Error Rates and Channel Capacities in Multipulse PPM

It is now possible to compare expected performances of candidate modulation schemes.

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A method of computing channel capacities and error rates in multipulse pulse-position modulation (multipulse PPM) has been developed. The method makes it possible, when designing an optical PPM communication system, to determine whether and under what conditions a given multipulse PPM scheme would be more or less advantageous, relative to other candidate modulation schemes.

In conventional $M$-ary PPM, each symbol is transmitted in a time frame that is divided into $M$ time slots (where $M$ is an integer $>1$), defining an $M$-symbol alphabet. A symbol is represented by transmitting a pulse (representing “1”) during one of the time slots and no pulse (representing “0”) during the other $M - 1$ time slots. Multipulse PPM is a generalization of PPM in which pulses are transmitted during two or more of the $M$ time slots. If the number of pulses per symbol is $n$, then the number of symbols in the alphabet is given by the binomial coefficient

$$C_n^M = M!/[n!(M-n)!].$$

The method is based partly on an analysis of the conditional probability, $p_1(y)$ or $p_0(y)$, that the actual value, $y$, of the noisy signal detected in a receiver during a given time slot represents a transmitted 1 or a transmitted 0, respectively. For purposes of the analysis, the signal-propagation channel is assumed to be memoryless. The analysis includes consideration of $L(y) = p_1(y)/p_0(y)$, defined as the likelihood ratio for receiving value $y$ during the time slot. It is assumed that $L(y)$ is finite and, as is true for many channels, most likely to have been transmitted, the digitized value of the detected signal (e.g., the number of detected photons) in a “1” slot is less than or equal to $s$, and $p_0(s)$ is the probability that the digitized value of the detected signal in a “0” slot is less than or equal to $s$. The figure presents an example of SER values calculated by use of these equations.

Next, a comparative analysis of throughput achievable in conventional and multipulse PPM under bandwidth, average-power, and peak-power constraints leads to the following equation for the channel capacity:

$$C = \frac{n}{M}E_{x=1} \log \frac{p_0(Y)}{p(Y)} + \frac{M-n}{M}E_{x=0} \log \frac{p_0(Y)}{p(Y)} \text{ bits/slot},$$

where $p(Y) = (s/M)p_1(Y) + [(M-n)/M]p_0(Y)$ is the probability mass function for a randomly chosen slot and $E_{x=1}$ or $E_{x=0}$ is the expected value of signal level $Y$ in a slot for which the transmitted signal value, $x$, was 1 or 0, respectively.

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SER Values were calculated for 16-ary PPM using several different values $n$ and two different noise levels in a Poisson channel.