Directivity of a Sparse Array in the Presence of Atmospheric-Induced Phase Fluctuations for Deep Space Communications

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Abstract

Widely distributed (sparse) ground-based arrays have been utilized for decades in the radio science community for imaging celestial objects and for various astrometric measurements. By correlating measurements taken by several widely separated antennas, an effective aperture area of the distance between the furthest separated antennas is created, drastically increasing the spatial resolution of the system. However, the natural "seeing" ability of a particular site will be fundamentally limited by the local atmospheric-induced phase fluctuations imposed on the signal. This phenomenon is a result of large amounts of inhomogeneous distributions of water vapor exposed to turbulent air flow conditions in Earth's upper atmosphere (troposphere), which directly leads to variations in the effective electrical path length (phase) of a received signal. Such variations are seen as 'phase noise' and will inherently degrade the resolution of radio arrays.

The same issues arise when developing sparse arrays for intrastellar communications. But whereas radio science applications impose only receive-mode requirements (i.e., imaging) and observations are made on the order of minutes to hours (long integration times), communications applications require both transmit and receive capabilities, as well as real-time corrections over time scales as short as fractions of a second. In the receive case, adaptive techniques have been utilized by the DSN since the 1980s to compensate for the atmosphere at frequencies up to X-band (Ref. 1). More recently, uplink arraying of a 7.15 GHz signal was successfully demonstrated in an experiment with the Mars Global Surveyor (Ref. 2). However, since this atmospheric phase noise scales with frequency, at Ka-band (the frequencies of interest for future NASA DSN operations) the problem becomes much more severe and has yet to be successfully resolved, particularly in the uplink. In this paper, an analysis of the performance of a sparse antenna array, in terms of its directivity, is performed to derive a closed form solution to the expected array loss in the

1.0 Introduction

Widely distributed (sparse) ground-based arrays have been utilized for decades in the radio science community for imaging celestial objects and for various astrometric measurements. By correlating measurements taken by several widely separated antennas, an effective aperture area of the distance between the furthest separated antennas is created, drastically increasing the spatial resolution of the system. However, the natural "seeing" ability of a particular site will be fundamentally limited by the local atmospheric-induced phase fluctuations imposed on the signal. This phenomenon is a result of large amounts of inhomogeneous distributions of water vapor exposed to turbulent air flow conditions in Earth's upper atmosphere (troposphere), which directly leads to variations in the effective electrical path length (phase) of a received signal. Such variations are seen as 'phase noise' and will inherently degrade the resolution of radio arrays.

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presence of atmospheric-induced phase fluctuations. The theoretical derivation for array directivity
degradation is validated with interferometric measurements for a two-element array taken at Goldstone,
California. Excellent agreement between theory and measurement is observed for the case of a two-
element array. With the validity of the model established, an arbitrary 27-element array geometry is
defined at Goldstone, California, to ascertain its theoretical performance in the presence of phase
fluctuations.

2.0 Theory

Consider a widely distributed (d >> λ) array of N elements arbitrarily spaced on a plane, whose
geometry is defined in Figure 1, where \( \hat{a}_r \) is a unit vector in the direction of propagation, \( \vec{p}_m \) is the
distance from the array origin to the mth element, \( R \) is the distance the signal travels from the array origin
to the receiver, and \( r_m \) is the distance the signal travels from the mth element to the receiver.

Let us assume that each element has its maximum radiation intensity in the +z direction and has zero
radiation in the lower half-space (z < 0). Further, we will utilize the well known pattern approximation of
\( \cos^q(\theta) \) for the E- and H- planes of the antennas to simplify the analysis. Thus, for an arbitrarily polarized
antenna element, we can represent its far field electric field, \( \vec{E}_m \), in the upper half-space \((0 \leq \theta \leq \pi/2)\) by

\[
\vec{E}_m(r, \theta, \phi) = I_m \left( \frac{e^{-jkR}}{4\pi R} \right) [\hat{\Theta}V_{Em}(\theta)(ae^{jw} \cos \phi + b \sin \phi) + \hat{\Phi}V_{Hm}(\theta)(-ae^{jw} \sin \phi + b \cos \phi)]
\]

where,

\[
I_m = c_m e^{j \psi_m} = \text{complex excitation coefficient}
\]

\( k = 2\pi/\lambda = \text{wavenumber} \)

\( V_{Em}(\theta) = \cos^q(\theta) = \text{E—plane voltage pattern approximation} \)

\( V_{Hm}(\theta) = \cos^q(\theta) = \text{H—plane voltage pattern approximation} \)

\((a, b, \psi) = \text{polarization coefficients (see Table I) }\)

| TABLE I.—VARIOUS FEED POLARIZATIONS |
|-----------------|-----|-----|
| Linear X       | 1   | 0   | 0   |
| Linear Y       | 0   | 1   | 0   |
| RHCP           | \( \sqrt{2} \) | \( \sqrt{2} \) | \( \pi/2 \) |
| LHCP           | \( \sqrt{2} \) | \( \sqrt{2} \) | \( -\pi/2 \) |

If we translate this analysis to an array environment, the array of antennas will produce a far field
electric field that will consist of the superposition of each individual element field plus a propagation
delay, as determined by the geometry, relative to some specified origin (see inset of Fig. 1). This will
result in an array far field, \( \vec{E}_{array} \), which can be represented by the product of the individual element far
field and an array factor.

\[
\vec{E}_{array}(r, \theta, \phi) = \hat{f}(\theta, \phi) \left( \frac{e^{-jkR}}{4\pi R} \right) \sum_{m=1}^{N} I_m e^{-jk\hat{a}_r \cdot \vec{p}_m}
\]

where,

\[
\hat{f}(\theta, \phi) = \hat{\Theta}V_{Em}(\theta)(ae^{jw} \cos \phi + b \sin \phi) + \hat{\Phi}V_{Hm}(\theta)(-ae^{jw} \sin \phi + b \cos \phi)
\]

\( \hat{a}_r = \text{unit vector in direction of propagation} \)

\( \vec{p}_m = \text{position vector of element relative to array origin} \)
From the far field array pattern, the radiation intensity, $U(\theta, \phi)$, and radiated power, $P_{rad}$, of an array can be determined, whose ratio defines the array directivity. The complete derivation of the array directivity, which involves reducing Bessel functions of 2nd order (Ref. 3), are omitted here for conciseness, but if we assume the antenna elements are identical, spaced many wavelengths apart, and there are no pointing losses to consider, then the peak directivity, $D_{peak}$, as a function of deterministic phase, $\phi$, can be approximated by

**Peak Radiation Intensity:**

$$U(\theta_{peak}, \phi_{peak}) = \frac{r^2}{2\pi} |\mathbf{E}(r, \theta_{peak}, \phi_{peak})|^2 = \frac{1}{32\pi^2} \sum_{m=1}^{N} \sum_{n=1}^{N} e^{i(\phi_m - \phi_n)}$$

**Radiated Power:**

$$P_{rad} = \frac{1}{16\pi n} \left[ \frac{q_{E} + q_{H} + 1}{(2q_{E} + 1)(2q_{H} + 1)} \right] N$$

**Peak Directivity:**

$$D_{peak} = \frac{4\pi U(\theta_{peak}, \phi_{peak})}{P_{rad}} = D_{element} \frac{1}{N} \sum_{m=1}^{N} \sum_{n=1}^{N} e^{i(\phi_m - \phi_n)}$$

where,

$$D_{element} = \frac{2(2q_{E} + 1)(2q_{H} + 1)}{q_{E}q_{H} + 1}$$

### 2.1 Array Directivity in Presence of Random Phase

The above analysis assumes a constant deterministic phase between elements, but if we now suppose that random phase fluctuations are present during signal transmission, such as those induced by water vapor in the atmosphere, the statistical distribution of the random process can be utilized to obtain a closed-form solution. Let us assume (and later confirm this assumption) that the phase fluctuations induced by the atmosphere are normally distributed over the time scale of interest with mean zero and variance $\sigma^2$. The average peak directivity of an array, in terms of the statistical ensemble average of the random phase fluctuations between elements $m$ and $n$, can be determined, in closed form, as
\[
\langle D_{\text{peak}} \rangle = D_{\text{element}} \frac{1}{N} \sum_{m=1}^{N} \sum_{n=1}^{N} (e^{j\phi_{mn}})
\]

\[
= D_{\text{element}} \frac{1}{N} \sum_{m=1}^{N} \sum_{n=1}^{N} (\cos \delta_{mn} + j\sin \delta_{mn})
\]

\[
= D_{\text{element}} \frac{1}{N} \sum_{m=1}^{N} \sum_{n=1}^{N} \int_{-\infty}^{\infty} \cos \delta_{mn} \frac{1}{\sqrt{2\pi}\sigma_{mn}} e^{-\frac{\delta_{mn}^2}{2\sigma_{mn}^2}} d\delta
\]

\[
= D_{\text{element}} \frac{1}{N} \sum_{m=1}^{N} \sum_{n=1}^{N} e^{-\frac{\sigma_{mn}^2}{2}}
\]

where we have defined \( \delta_{mn} \equiv \phi_m - \phi_n \). The average array loss can then be described by the difference (in dB) of the ideal directivity and the actual directivity achieved in the presence of the phase fluctuations.

\[
\langle D_{\text{loss}} \rangle = 10 \log(ND_{\text{element}}) - 10 \log(\langle D_{\text{peak}} \rangle)
\]

For a two-element \((N=2)\) array, the above equation simplifies to

\[
\langle D_{\text{loss}} \rangle = 10 \log(2) - 10 \log \left( \frac{1}{2} \sum_{m=1}^{2} \sum_{n=1}^{2} e^{-\frac{\sigma_{mn}^2}{2}} \right)
\]

\[
= 10 \log(2) - 10 \log \left( \frac{1}{2} \left( e^{-\frac{\sigma_{11}^2}{2}} + e^{-\frac{\sigma_{12}^2}{2}} + e^{-\frac{\sigma_{14}^2}{2}} + e^{-\frac{\sigma_{22}^2}{2}} \right) \right)
\]

\[
= 10 \log(2) - 10 \log \left( 1 + e^{-\frac{\sigma_{12}^2}{2}} \right)
\]

3.0 Validation

The above theoretical derivation for array loss can be considered exact, so long as the original assumption that differential phase fluctuations induced by the atmosphere are zero-mean, normally distributed random variables is true. Therefore, before further analysis can proceed, we first validate this assumption by investigating the probability distribution function (PDF) of differential phase fluctuations, as measured by a two-element site test interferometer currently deployed in Goldstone, California.

The two-element interferometer developed by NASA Glenn Research Center utilizes a digital I and Q receiver which monitors an unmodulated beacon signal at 20.199 GHz from a geostationary satellite, Anik F2, with a baseline separation distance of 256 m and an elevation angle of 48.5°. A localized 10 MHz GPS-disciplined rubidium oscillator provides the reference timing for all operations and data collection. A more detailed description of the system hardware and setup can be found in (Ref. 4). The signal is sampled at 3.64 MHz with an integration time of 144 ms and recorded every second. Since our measurements limit the resolution to which we can observe phase fluctuations to time scales greater than 1 second, we must make several assumptions as to the characteristics of these fluctuations at time scales comparable to a symbol period (<< 1 sec), the time scale of interest for communications applications.

3.1 Statistics of Phase Fluctuations

A representative plot of the calibrated differential phase, as measured by our two-element interferometer, is shown in Figure 2 for September 1, 2007.\(^1\)

\(^1\)Note: This day was arbitrarily chosen from a set of data blocks that possessed no erroneous data points. The statistics derived from this data is representative of all data collected, thus far.
We analyze the statistics for a block of data from 02:00 to 03:00 GMT (where the atmosphere appears to be mildly turbulent) and 12:00 to 13:00 GMT (where the atmosphere appears to be calmer) at different time scales. Figure 3 shows the PDF at time scales of 1 hr, 30 min, and 10 min for each of these times. From the plot, we observe that the statistics do appear to follow a zero-mean normal distribution as we extend to shorter and shorter time periods, but notice that the rms of the phase fluctuations as we move towards shorter time scales does possess some variance. Since the rms phase is the metric by which array degradation is measured, characterizing these changes at time scales comparable to a symbol period is paramount to determining array performance for communications applications.

From this analysis, we validate that the distribution of phase fluctuations induced by the atmosphere is indeed Gaussian for time scales larger than approximately 10 min. Due to the low sampling frequency, we assume that the trend observed in the statistics for this process continue to extend to shorter time scales; that is, at time scales comparable to a symbol period, the PDF of phase fluctuations remains a zero-mean, normally distributed process. To truly verify this assumption, the sampling rate of recorded phase data would need to be increased to obtain enough information at sub-second intervals.

### 3.2 Two-Element Array Loss: Measurement Versus Predicted

To compare the theoretically predicted array loss with measured array loss for a two-element array, we must begin with the assumption that the phase difference induced by the atmosphere is an ergodic random process. In this way, we can take the time-averaged directivity and directly compare it to the ensemble average directivity loss determined above. From the equation for peak directivity for a two-element array,

\[
D_{\text{peak}}(t) = D_{\text{element}} \frac{1}{2} \sum_{m=1}^{2} \sum_{n=1}^{2} e^{j(\varphi_m(t) - \varphi_n(t))}
\]

The time-averaged directivity for a given time interval, \(\Delta T\), can be described by

\[
\overline{D_{\text{peak}}} = D_{\text{element}} \frac{1}{2\Delta T} \int_{t_k}^{t_k + \Delta T} \left[ 2 + 2e^{j(\varphi_2(t) - \varphi_1(t))} \right] dt
\]

\[
= D_{\text{element}} \left[ 1 + \frac{1}{\Delta T} \int_{t_k}^{t_k + \Delta T} e^{j\delta_{21}(t)} dt \right]
\]

where

\[
\delta_{21}(t) \equiv \varphi_2(t) - \varphi_1(t).
\]

Figure 2.—Differential phase time series measured by the two-element site test interferometer at Goldstone, California, on September 1, 2007.
Figure 3.—PDF’s of phase fluctuations for 1 hr (top left, 1), 30 min (top right, 2), and 10 min (bottom center, 6) at (a) 02:00 to 03:00 GMT and (b) 12:00 to 13:00 GMT on September 1, 2007.

Figure 4.—Measured versus theoretical array loss for varying rms phase.

The plot of Figure 4 shows the comparison between the predicted ensemble average array degradation, \( \langle D_{\text{loss}} \rangle \), for a given rms phase and the measured time-averaged directivity loss, \( D_{\text{loss}} \), for a two-element array. As 10 min is the smallest interval in which we possess enough data points to establish a normal distribution, 10 min averages were used. From the plot, we observe extremely good agreement between the two curves, indicating the correctness of the theoretical derivation for array loss in the presence of atmospheric-induced phase fluctuations, as well as confirming the ergodicity of the atmospheric-induced random process.
4.0 Typical Array Loss at Goldstone, California

To determine the typical array loss at Goldstone, California, we generate the cumulative distribution function (CDF) of the phase rms based on 1-yr data collected (2007 to 2008). To normalize our analysis, the data has been transformed to zenith and an operating frequency of 32 GHz, where the transformation to zenith is performed by multiplying by $1/\text{air mass}$ ($\sin(\text{elevation angle})$) and the rms phase scales linearly with frequency. The baseline separation distance is 256 m. The resulting zenith rms phase CDF is shown below in Figure 5.

From the CDF, we observe that 90 percent of the time, the rms phase is better than $35.2^\circ$ (at zenith), which corresponds to an array loss of only 0.39 dB (2.2 dB at 20$^\circ$ elevation). Though this value appears low, recall that this result is for a simple two-element array. We can extrapolate this value to $N$ elements, given a particular array geometry, by scaling the phase rms to different baselines (Ref. 5). Note that the results of this analysis will be extremely geometry dependent (Ref. 6). As a simple example, let us consider an array geometry similar to the Very Large Array (VLA) in Socorro, New Mexico, first, with an antenna spacing of 250 m between individual elements (circles in Fig. 6(a)), and one with a spacing of 50 m (x’s in Fig. 6(a)). Since maximum directivity is only a function of number of elements (in widely-spaced arrays), these two geometries can be readily compared. In our analysis, we assume the theoretical Kolmogorov turbulence root phase structure function exponent of $5/6$ ($d < 1 \text{ km}$) and $1/3$ ($d > 1 \text{ km}$) to scale the phase rms to different baselines (Ref. 7). We further assume that the average rms phase between antenna elements is similar for identical baseline separations, regardless of orientation or reference. Calculating the array loss curve based on the theoretical derivation (Fig. 6(b)), we observe that for the 250-m baseline array geometry in Goldstone, California, we will need a margin of approximately 2.8 dB at zenith (12.3 dB at 20$^\circ$ elevation) to maintain 90 percent availability. This margin can be reduced, by reducing the baseline separation to the 50-m geometry, which only requires 1.1 dB at zenith (5.2 dB at 20$^\circ$ elevation). This is due entirely to the fact that small-scale fluctuations will contain much less energy than larger scale fluctuations, which would directly impact large baseline arrays. Thus, for communications applications, it will be desirable to maintain the most compact geometry possible to minimize array loss due to atmospheric phase fluctuations, as the furthest extent of the array will dominate this factor.

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2Where the air mass ($1/\sin \theta$) is used to convert rms phase at zenith to another elevation angle.
Figure 6.—(a) Model array geometries for Goldstone, California, array loss calculation example with 250-m baseline geometry (o) and a 50-m baseline geometry (x), and (b) resulting theoretical array loss versus rms phase curve.

Figure 7.—(Top) Time series phase fluctuations, (middle) rms phase time series, and (bottom) calculated array loss for model arrays for July 25, 2007 measured data at Goldstone, California.

To investigate the transient behavior of the model array, we can analyze the time-domain array performance for a particularly turbulent day at Goldstone, California. Figure 7 shows the phase fluctuations observed by the two-element interferometer on July 25, 2007, as well as the resulting rms
phase and calculated array loss (for both the wide and compact array geometries described above). During extremely turbulent times (beginning of the day), array degradation can exceed 10 dB with a mean array loss of 4.4 dB for the entire day (250-m baseline geometry). An approximate 2 dB improvement, on average, can be realized for the more compact array design (50-m baseline geometry).

5.0 Conclusions

Herein we report on the theoretical performance of a sparse array whose signal degradation is primarily due to atmospheric-induced phase fluctuations. The ensemble average directivity of an $N$ element array in the presence of phase noise was derived theoretically and validated with measured data. Further, it is shown that the measured phase differential between two elements is indeed normally distributed (to the resolution limits defined by the experimental setup) and ergodic, the fundamental assumption which allows the prediction of a theoretical array’s performance. It is observed that the performance of an array in the presence of atmospheric-induced phase fluctuations is limited by the furthest extent of the array elements, and, for communications applications, this geometry should remain as compact as possible. Finally, the time series performance of an arbitrary array is shown for a particularly turbulent atmospheric day. For the geometries described, there is still significant array losses observed and to prevent these losses, some form of compensation is necessary, particularly during transmit.

References


$^3$Recall that the phase data recorded at Goldstone, California, is for an elevation angle of 48.5°
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Prescribed by ANSI Std. Z39-18