby anomaly was made. A more comprehensive investigation has since been published [2], although it was directed at the Pioneer anomaly and possible effects of universal rotation. The general subject of Solar System anomalies will be examined here from the point of view of planetary science.

**Cosmological Effects on Local Systems:** Following the discovery of the expansion of the Universe, the question arose whether universal expansion necessarily means that bound systems like the Solar System and galaxies within it also expand due to the Hubble expansion of space-time [3]. The basic issue remains unresolved today. The consensus of opinion has been negative, namely that the expansion causes miniscule changes in local dynamics that are too small to be measured if they occur at all. Another opinion is that local cosmological effects are in fact not measurable quantities – a point of view following from quantum physics.

The question indirectly involves the cosmological constant problem (CCP) in General Relativity (GR), having to do with the disparity between the vacuum energy density predicted by Einstein and found in astrophysics, when compared to that required in the hadron model of particle physics. The CCP amounts to a fine-tuning problem on the order of ~$10^{56}$ orders of magnitude. This has led to a new strategy known as dark energy (matter), and an attribute called quintessence where the cosmological constant $\Lambda$ is an *ad hoc* function of time, $\Lambda(t)$ [4]. Obviously, contemporary physics is inconsistent if $\Lambda$ has to be tuned by such a great amount.

If $\Lambda$ is set to zero, many of these inconsistencies vanish. However, $\Lambda$ is not zero because the Universe is currently measured to be accelerating [5]. The term also appears in the precession of planetary orbits predicted by Einstein's theory of gravitation – hence its debut in planetary science. Although mathematically present, the miniscule $\Lambda$-term is simply not measurable on planetary scales of Keplerian conic physics. Regardless, it definitely plays a role in planetary science.

In addition, the failure to explain the Pioneer anomaly [6-8] as an experimental measurement of spacecraft orbits on planetary scales has inspired an interesting discussion in theoretical physics regarding the same long-standing questions about $\Lambda$, dark energy, and the expanding Universe expressed in terms of the Hubble expansion parameter $H$ in cosmology. Do atomic and planetary orbits change with universal expansion or not, and what does this have to do with $H$? This subject has been addressed with renewed interest [9-15, 2], and a brief summary of its derivation follows.

**The Two-Body Problem in an Accelerating, Expanding Universe:** Suppose that one wants to study the local dynamics of a closed, bound state such as a solar system or a satellite in Earth orbit. The equation of motion involved has been derived from GR using the method of geodesic deviation [2, §3.2, Eq. (3c)],

$$\frac{dx^j}{d\tau^2} = -\nabla \Phi_{Eff} - {}^F R^j_{0k0}\, x_F{}^k \qquad (1)$$

where the first term on the right-hand side is the standard expression of Newton's law and the second term represents the Riemann curvature effects (classically speaking, the gravitational gradients) produced by the background geometry of space-time.

To discuss cosmological effects, one must next define a background metric in order to derive the second term. For the standard model of Big Bang cosmology, the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric is adopted and the details can be found elsewhere [2, §3.3]. The Riemann curvature tensor in (1) becomes ${}^F R^j{}_{0k0} = \ddot{a} / a$ where $a$ is the FL scale factor of expansion, and the dots represent differentiation with respect to time. Hence $\ddot{a}$ and $\dot{a}$ represent the acceleration and velocity of $a$ respectively. When a distribution of matter is specified [2, Eq. (7b)], $\ddot{a} / a$ is determinable from GR. It necessarily involves $\Lambda$.

Going to spherical coordinates $(r,\theta,\varphi)$ for an object $m$ in orbit about a much larger mass $M$ ($m<<M$), the result is a bound state whose orbital eccentricity is $e<1$. For the sake of simplicity, (1) for a circular orbit ($e=0$) of radius $r$ is

$$\ddot{\vec{r}} = -\frac{GM}{r^2}\hat{r} + \frac{\ddot{a}}{a}\vec{r} \quad , \qquad (2)$$

which basically is Newton's law for the Kepler two-body problem, but it is modified by an acceleration term arising from universal FLRW expansion. (2) is the equation of motion for an object such as Pioneer on a

trajectory in the Solar System, now modified by the expanding Universe. Here $\hat{r}$ is a radial unit vector, $G$ is Newton's gravitational constant, and $M$ is the source mass of the gravitational Newtonian potential.

**Hubble Parameter In Planetary Dynamics:** If instead of $\ddot{a}/a$ one focuses upon $\dot{a}/a$, where $\dot{a}$ is the velocity of change of $a$, Hubble's law results. It states that the velocity of expansion is proportional to the distance $a$: $\dot{a} = Ha$. Obviously $H$ is not constant, rather it is known as the Hubble parameter. Using the subscript naught or cipher for the current epoch of cosmology with $H_o$ = 70 km Mpc$^{-1}$ s$^{-1}$ as the present value of $H$, the age of the FL universe is $t = t_o$= 13.5 Gyr, and $\Lambda$ = 1.2 x 10$^{-52}$ cm$^{-2}$ [2].

Convention in FLRW cosmology also defines a deceleration parameter $q$. This term is expressed as $q = -\ddot{a}a/\dot{a}^2$, where $q \rightarrow q_o$ represents the value of $q$ today. To visualize $q$, it has been plotted in Figure 1. Throughout the age $t_o$ = 13.5 Gyr of the FL Universe, $q$ is seen to vary considerably. This universe even undergoes a coasting transition when $q$~0 and $t$ ~ 7.55 Gyr old.

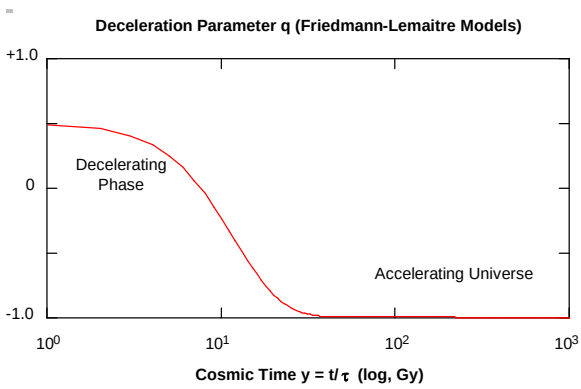


Figure 1. The deceleration parameter $q$ in FL models. Adapted from [5] where $\tau$ is a scaling parameter.

A simple calculation using $q$ and $H$ shows that one has the relation $\ddot{a}/a = -qH^2$ whereby (2) becomes

$$\ddot{\vec{r}} = -\frac{GM}{r^2}\hat{r} - qH^2\vec{r}\,, \tag{3a}$$

or today ($t \rightarrow t_o$)

$$\ddot{\vec{r}} = -\frac{GM}{r^2}\hat{r} - q_o H_o^{\,2}\vec{r}\,. \tag{3b}$$

The deceleration parameter can likewise be expressed as $q_o = (\frac{1}{2}\Omega_m - \Omega_\Lambda)$ in terms of the density parameter for matter ($\Omega_m$) and the cosmological constant ($\Omega_\Lambda$). Note that in FLRW cosmology (3) has the wrong sign for the Pioneer effect if the Universe is accelerating. The correct sign exists only if we are living in a decelerating Universe, or possibly a low density open Universe with non-accelerated expansion ($\Lambda<0$) [16-18].

**Cosmology or Coincidence?:** It may be a curious coincidence that the Solar System formed 4.6 Gyr ago around the same epoch that the FL universe became $\Lambda$-dominated at a redshift $z_{eq}$ = 0.33. The transition to an accelerated expansion began earlier at redshift $z_*$ = 0.67. With $H_o$ = 71 km Mpc$^{-1}$ s$^{-1}$, $\Omega_m$ = 0.3, and $\Omega_\Lambda$ = 0.7, it follows that this happens at $t_*$ = 7.2 Gyr and $t_{eq}$ = 9.2 Gyr for an accelerating FL model with Euclidean space. This means that the following relation holds: $t_* < t_{eq} < t_o$ = 13.7 Gyr.

The shape of a given trajectory in space is found by the solution of Binet's equation which can be derived from (3) using the conservation of angular momentum. Its first integral is known as Clairaut's equation and these results will be published elsewhere.

**Conclusions:** The cosmological effects on the local dynamics of planetary systems has been derived from FLRW cosmology, resulting in (3). This effect involves an acceleration/deceleration term that is regulated both by the deceleration parameter $q_o$ and the Hubble parameter $H_o$. One can see in Figure 1 the dramatic effect of $q$ and $H$ in (3), even during the formation of galaxies and solar systems in the latter stages of the evolution of the FL universe when $q \neq q_o$ and $H \neq H_o$, that is when planetary science became a reality.

**References:** [1] Blome H.-J., Wilson T.L. (2008) *LPS, 40,* 1704. [2] Wilson T.L., Blome H.-J. (2009), *Adv. Spa. Res., 44,* 1345-1353. [3] McVittie G.C. (1933), *Mon. Not. Roy. Astron. Soc., 93,* 325-339. [4] Peebles P.J.E., Ratra B. (2003), *Rev. Mod. Phys., 75,* 559-606. [5] Blome H.-J., Wilson T.L. (2005), *Adv. Spa. Res., 35,* 111-115. [6] Anderson J. D. et al. (2008) *Phys. Rev. Lett. 100,* 091102. [7] Anderson J. D. et al. (2002) *Phys. Rev. D 65,* 082004. [8] Anderson J. D. et al. (1998) *Phys. Rev. Lett. 81,* 2858-2861. [9] Cooperstock F.I., Faraoni V., Vollick D.N. (1998), *Ap. J., 503,* 61-66. [10] Adkins G.S., McDonnell J., Fell R.N. (2007), *Phys. Rev. D, 75,* 064011. [11] Anderson J.L. (1995), *Phys. Rev. Lett., 75,* 3602-3604. [12] Mena F.C., Tavakol R., Vera R. (2002), *Phys. Rev. D, 66,* 044004. [13] Bonnor W.B. (1999), *Class. Quant. Grav., 16,* 1313-1321. [14] Bonnor W.B. (1996), *Mon. Not. Roy. Astron. Soc., 282,* 1467-1469. [15] Bonnor W.B. (1987), *Ap. J., 316,* 49-51. [16] Coles P., Ellis G.F.R. (1994), *Nature, 370,* 609-615. [17] Coles P., Ellis G.F.R. (1997), *Is the Universe Open or Closed? The Density of Matter in the Universe,* Cambridge University Press, 236p. [18] Coles P. (2005), *Nature, 433,* 248-256.