GLOBAL INSTABILITY
OF LAMINAR SEPARATION BUBBLES

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Introduction and Background
COMMON WISDOM ON LSB

- Separation under adverse pressure gradient
- Reversed flow region enclosed ➔ Shear layer
- Kelvin-Helmholtz instability ➔ transition to turbulence
- Turbulent strong mixing forces re-attachment

(Horton 1968)
A MORE RECENT VIEW

- **Two different classes** of eigenmodes:
  - Shear-layer/KH
  - Global mode

Shear-layer/KH modes are represented by two curves, one for each value of $\beta$, and the DNS simulation. The global mode is shown in a contour plot with a color scale indicating the magnitude of the velocity $u_r$. The plots are based on the work of Th, Hein & Dallmann (Phil. Trans. R. Soc. Lond. A, 2000) pp. 3229-3246.
Amplification of the wake mode gives rise to the 2D time-periodic basic state

\[ q(x, y, t) = \bar{q}(x, y) + \varepsilon \hat{q}(x, y) \exp \Theta \]
The bubble mode is stronger damped than the wake mode is.

Abdessemed, Sherwin, Th, JFM (2009) 628: 57-83
A UNIFYING PERSPECTIVE

THE GLOBAL MODE OF LSB

• The structure of the amplitude functions of the *minimally damped* global eigenmode of the LPT blade…

  spanwise \( w(x,y) \)

…is analogous to that on the flat-plate and the NACA 0012 aerofoil

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• Dallmann’s conjecture on the topological flowfield changes due to the global mode
Amplification of the global mode leads to three-dimensionalization of the primary reattachment line in 3d…

…while it leaves the primary separation line unaffected
The present contributions
Incompressible Flow

- Some numerics on the direct/adjoint EVP
- U-separation on a flat-plate
- Stall cells on a NACA0015 airfoil
- Receptivity and Sensitivity of LSB on a flat plate

Compressible Flow

- The Howarth/Briley bubble in subsonic flow
- Shock/Boundary-Layer Interaction (SBLI) on a semi-infinite plate at Mach 2
  - SBLI on a 45º wedge

Focus on the (stationary, self-excited) global mode alone
Numerical considerations
• Ten years later…

- Incoming TS waves at the inflow boundary
  (besides the homogeneous Dirichlet BCs used in THD)

- 400x100 points per spatial direction used to resolve the amplitude function of the eigenmodes
  (as opposed to 60x40 in THD)

- 25k CPU hrs ( = 25 hrs x 1000 procs) per wavenumber
  (as opposed to 24 CPU hrs serial computing time on a single processor in THD)

...permit solving substantially larger (but not all) problems,

• on Mare Nostrum at BSC and (the Petaflop) Blue-Gene/P at FZJ
  (but Jürgen Seidel finally got access to DoD HPC machines)
Classification of global modes of LSB flow
GLOBAL MODES IN LSB FLOW ON A FLAT PLATE

Case: $Re_{\delta^*}=500$ at inflow, $Re_{\delta^*}=700$ at outflow, Separation Bubble
Boundary conditions: Dirichlet at inflow & Extrapolation at outflow
• Composed of discrete and continuous branches
• Focus discussion on two traveling (and the leading stationary) global eigenmodes
Comparison of traveling modes with OSE

- Extract "spatial amplification" from successive peaks of the (2d) global eigenfunction;
- Run a spatial OSE at successive downstream locations;
- Compare
Extracting physics out of eigenspectra:

- **U-separation** on a flat plate
Fluid Mechanics Group

Walter Bassett Aerodynamics Laboratory

Department of Mechanical Engineering, University of Melbourne, Australia

The above diagram represents a local Taylor series expansion solution of the Navier-Stokes equation for a simple U-separation.


Academic Staff

Research Staff
FIG. 17: Perspective view of the free stream surface sheets and wall streamlines of simple U-shaped separation
For example, Fig. 6 shows a three-dimensional separation pattern that has been classified as a U separation (Perry and Hornung\textsuperscript{17}). The flow field has been obtained by solving the Navier–Stokes equations locally (see Perry and Chong\textsuperscript{18}) using a third-order Taylor series expansion, and assuming that the flow pattern is symmetrical (i.e., $x_1$ and $x_3$ are as-
TYPICAL FLAT-PLATE GLOBAL EIGENSPECTRUM

- Composed of discrete and continuous branches
- Focus on (two traveling and) the leading stationary global eigenmode
Critical Points of \(( BF + \varepsilon A )\)

- Reconstruct a composite flowfield, using the steady basic state and the leading global eigenmode
- Identify critical points and their nature

S: Saddle  N: Node  F: Focus  C: Center
(s): stable  (u): unstable
Critical Points of \(( BF + \varepsilon S )\)

- **Critical point analysis** of composite flow

Eigenvalue spectrum

BF + \(\varepsilon\) (Global Mode)
U – SEPARATION

Present computations

Bippes (198?)

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Extracting physics out of eigenspectra:

- **Stall Cells** on an Airfoil
**Critical point analysis** of composite flow

Eigenvalue spectrum

Leading stationary eigenmode

- **u-perturbation**
- **pressure perturbation**
• **Stall-cells** appear on the airfoil, as result of linear modal amplification of the leading stationary global mode.
Flow Control of LSB


- The kernel of the direct/adjoint coupling (Luchini 2003) permits identifying the region of maximal feedback region...

- ... it coincides with the primary recirculation center
- (a result not unexpected, really)
• Solution of the adjoint eigenvalue problem permits identifying the maximally receptive region (dominant streamwise velocity perturbation)

• ... it coincides with the primary separation line

• NB. Analysis is meaningful due to homogeneous Dirichlet BCs imposed upstream (direct) / downstream (adjoint) EVP
• Predict the actuator placement without prior knowledge (or ad-hoc assumptions)
Compressibility effects – subsonic flow
MACH # EFFECT ON HOWARTH/BRILEY LSB

- Linear pressure increase (compressible) vs linear deceleration (incompressible) in the free-stream

- No major differences in integral quantities have been found, i.e.
- Quantitative but no qualitative differences of instability results
- Work in progress, in collaboration with Stanford U.
Compressibility effects – supersonic flow
• Typical LES result at $Re_\delta = 10k$

• Origin of unsteadiness?
• Origin of spanwise periodic structures?
• Basic states obtained by high-order accurate, low-diffusion WENO schemes (Ekaterinaris)
• Flat plate leading edge included at upstream end of both domains
• Finite-angle wedge in compression corner (no triple-deck)
• Both flows at $5000 < \text{Re}_L < 20000$, $\text{Ma} = 2$ and 2.5
• Unsteadiness identified by DNS at $\text{Re}_L \approx 25k$
GLOBAL “MODES”

- Compressible BiGlobal EVP solved at $5000 < Re_L < 20000$ by
  - Spectral collocation (SBLI on flat plate)
  - Immersed boundary (45° compression corner)
- Entire BF domain considered – shocks not treated (≠ Crouch)
- Qualitatively reasonable results,…

- …but spectra are not converged (despite very large resolutions)
- Work in progress, in collaboration with Patras U.

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SUMMARY ( I / II )

• In the last 3 years, global linear instability of LSB has been revisited, using state-of-the-art hardware and algorithms.

• Eigenspectra of LSB flows have been understood and classified in branches of known and newly-discovered eigenmodes.

• Major achievements:
  • World-largest numerical solutions of global eigenvalue problems are routinely performed.
  • Key aerodynamic phenomena have been explained via critical point theory, applied to our global mode results.

• Theoretical foundation for control of LSB flows has been laid.
SUMMARY (II/II)

- Global mode of LSB at the origin of observable phenomena
  - U-separation on semi-infinite plate
  - Stall cells on (stalled) airfoil
- Receptivity/Sensitivity/AFC feasible (practical?) via
  - Adjoint EVP solution
  - Direct/adjoint coupling (the Crete connection)
- Minor effect of compressibility on global instability in the subsonic compressible regime
- Global instability analysis of LSB in realistic supersonic flows apparently quite some way down the horizon

(Saric joke # 431: “an imaginary, ever-receding line”)

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FUTURE CHALLENGES AND WORK DIRECTIONS

• From a **physical (and theoretical)** point of view
  • Continue with the analysis of EVP results
  • Instability analysis of **turbulent flows** (obtain EVP spectra of 2.5D flows)
  • Embed instability analysis into (AFC) **flow control** concepts (for 2.5D flows)
  • Extend global instability tools in **3D flows**

• From a **numerical** point of view
  • Devise new algorithms for the **efficient parallel** solution of (massive) eigenvalue problems
Thank You
Crete, Greece, Sept 28 – Oct 2, 2009
ADDITIONAL MATERIAL
Boundary Layer transformation:

\[ \xi = \frac{x}{L}, \quad \eta = y \sqrt{\frac{U_e}{\nu x}}, \quad \Psi = \sqrt{U_e \nu x} f(\xi, \eta) \]

Separated states recovery:

- Reyhner and Flügge-Lotz approximation
- Displacement thickness imposed
COMPARISON WITH TEMPORAL OSE

Basic Flow: Artificial parallel Blasius
Mack's Case: Re = 580, $\alpha = 0.179$  (Mack JFM 1976)
Streamwise extension: $L = 10 \times 2\pi / \alpha$
Periodic boundary conditions

![Graph](image)
FURTHER COMPARISONS WITH TEMPORAL OSE

Basic Flow: Artificial parallel Blasius
Mack’s Case: Re = 580, $\alpha = 0.179$ (Mack JFM 1976)
Streamwise extension: $L = 10 \times 2\pi / \alpha$

Boundary conditions:
- X Periodicity - Fourier
- + Periodicity - Chebyshev
- O Robin inflow & outflow
- □ Robin inflow & Extrapolation outflow
Comparisons with Spatial OSE

Case: $Re_{\delta^*}=500$ at inflow, $Re_{\delta^*}=700$ at outflow

Analysis 1:
- **Basic Flow**: Artificial parallel Blasius
- **Boundary conditions**: Robin* at inflow & outflow

Analysis 2:
- **Basic Flow**: Artificial parallel Blasius
- **Boundary conditions**: Robin* at inflow & extrapolation at outflow

Analysis 3:
- **Basic Flow**: Real Blasius boundary layer
- **Boundary conditions**: Robin* at inflow & extrapolation at outflow

* Robin boundary condition of Ehrenstein & Gallaire JFM 2005:

$$\frac{\partial \hat{q}}{\partial x} = i \left( \alpha_r,0 + \frac{\partial \alpha_r}{\partial \omega_r} (\omega_0) \cdot (\omega - \omega_0) \right) \hat{q}$$
FURTHER COMPARISONS WITH SPATIAL OSE

Case: $Re_{\delta^*} = 500$ at inflow, $Re_{\delta^*} = 700$ at outflow
Robin boundary condition evaluated at: $\omega_0 = 0.13$

Analysis:

1. $\times$ Parallel Blasius, Robin + Robin
2. $+$ Parallel Blasius, Robin + Extrapolation
3. $\circ$ Real Blasius, Robin + Extrapolation
Journals

• [J-4] Rodríguez, D., Theofilis, V. 2009 Massively Parallel Numerical Solution of the Bi-Global Linear Instability Eigenvalue Problem Using Dense Linear Algebra AIAA Journal, accepted for publication, DOI: 10.2514/1.42714

Book Chapters


Conferences

• U of Arizona
  • Theofilis Visiting/Adjunct Professor since 08
  • Rodríguez visited in 08, will return in 09 (one month)
  • Joint publications in preparation with
    • Tumin (on characterization of eigenspectra)
    • Fasel (on global instability of 3d bubble flows)
• Stanford U
  • Joint publication in preparation with
    • Marxen (on laminar separation bubbles)
• U of Patras
  • Provision of 2d shock-induced laminar separated basic flows
  • Joint publication (IUTAM 2009) with
    • Ekaterinaris (on preliminary global instability analyses)