



## Closed-loop control of vortex formation in separated flows

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**TeXPoint fonts used in EMF.  
Read the TeXPoint manual before you delete this box.: A**



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### Challenges for MAV

- Low Re (100 to  $10^5$ ) aerodynamics.
  - Three-dimensional geometry
  - Unsteady flow
  - Laminar separations
  - Instabilities
- Closed-loop control
  - Nonlinearity
  - High-dimensional systems.
  - Feedback to achieve flow states not available in open loop

Re ~ 100,000

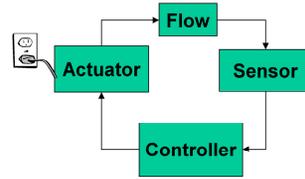


AeroVironment  
Black Widow MAV

Re ~ 10,000

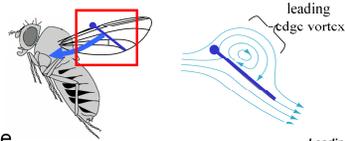


Re ~ 100

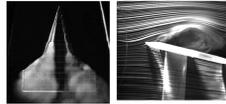


## Opportunities

- Birds, insects, and bats offer enticing performance benchmarks:
  - hover, low cruising speed
  - agility
  - propulsive efficiency
  - robustness, gust resistance
  - sensor-based control
- High-lift: stable leading edge vortex
- Maneuverability: control/synchronization of vortex formation/shedding



Leading-edge vortex lifts swifts.  
Videler, Stamhuis & Povel, *Nature* 306, 2004

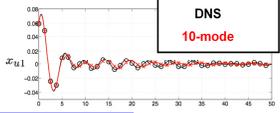


Can closed-loop control be used to mimic some of these behaviors on a (more) conventional airfoil?

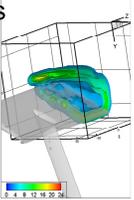


### MURI

- Simulation and Reduced-order modeling
- Unsteady wind tunnel experiments at moderate Re



- Oil tunnel experiments at low Re



Control law development:

- Stabilize LEV (enhance lift)
- Suppress vortex shedding
- Synchronize vortex shedding
- Lift cancellation in gusting flow
- Maneuver (perched landing)



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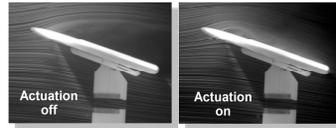
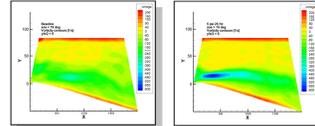
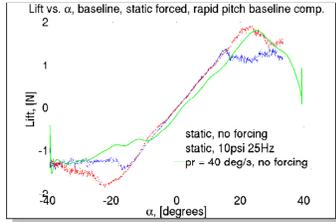
### LEV control, semi-circular wing, Re=68000

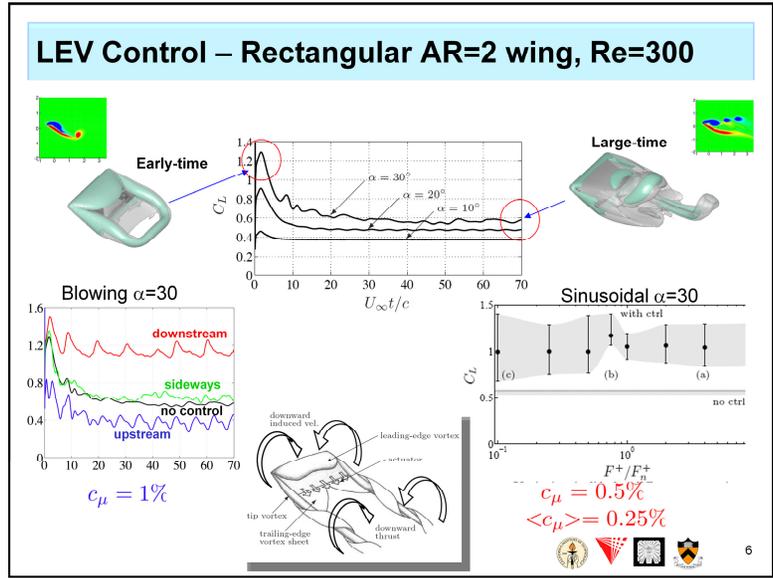


Pulsed-blowing actuation concentrates vorticity at leading edge.

$$F^+ = fc/U = 1.1$$

$$C_{l1} = .0074$$





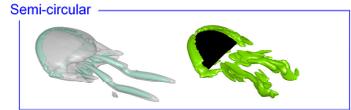
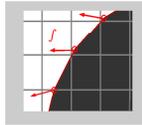
## Numerical Simulation (DNS)

- Re  $O(10^2)$  to  $O(10^3)$
- Data for modeling efforts & test control in simulation
- Validated Immersed boundary projection method
- Fast algorithm (FFT+multi-domain far-field BC)
- Wrappers for:
  - Linearized/adjoint
  - Linear stability
  - Continuation (stable/unstable steady states)
  - Snapshots for POD/BPOD/Galerkin Projection

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u} + \int_{\xi} \mathbf{f}_B(\xi) \delta(\mathbf{x} - \xi) d\xi$$

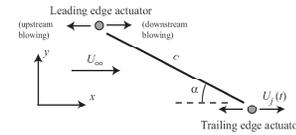
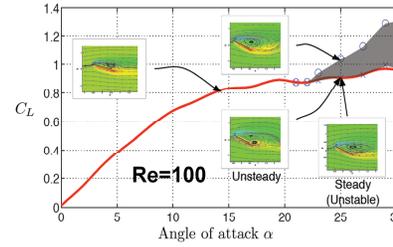
$$\nabla \cdot \mathbf{u} = 0$$

$$\mathbf{u}(\xi) = \int_{\mathbf{x}} \mathbf{u}(\mathbf{x}) \delta(\xi - \mathbf{x}) d\mathbf{x} = \mathbf{u}_D(\xi)$$



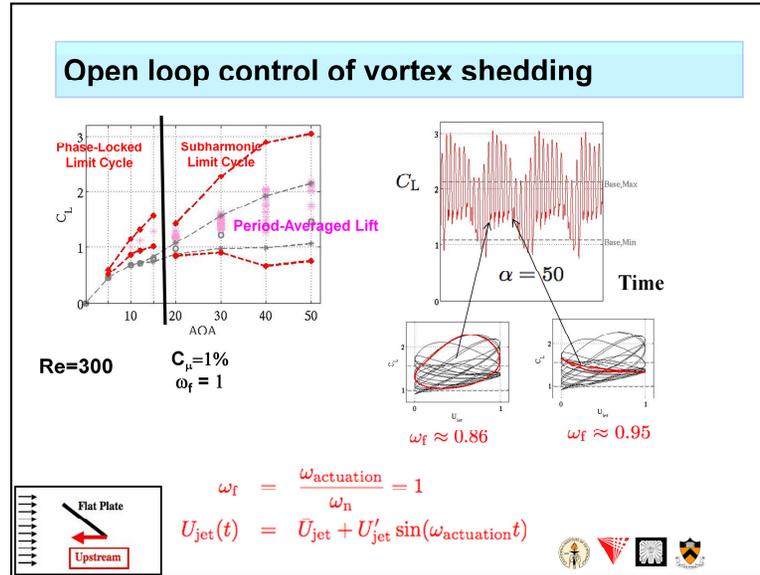
### Model problem: 2D flat plate at angle of attack

- Economical test case displaying vortex shedding behavior
- Close to bifurcation point (with Re or AOA)
- Use actuation and feedback to:
  - Stabilized otherwise unstable equilibria
  - Phase-lock lift fluctuations to actuation
  - Actuation modeled as localized body force near leading or trailing edge



AFOSR Math Program  
Review 2009





The flow becomes phase-locked to the actuation upto 15deg, but at higher alpha, it displays a more complicated limit cycle, at which subharmonic to the forcing frequency is excited.

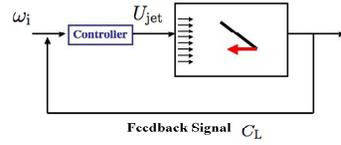
PINK stars represent the period-averaged lift over each actuation period.

And they have the same value if the flow is phase-locked, as in the case of lower alpha.

Lift history over time on the RHS show the example of these subharmonic behaviors at alpha=50  
Each subharmonic limit cycle is consisted of several actuation periods associated with a particular phase shift between the forcing signal and the lift, and with different period-averaged values, as shown as variation of PINK stars on the LHS for high alpha.

### Phase lock loop

- Can we obtain phase lock of 'good' limit cycles without searching for the right forcing frequency?
- Can we achieve lock on starting from arbitrary IC (or in presence of noise)?
- Adjust (slightly) forcing frequency and phase to attempt to match specified phase shift with output lift signal

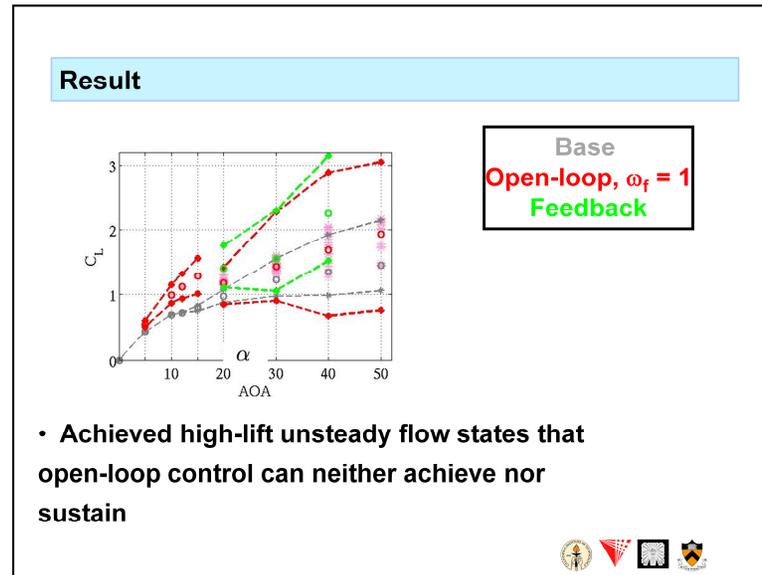


$$a_1(t) = \frac{2}{T_i} \int_{t-T_i}^t C_L(t') \cos(\omega_i t') dt'$$

$$b_1(t) = \frac{2}{T_i} \int_{t-T_i}^t C_L(t') \sin(\omega_i t') dt'$$

$$U_{jet}(t) = a_0 + K_p [a_1(t) \cos(\omega_i t + \phi_i) + b_1(t) \sin(\omega_i t + \phi_i)]$$





In order to phase lock the flow at the desired shedding cycle, particularly at  $\Phi_{best}$ , We designed a feedback compensator.

(Even though the open-loop forcing at  $\omega_f$  below  $\omega_n$  can lead to phase-locked limit cycles with a high average lift,)

This feedback controller resulted in the phase-locked limit cycles that the open-loop control could not achieve for  $\alpha=30$  and  $40$

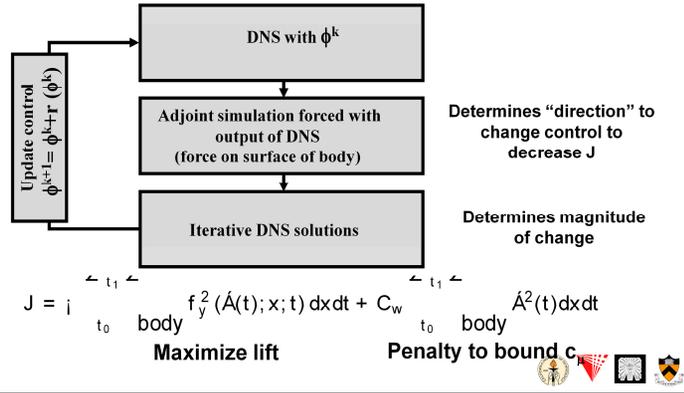
Particularly for  $\alpha=40$ , the feedback was able to stabilize the limit cycle that was not stable with any of the open-loop periodic forcing.

This results in stable phase-locked limit cycles for a larger range of forcing frequencies than the open-loop control.

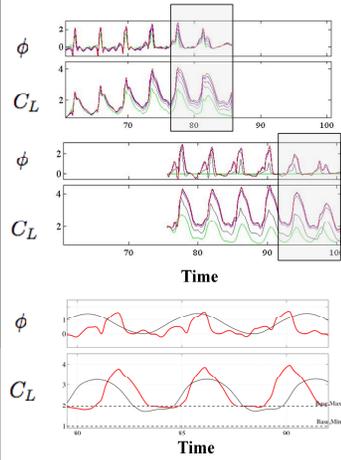
Also, it was shown that the feedback achieved the high-lift unsteady flow states that open-loop control could not sustain even after the states have been achieved for a long period of time.

## Optimized waveform

- What is the ideal control ( $\phi = U_{jet}$ ) to maximize lift?



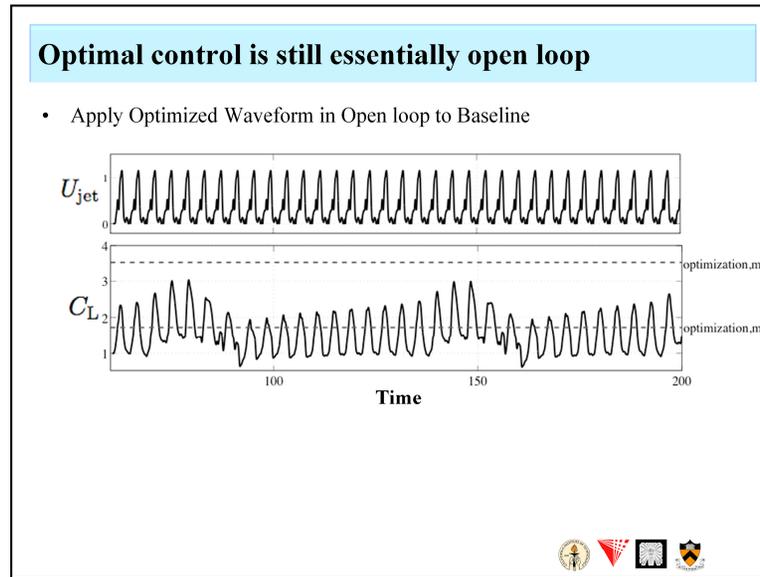
### Receding horizon approach



- Problems get exponentially harder to optimize as prediction horizon is increased.
- Controls near the end of each optimization horizon discarded.
- Chess: re-evaluation of the game plan after each move played.

**Specific example achieving (about) the same period-averaged lift as sinusoidal, but with 40% of the energy**





Optimization provides a periodic control waveform that achieves period limit cycle after 4~5 transient periods.

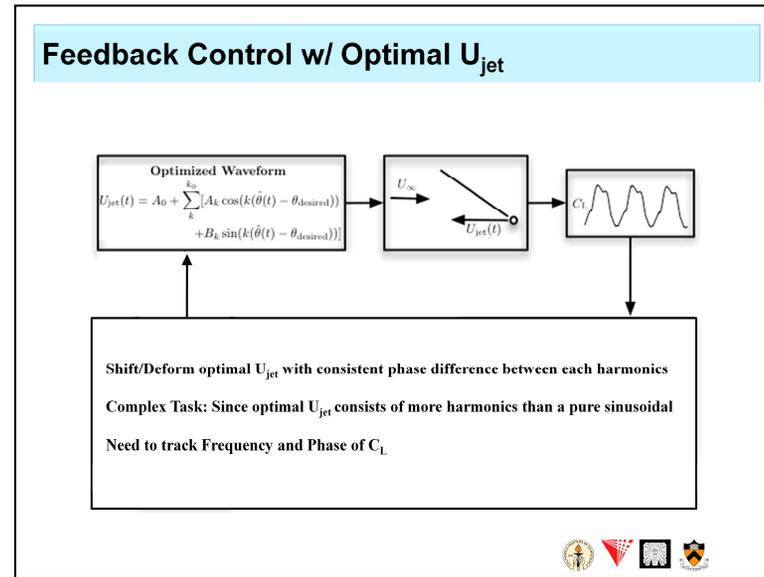
However, it'd be practically implementable only if we can reproduce the high lift limit cycles without the transient signal started from any phase of the baseline cycle.

While it is straight forward to extract a single period of the optimal waveform

If optimized waveform is applied in open loop to the baseline flow, the flow fails to lock to the forcing and the performance can be significantly degraded as shown in the figure.

This calls for designing a feedback algorithm

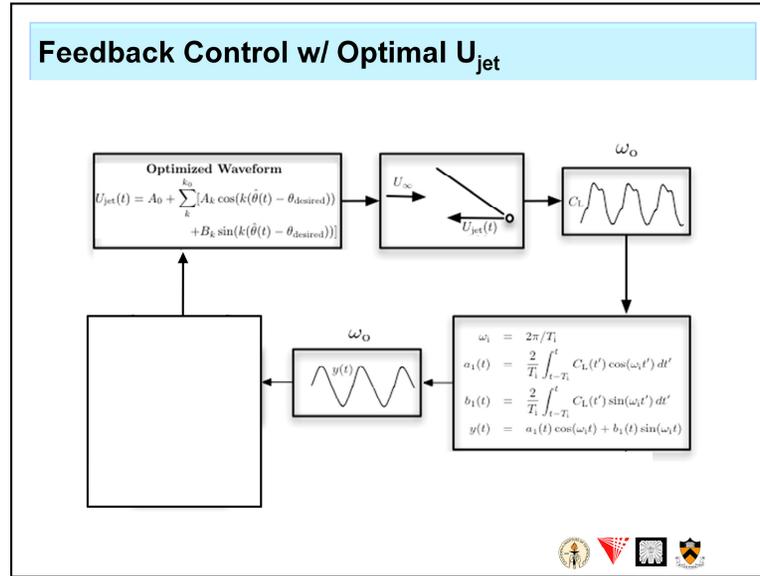
To achieve phase lock between the lift and the optimal waveform.



We feedback lift again as an attempt to march along optimized  $U_{jet}$  accordingly.

The goal is to shift and deform optimal  $U_{jet}$  with consistent phase difference between each harmonics.

However, this is a complex task since optimal  $U_{jet}$  consists of more harmonics than a pure sinusoidal.

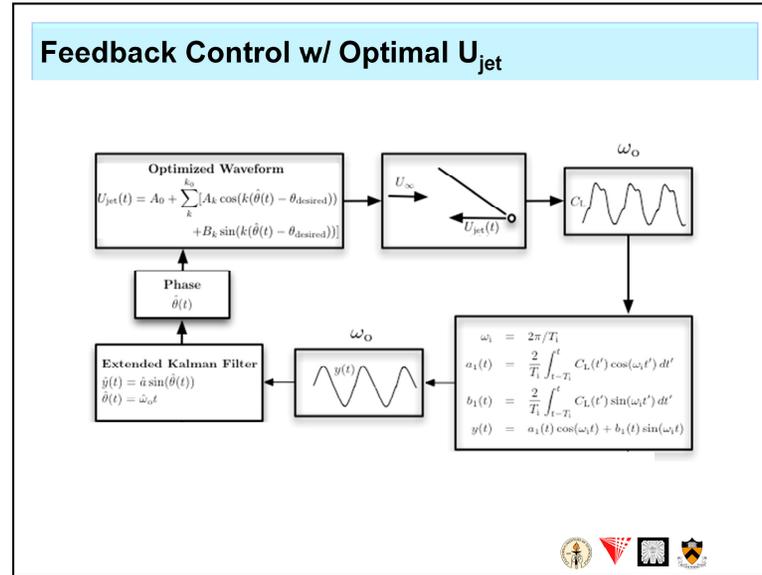


Recall from the previous implementation of feedback with sinusoidal waveform; feedback controller demodulated the lift signal, applied a low-pass filter, shifted the phase by a specific amount, and remodulated the signal in order to produce a sinusoidal output locked by a specified lag to the lift signal.

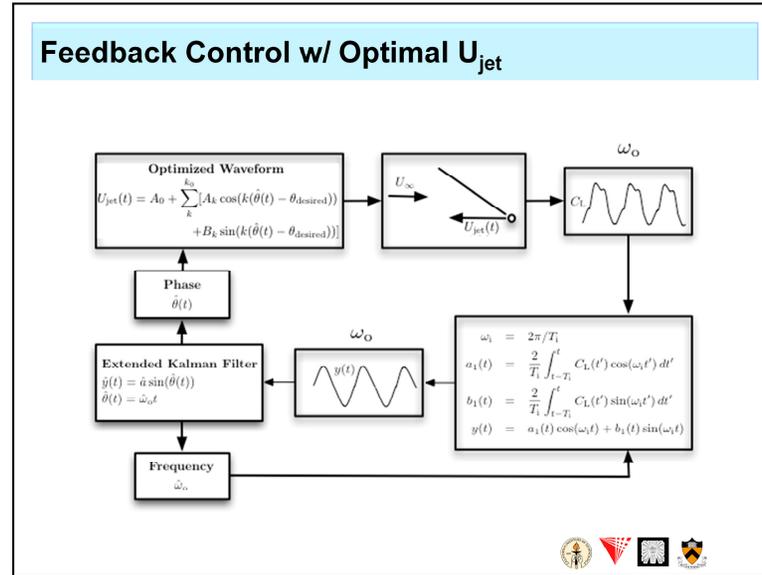
This can be used as a narrowband filter if no phase shift is added.

The output signal,  $y(t)$  is now a sinusoidal which retains the dominant frequency of CL ,  $\omega_0$  and filters out higher harmonics.

Since we're interested in marching optimal  $U_{jet}$  at the dominant frequency of CL , it is easier to track the frequency of  $y(t)$  instead of CL .



we use Extended Kalman Filter (EKF) to dynamically estimate frequency  $\omega_o$ , and the phase  $\theta(t)$ . Based on the phase estimate  $\theta(t)$ , we can then march along optimal  $U_{jet}$  as shown. Where  $\theta_{desired}$  is an additional (specific) phase shift relative to the lift signal. ( $\theta_{desired}=0$ , in optimization case)



Also when computing  $y(t)$ , note that if demodulation frequency,  $\omega_i$  is not equal to the actual frequency of the lift signal,  $\omega_0$ , then  $y(t)$  will not be in phase with  $CL(t)$ ,

But they will be in phase only when  $\omega_i = \omega_0$ .

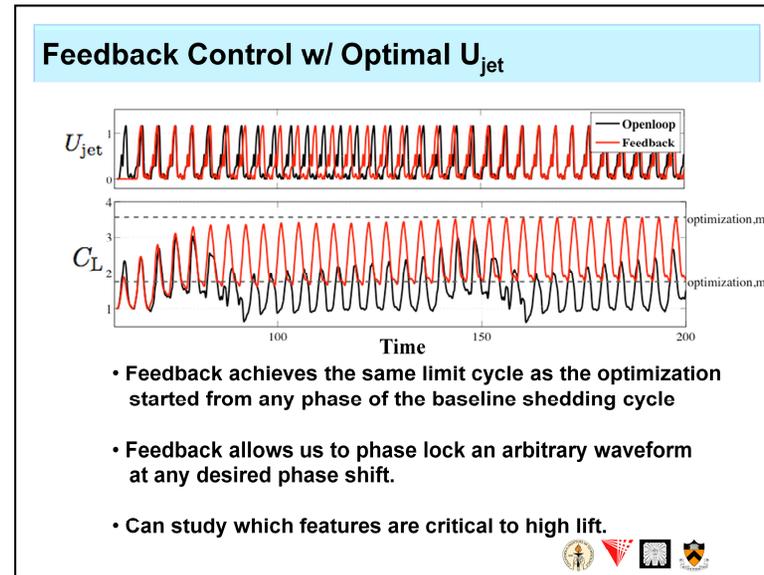
Thus, it is necessary to update  $\omega_i$  periodically with  $\hat{\omega}_0$ , the frequency estimated by an EKF.

Now,  $y(t)$  will be in phase with  $CL(t)$  with the same dominant frequency.

Based on the estimated phase  $\hat{\theta}$ , we can then march along the optimized waveform as,

To summarize :

1. Narrowband filter is used on the lift cycle to obtain a sinusoidal signal.
2. Filtered lift signal is used as input to frequency tracking Extended Kalman Filter (EKF) to estimate phase,  $\hat{\theta}(t)$  of the lift signal.
3. EKF frequency estimate is used to tune the filter to avoid introducing phase lag.
4. Finally, phase estimate  $\hat{\theta}(t)$  from EKF is used to march along  $\varphi_{optimal}$ .

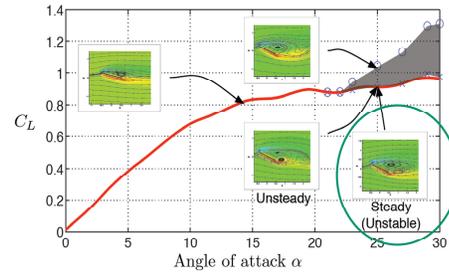


Now we can start from any phase of the baseline flow,  
and the feedback (in red) phase-locks the flow at the same phase shift, and achieves the same high-lift limit cycle as the optimization.

The feedback controller now allows us to phase-lock an essentially arbitrary waveform at any desired phase shift,

Thus we can utilize this fact to investigate which features of the optimized waveform are critical to high lift.

Stabilize unstable equilibrium (optimal control)?

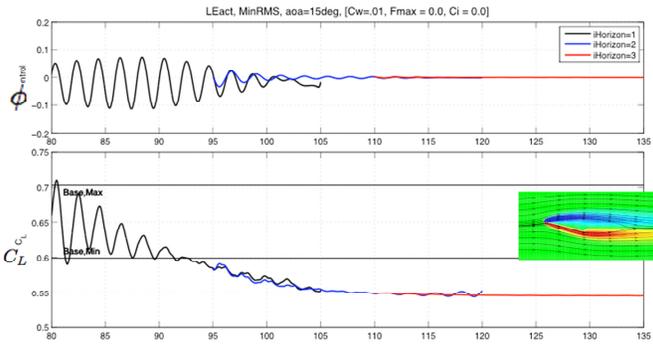


$$\mathcal{J} = \int_{t_0}^{t_1} \int_{\Omega} (\bar{f}_y(\phi(t), x, t))^2 dx dt + C_w \int_{t_0}^{t_1} \int_{\mathcal{E}} (\phi(t))^2 dx dt$$

Minimize lift



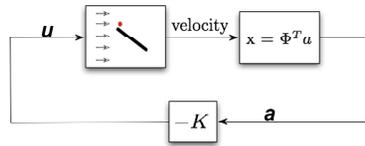
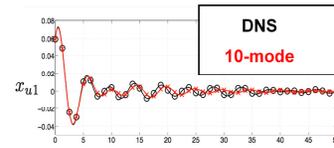
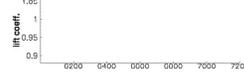
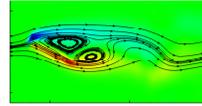
### Stabilize Steady State at $\alpha = 15\text{deg}$



## Model-based feedback control using low-order models

S. Ahuja and C. Rowley, Princeton University

- 10 mode BPOD model
- Full state feedback (also possible with 2 observers)
- LQR to determine K
- Large domain of attraction even in the full NL system
- Controller suppresses the vortex shedding



## Summary

- Open loop LEV control shows lift enhancement and flow structure similar to dynamic stall vortex
  - Extra lift used with dynamic physics-based models to cancel lift fluctuations in gusting flow (D. Williams)
- 2D vortex shedding
  - Post-stall open-loop forcing gives complex, subharmonic resonance
  - Individual periods of forcing appear to be unstable periodic orbits
  - Phase Lock Loop able to phase lock lift fluctuations to forcing, stabilize periodic orbits (e.g. max lift)
  - Optimization of waveforms using gradient-based optimal control, implemented in closed-loop with PLL
  - Suppression of vortex shedding with BPOD/linear instability/LQR/observer models
    - Extension of the BPOD approach to unstable period orbits



## Model-based feedback control using low-order models

- Computationally tractable implementation of balanced truncation
- Balanced POD based on snapshots of impulse (forward) response, adjoint response
- Bi-orthogonal set of forward/adjoint modes
- Galerkin Projection, retain small number of modes
- *A priori* error bounds for the ROM
- Extension to unstable equilibria
  - BPOD only on stable manifold
  - Unstable manifold (2 modes) treated exactly
  - Similar extension to periodic orbits

