

Analytic Development of a Reference Profile for the First Entry in a Skip Atmospheric Entry

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I. Introduction

For entry vehicles with relatively low lift-to-drag (L/D) ratios, a known strategy for achieving long down-range is to allow the vehicle to skip out of the atmosphere.¹ During this high-altitude and low-drag skip phase of the entry, the vehicle can dramatically increase its range. This note is exclusively focused on entries from super-circular velocities that require an exo-atmospheric phase to reach the landing site. Various methods have been suggested and implemented for atmospheric guidance of such trajectories. Most fall into one of three broad categories:^{2,3} numeric predictor-correctors,^{4–9} analytic predictor-correctors,^{10,11} and reference-following controllers.^{12–16} This note presents a method, primarily intended for its use under the latter category, that generates analytically a reference drag profile for the first entry portion of a skip entry when the exit conditions (and the initial conditions) are known. The analytic generation of such a reference profile has not been attempted before. In this way, this note intends to contribute to the effort of developing a, yet unaccomplished, complete analytic solution to the skip entry guidance problem. Note that a complete analytic solution to the skip entry problem should determine the conditions at exit (range to go, velocity $[V]$, and flight path angle $[\gamma]$) that render the entry conditions for the final phase. The central difficulty in determining analytically the exit conditions lies in the limitations that various approximations and linearization assumptions have been found to have in estimating the range flown at low drag altitudes.^{2,17–19} The analytic determination of the exit conditions remains an elusive problem whose resolution is not intended in this note.

The analytic development of a drag reference profile for a sub-circular entry is the basis to the Space Shuttle Orbiter guidance logic.²⁰ This idea is based on the fact that the range to be flown during entry is a unique function of the drag acceleration maintained throughout the flight. This range is predictable using analytic techniques for simple geometric drag acceleration functions of the relative velocity (quadratic, linear and constant, in the case of the Orbiter), provided the local flight path angle is small, which is the case at high speeds. Flight throughout the entry corridor can be achieved by linking these geometric functions together in a series. It is conceivable to divide the first entry in a skip trajectory in segments with linear and/or quadratic drag functions as is the case in the Space Shuttle entry guidance; however, this approach will not be pursued in this note. It is proposed in this note to express the drag reference of the complete first entry as a polynomial function of the velocity, with degree *higher* than two. In addition, the generic method proposed to obtain the drag reference profile will be further simplified by thinking of the drag as the *probability density function* of the velocity or, conversely, by thinking of the velocity as the *distribution function* of the drag. With this notion it will be shown that the reference drag profile can be generated by solving a system of linear algebraic equations.

For completeness, the drag profiles generated with this method will be tracked through the implementation of the feedback linearization method of differential geometric control as a guidance law with the error dynamics of a second order homogeneous equation in the form of a damped oscillator.²¹ Although this approach was first proposed as a revisited version of the Space Shuttle Orbiter entry guidance to demonstrate the commonality of both guidance laws, it has never been used to fly a skip-like entry trajectory, where the drag profile for the first entry does not fit a quadratic polynomial of the velocity.

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A number of different approaches to skip entry guidance for the Orion CEV (Crew Exploration Vehicle) spacecraft have been under evaluation at the Flight Mechanics and Trajectory Design Branch at the NASA's Johnson Space Center.²² A guidance algorithm called NSEG¹⁹ (Numerical Skip Entry Guidance), that combines features of the original Apollo Guidance algorithm^{23,24} with a numerical scheme for computing a real-time long-range skip trajectory, was found to provide very reliable means of meeting the skip entry range requirement. Out of a comprehensive set of 60,000 skip entry cases (20 nominal and 59,980 dispersed) that have been simulated for the CEV using NSEG, the 20 nominal cases will be used as test cases for this note. As explained before, obtaining the skip out exit conditions from the knowledge of the landing site and entry interface is not the objective of this note, therefore, the initial and final conditions for testing purposes will be those pertaining to the 20 nominal trajectories. The 20 cases are subdivided into 4 groups. Each group is composed of 5 trajectories that have a common entry interface (EI) (figure 1), but different target landing sites in the western continental United States, and different L/Ds (0.3, 0.33, and 0.35).

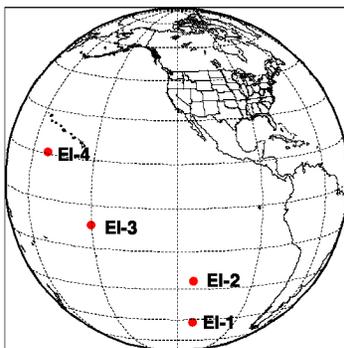


Figure 1. Location of the Test Entry Interfaces With Respect to the U.S.

II. Equations of Motion

The Earth-relative longitudinal translational state of the spacecraft is represented by the variables R (range), h (altitude above the Earth's surface), V (Earth relative velocity), and γ (Earth relative flight path angle). The equations of motion in this document use a coordinate system with one axis oriented along the Earth-relative velocity vector, one axis perpendicular to the plane formed by the position and Earth-relative velocity vectors, and a third axis completing the right-hand coordinate system. The equations of motion are as follows

$$\dot{R} = V \cos \gamma \quad (1)$$

$$\dot{h} = V \sin \gamma \quad (2)$$

$$\dot{V} = -D - g \sin \gamma \quad (3)$$

$$\dot{\gamma} = \frac{1}{V} \left[L \cos \phi + \left(\frac{V^2}{r_e + h} - g \right) \cos \gamma \right] \quad (4)$$

where r_e is the mean Earth radius, g is the gravity acceleration, and ϕ is the bank angle. These equations of motion neglect the Coriolis and centripetal accelerations due to Earth's rotation because these accelerations are small compared to the aerodynamic acceleration.

The specific drag and lift are given by

$$D = \rho V^2 \left(\frac{S_r C_D}{2m} \right) \quad (5)$$

$$L = \rho V^2 \left(\frac{S_r C_L}{2m} \right) \quad (6)$$

where S_r and m are the reference surface and mass of the vehicle, respectively. The drag and lift aerodynamic coefficients (C_D and C_L) are assumed constant since only hypersonic velocities are involved in the phase of interest.

An exponential atmospheric density model with constant atmospheric density at base altitude is assumed for this study

$$\rho = \rho_0 e^{-\frac{h}{h_s}} \quad (7)$$

where h_s is the atmospheric density scale height.

III. Generation of the Reference Drag Profile

The phase under consideration is the first entry in a skip atmospheric entry. The origin for this phase is an initial velocity and flight path angle, and the destination is a set of specific exit conditions in terms of velocity and flight path angle at a desired range. A reference drag profile as a polynomial expression of the velocity is assumed

$$D(V) = \sum_{n=0}^m a_n V^n \quad (8)$$

The $m + 1$ equations required to determine the coefficients a_n are then resolved. The problem is how to define the a_n coefficients. The solution is to find trajectory constraints that can be functionally related to the drag. There are five such basic constraints in all: four conditions on the initial and final (or exit) velocity and flight path angle, and a fifth condition on the range flown.

A. Defining the Basic Constraint Equations

Two equations relate the initial and final velocities to the initial and final drags

$$D_{i,f} \equiv D(V_{i,f}) = \sum_{n=0}^m a_n V_{i,f}^n \quad (9)$$

where i and f refer to the initial and final (exit) state, respectively.

The relationship between the drag and the flight path angle is derived using the equation for the atmospheric density. Differentiating Eq. (7) with respect to time produces the following equation

$$\frac{\dot{\rho}}{\rho} = -\frac{\dot{h}}{h_s} \quad (10)$$

Differentiating Eq. (5) with respect to time, dividing by D , and combining with Eq. (10) results in

$$\frac{\dot{D}}{D} = -\frac{\dot{h}}{h_s} + \frac{2\dot{V}}{V} \quad (11)$$

Provided that $\dot{V} \approx -D$ for small flight path angles, that $\dot{D} = \dot{V} dD/dV \approx -D dD/dV$, and that $\dot{h} = V \sin \gamma$, Eq. (11) results in

$$-\frac{dD}{dV} + \frac{2D}{V} = -\frac{V \sin \gamma}{h_s} \quad (12)$$

Hence, the other two equations relating the initial and final velocities and flight path angles to the initial and final drags are

$$D'_{i,f} \equiv \left. \frac{dD}{dV} \right|_{i,f} = \frac{V_{i,f} \sin \gamma_{i,f}}{h_s} + \frac{2D_{i,f}}{V_{i,f}} = \sum_{n=0}^m n a_n V_{i,f}^{n-1} \quad (13)$$

A specific range R must be covered between the initial and final conditions. For small flight path angles, combining Eqs. (1) and (3), separating variables and integrating yields the equation on the range and, thus, the last of the five basic equations

$$R = - \int_{V_i}^{V_f} \frac{V}{D(V)} dV = \int_{V_i}^{V_f} \frac{V}{\sum_{n=0}^m a_n V^n} dV = \sum_{j=1}^m \frac{r_j}{\sum_{n=1}^m n a_n r_j^{n-1}} \log \left(\frac{V_i - r_j}{V_f - r_j} \right) \quad (14)$$

where r_j are the roots of the polynomial $D(V)$. Having the drag expressed as a polynomial of the velocity implies that the integral in Eq. (14) can be solved analytically.²⁵

By solving the system composed of Eqs. (9), (13) and (14), a drag reference profile expressed as a degree 4 ($m = 4$) polynomial of the velocity can be generated. This system is comprised of 4 linear equations, Eqs. (9) and (13), and one non-linear equation, Eq. (14). The form of the equation on range, Eq. (14), implies that numerical methods need to be used to find one of the coefficients in the drag polynomial from which the others could be derived analytically. It would be highly desirable to find a relation such that the set of equations to obtain the coefficients of the drag polynomial could be solved as a linear system. The following development derives a replacement for Eq. (14) such that the system containing the five basic equations becomes a system of linear algebraic equations.

One way to achieve the desired linear equation is by relating the range to the integral of the drag along the velocity

$$\int_{V_i}^{V_f} D(V) dV = - \sum_{n=0}^m \frac{a_n}{n+1} \left(V_f^{n+1} - V_i^{n+1} \right) \quad (15)$$

However, in principle, it is only known how to relate range and drag through Eq. (14). The following steps are proposed to solve the integral in Eq. (15). Multiplying both sides of $dV \approx -Ddt$ by the drag and integrating results in

$$\int_{V_i}^{V_f} D dV = - \int_{t_i}^{t_f} D^2 dt \quad (16)$$

Expressing the integral on the right hand side of Eq. (16) in terms of means and increments yields

$$\int_{V_i}^{V_f} D dV = - \langle D^2 \rangle \Delta T \quad (17)$$

If a mean drag is associated to the phase under consideration, integrating Eq. (14) would result in $\langle D \rangle = (V_i^2 - V_f^2)/2R$. From the mean of the drag, the time duration of this phase can be found: $\Delta T = -\Delta V / \langle D \rangle$. In order to calculate the integral of interest, the mean of the square of the drag must also be found. The next method is proposed to find $\langle D^2 \rangle$.

Noting that a probability density function f is defined in terms of its distribution function F as $f(x) = dF(x)/dx$.²⁶ The relation $dV \approx -Ddt$ could be understood as the relation between a distribution function (V) and its probability density function (D) that have been specifically scaled and initialized. For instance, if it were not for the negative sign, the velocity *distribution* of the drag *probability density* would be very similar to the Maxwell distribution and density functions²⁶ (figure 2).

Assume that the drag in terms of time can be considered as a probability density function. In that case, $\langle D^2 \rangle$ and $\langle D \rangle^2$ are related through the variance because the variance of a probability distribution is defined as $\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$. Provided that in the Maxwell distribution the variance is proportional to the square of the mean,²⁶ it is reasonable to think that $\langle D^2 \rangle$ and $\langle D \rangle^2$ could ultimately be related through a linear function. From the 20 test cases referred to in the introduction, the relationship between $\langle D^2 \rangle$ and $\langle D \rangle^2$ can be found empirically and checked if it follows a specific relation. Figure 3 presents that relation. A data correlation coefficient of 0.998 for the 20 nominal cases shows that $\langle D^2 \rangle$ and $\langle D \rangle^2$ are, in fact, highly correlated (this analysis was also carried out for the 59,980 dispersed cases. For the dispersed cases, the correlation coefficient was found to be 0.985).

From figure 3 it is deduced that

$$\langle D^2 \rangle = k_{d1} \langle D \rangle^2 + k_{d2} \quad (18)$$

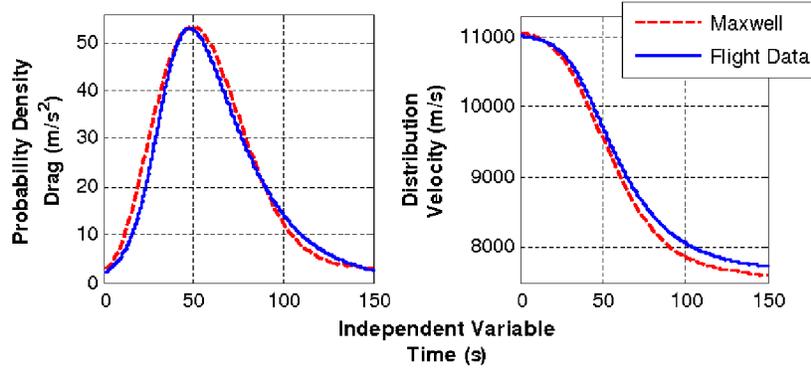


Figure 2. Example of Drag and Velocity Signals During the First Entry in a Skip Entry Compared to Maxwell Probability Density and Distribution Functions that have been properly scaled and initialized for comparison purposes.

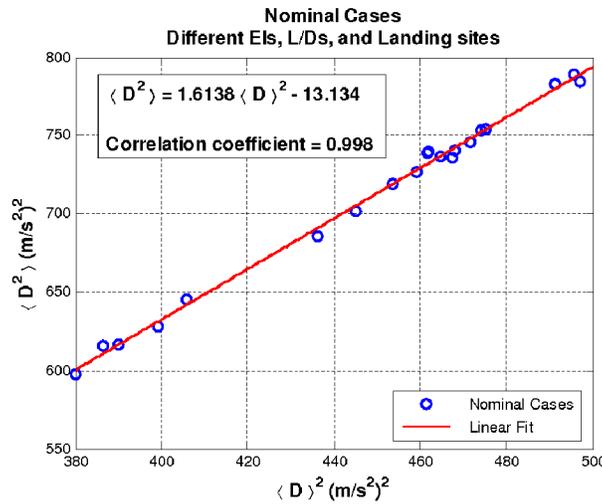


Figure 3. Relation Between $\langle D^2 \rangle$ and $\langle D \rangle^2$ in the Phase of Interest for the 20 Nominal Skip Entries Flown Using NSEG.

where k_{d1} and k_{d2} are the regression constants. The range equation, Eq. (14), can now be substituted with

$$\sum_{n=0}^m \frac{a_n}{n+1} (V_f^{n+1} - V_i^{n+1}) = (k_{d1} \langle D \rangle^2 + k_{d2}) \Delta T \quad (19)$$

which could be considered a pseudo-range equation because the range is indirectly accounted for through the terms $\langle D \rangle$ and ΔT .

B. An Additional Constraint Equation

Should it be desired to add constraint equations to the drag profile, the degree of the drag polynomial should be increased accordingly. In this section, one additional constraint equation will be generated. From the equations derived so far, the drag reference profile may turn out to have a maximum drag acceleration at an arbitrary velocity ($V_{D_{max}}$). It is desired to generate drag reference profiles whose shape is more in line with realistic shapes. From the cases simulated with NSEG a quite simple trend can be found by inspection of the drag profiles: the velocity at which the maximum drag occurs has a high correlation with the product of the initial and final velocities (figure 4). The correlation factor was found to be 0.991 for the 20 nominal test cases (for the 59,980 dispersed cases the correlation coefficient was found to be 0.851).

Therefore, the additional constraint equation will be given by

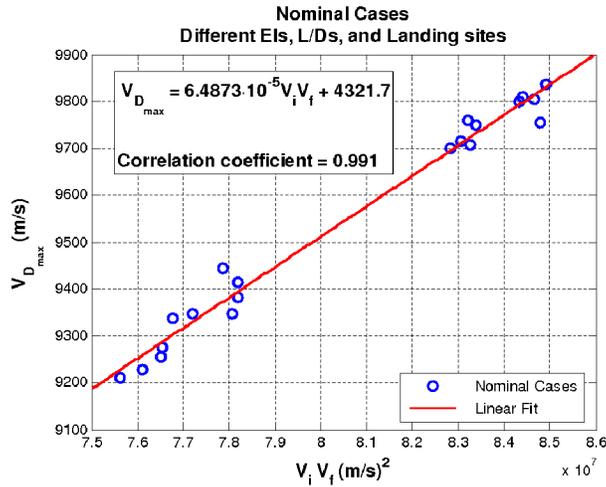


Figure 4. Relation Between the Velocity at Maximum Drag and the Product of the Initial and Final Velocities in the Phase of Interest for the 20 Nominal Skip Entries Flown Using NSEG.

$$D'_{D_{max}} = \sum_{n=0}^m n a_n V_{D_{max}}^{n-1} = 0 \quad (20)$$

where the velocity at maximum drag, $V_{D_{max}}$, can be expressed as

$$V_{D_{max}} = k_{v1} V_i V_f + k_{v2} \quad (21)$$

C. General Solution to Generate Drag Reference Profiles

From the results obtained in the previous subsections, the system of 5 or 6 linear equations (depending on whether the drag is expressed as a 4 degree or 5 degree polynomial, respectively) from where the reference drag profiles can be obtained is

$$\begin{pmatrix} 1 & V_i & V_i^2 & V_i^3 & V_i^4 & V_i^5 \\ 1 & V_f & V_f^2 & V_f^3 & V_f^4 & V_f^5 \\ 0 & 1 & 2V_i & 3V_i^2 & 4V_i^3 & 5V_i^4 \\ 0 & 1 & 2V_f & 3V_f^2 & 4V_f^3 & 5V_f^4 \\ \frac{V_f - V_i}{1} & \frac{V_f^2 - V_i^2}{2} & \frac{V_f^3 - V_i^3}{3} & \frac{V_f^4 - V_i^4}{4} & \frac{V_f^5 - V_i^5}{5} & \frac{V_f^6 - V_i^6}{6} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 1 & 2V_{D_{max}} & 3V_{D_{max}}^2 & 4V_{D_{max}}^3 & 5V_{D_{max}}^4 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} = \begin{pmatrix} D_i \\ D_f \\ \frac{V_i \sin \gamma_i}{h_s} + \frac{2D_i}{V_i} \\ \frac{V_f \sin \gamma_f}{h_s} + \frac{2D_f}{V_f} \\ \langle D^2 \rangle \Delta T \\ \dots \\ 0 \end{pmatrix} \quad (22)$$

Figure 5 shows the degree 5 and degree 4 drag reference profiles generated using Eq. (22) when the range and the initial and final conditions of the 20 nominal test cases are used. With generality, a skip entry is defined whenever the lofting acceleration drops below 0.2g during the lofting phase of the flight. Therefore, for the generation of the profiles, the final drag and, for simplicity, the initial drag, are chosen to be 0.2g.

D. Feasibility of the Generated Drag Profiles

Given a reference profile $D(V)$, can the existence of a feasible bank control command be guaranteed such that $D(V)$ can be tracked? The approach to derive the reference bank control consists of time differentiating the drag along the trajectory (i.e., taking the Lie derivative of the drag) until the first appearance of the control. This has been done in references 20 and 21, where the reference bank angle was found to be

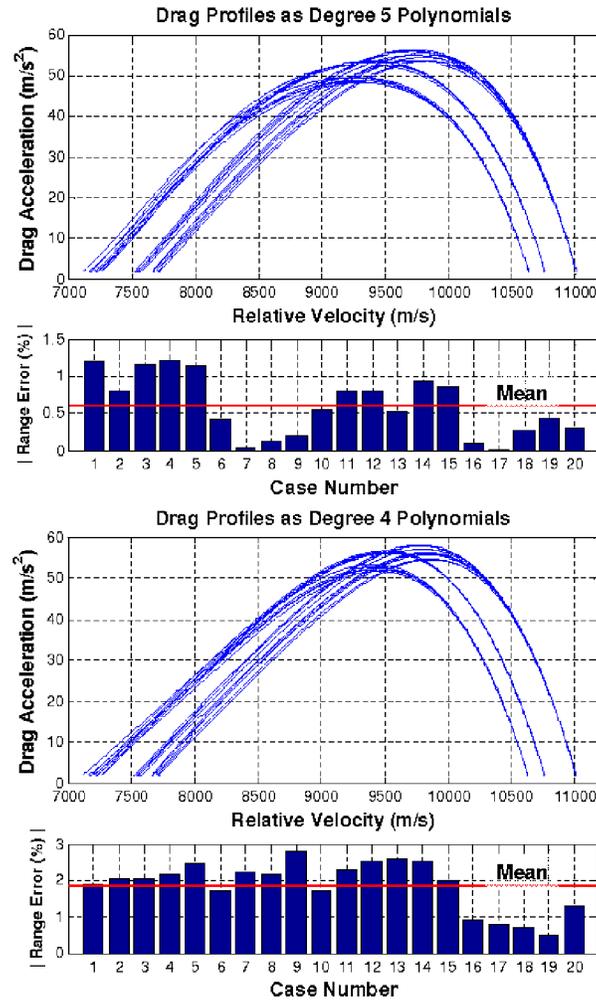


Figure 5. Drag Reference Profiles Generated Using Eq. (22) and Resulting Range Errors with Respect to the Ranges of the 20 Nominal Skip Entry Cases Flown Using NSEG.

$$\phi = \arccos \left\{ \left(\ddot{D} - \dot{D} \left(\frac{\dot{D}}{D} - \frac{3D}{V} \right) + \frac{4D^3}{V^2} + \frac{D}{\langle h_s \rangle} \left(\frac{V^2}{r_e + \langle h \rangle} - \langle g \rangle \right) \right) \left(\frac{-D^2}{(L/D) \langle h_s \rangle} \right)^{-1} \right\} \quad (23)$$

where the mean values of g , h_s and h have been used. In order to find the second time derivative of the drag to be substituted in Eq. (23), it is necessary to differentiate the equation $\dot{D} \approx -DdD/dV$ a second time. This operation results in

$$\ddot{D} = D \left(\frac{dD}{dV} \right)^2 + D^2 \frac{d^2D}{dV^2} \quad (24)$$

The resulting bank angle profiles, corresponding to the fifth and fourth order drag reference profiles depicted in figure 5, are shown in figure 6. A saturation at 180° exists at the beginning of the trajectories (high velocities) for a small velocity interval, when the drag is still fairly low. The following section will show how this feature is dealt with when the control law is implemented. Except for this undesired trait, the bank profiles remain completely feasible.

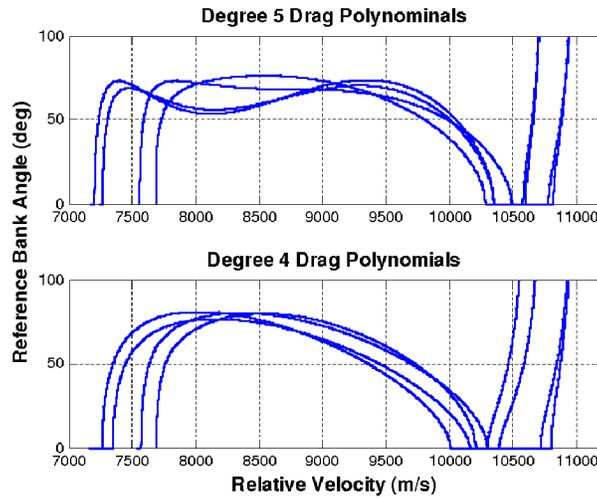


Figure 6. Reference Bank Angle Profiles Associated to the Fifth Order Drag Reference Profiles Generated Using Eq. (22).

IV. Reference Profile Tracking

For completeness, a bank-angle control law will be implemented for tracking the reference profiles. The control law is based on the guidance law for the Space Shuttle Orbiter revisited using nonlinear geometric methods, with the error dynamics of a second order homogeneous equation in the form of a damped oscillator. This work is detailed in reference 21 and, therefore, its theoretical development and background will not be presented here.

The control law presented in Ref. 21 guarantees perfect tracking of either a drag in the form of a linear function of the velocity or a quadratic drag with a constant error in steady state because it accounts for two integrators in the plant. The generated reference drag profiles (figure 5), although polynomials of the velocity of degree 4 or 5, at low velocities approximate a linear function of the velocity, which can be tracked with zero steady state error with a double integrator. It remains to be seen the performance of such controller at high velocities, where the drag profile does not have a linear behavior. This section will show that satisfactory performance is achieved using the double integrator control scheme.

The translational state is controlled by adjusting the vertical lift-to-drag ratio, $(L/D)\cos\phi$. Equivalently, the bank angle ϕ is taken to be the control in the following analysis. The control law derived in Ref 21 is given by

$$\phi_c = \arccos \left\{ \frac{1}{(L/D)b_r} (-a_r + \ddot{D}_r - \omega^2 \Delta D - 2\zeta\omega\Delta\dot{D}) \right\} \quad (25)$$

where the subindex r refers to the reference profile and where ω and ζ are the natural frequency and the damping ratio, respectively. The terms a_r and b_r are given by

$$\begin{aligned} a_r &= \dot{D}_r \left(\frac{\dot{D}_r}{D_r} - \frac{3\dot{D}_r}{V} \right) - \frac{4D_r^3}{V^2} + \left(\langle g \rangle - \frac{V^2}{r_e + \langle h \rangle} \right) \frac{D_r}{\langle h_s \rangle} \\ b_r &= -D_r^2 / \langle h_s \rangle \end{aligned} \quad (26)$$

The performance of the drag tracking control is evaluated on the 20 drag profiles with degree 5 shown in figure 5. The same set of control gains are used for all the cases regardless of the initial and final conditions. During the controlled simulation, the drag profile is not updated or modified to reduce the range error. Therefore, the target miss accumulated at guidance termination will be a measure of the control performance.

In all simulations, the control starts operating once the initial drag acceleration is higher than $1g$. Prior to that point, the commanded bank angle is constant and equal to 80 deg to avoid the saturation observed in the reference bank angle at the initiation of the entry (figure 6). Also, the bank angle rate of change and

acceleration were limited to $\pm 15 \text{ deg/s}$ and $\pm 6 \text{ deg/s}^2$, respectively. Each controlled simulation is divided in two phases, determined by the velocity and the curvature of the drag as a function of the velocity. Figure 5 shows that the drag reference profiles become almost linear at low velocities. It was determined that changing the set of control gains when the curvature was smaller than a certain threshold resulted in an improvement of the tracking performance. The curvature²⁷ is defined as $\kappa = (d^2D/dV^2) / (1 + (dD/dV)^2)^{3/2}$, and its threshold was chosen to be -0.05. The selected values of the control gains in phase 1 (high velocities) are $\zeta = 0.4$ and $\omega = 3/(80\zeta) = 0.0938$. In phase 2 (low velocities and $-0.05 < \kappa < 0$) the control gains are $\zeta = 0.68$ and $\omega = 3/(25\zeta) = 0.1765$. Note that the 5% criteria²⁸ (settling time equal to 3 times the time constant of the system) has been used for the calculation of ω .

Figure 7 shows, for each of the test cases, the absolute range and final flight path angle percentile errors with respect to the values in the 20 nominal cases flown using NSEG. Figure 7 depicts similar error values for the cases with the same entry interface. This suggests that specific sets of control gains could be selected for each group with the same entry interface to improve the tracking performance. Furthermore, the latter could imply that an optimal relation between control gains and the set of initial and final conditions could be found. Nevertheless, this possibility is not explored here because it is not the objective of this note to develop the control law to fly the drag profiles. Eventually, the control law could be as sophisticated as desired.

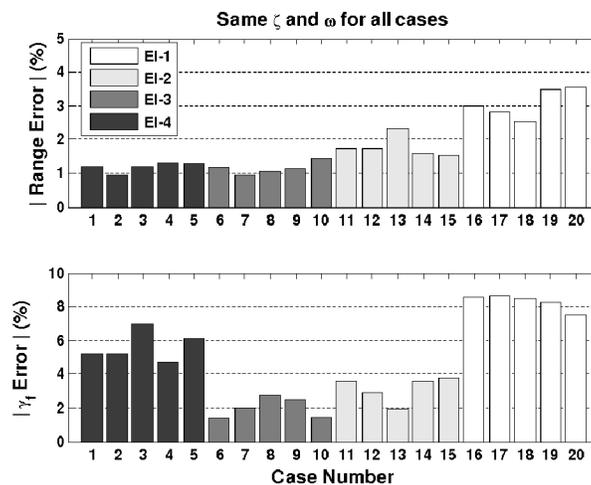


Figure 7. Absolute Range and Final Flight Path Angle Errors of the Controlled Trajectories with Respect to the Values of the 20 Nominal Skip Entry Cases Flown Using NSEG.

Figures 8 and 9 show the results of four controlled trajectories, one from each entry interface. Figure 10 shows the drag error signals for the same set of 4 trajectories.

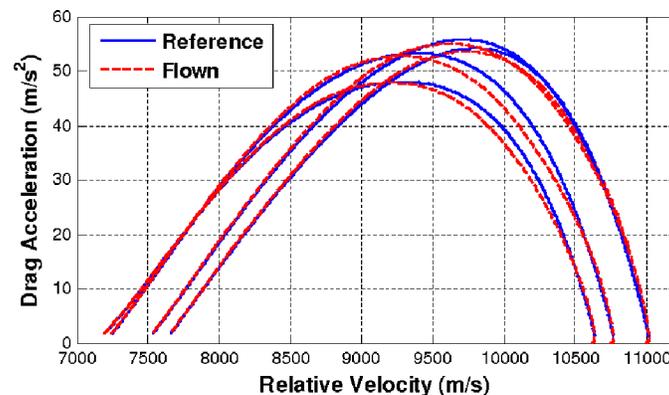


Figure 8. Examples of Drag vs Velocity Profiles of 4 Controlled Trajectories, One from Each Entry Interface.

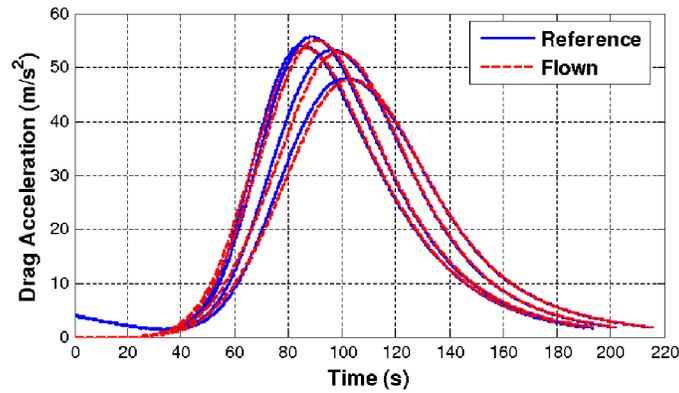


Figure 9. Examples of Drag vs Time Profiles of 4 Controlled Trajectories, One from Each Entry Interface.

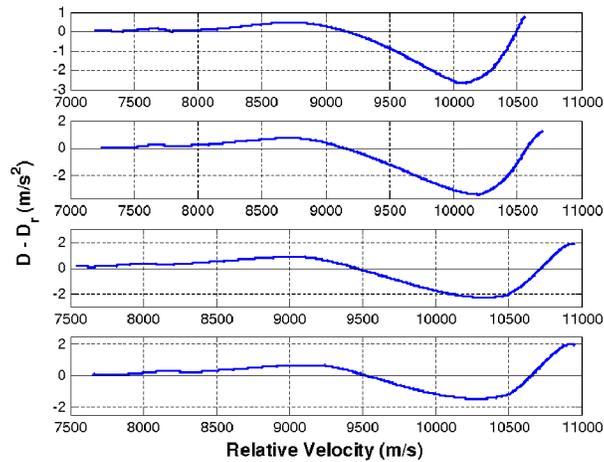


Figure 10. Examples of Drag Acceleration Error Signals of 4 Controlled Trajectories, One from Each Entry Interface.

V. Conclusion

This note shows that a feasible reference drag profile for the first entry portion of a skip entry can be generated as a polynomial expression of the velocity. The coefficients of that polynomial are found through the resolution of a system composed of $m + 1$ equations, where m is the degree of the drag polynomial. It has been shown that a minimum of five equations ($m = 4$) are required to establish the range and the initial and final conditions on velocity and flight path angle. It has been shown that at least one constraint on the trajectory can be imposed through the addition of one extra equation in the system, which must be accompanied by the increase in the degree of the drag polynomial.

In order to simplify the resolution of the system of equations, the drag was considered as being a probability density function of the velocity, with the velocity as a distribution function of the drag. Combining this notion with the introduction of empirically derived constants, it has been shown that the system of equations required to generate the drag profile can be successfully reduced to a system of linear algebraic equations.

For completeness, the resulting drag profiles have been flown using the feedback linearization method of differential geometric control as a guidance law with the error dynamics of a second order homogeneous equation in the form of a damped oscillator. Satisfactory results were achieved when the gains in the error dynamics were changed at a certain point along the trajectory that is dependent on the velocity and the curvature of the drag as a function of the velocity.

Future work should study the capacity to update the drag profile in flight when dispersions are introduced. Also, future studies should attempt to link the first entry, as presented and controlled in this note, with a

more standard control concept for the second entry, such as the Apollo entry guidance, to try to assess the overall skip entry performance. A guidance law that includes an integral feedback term, as is the case in the actual Space Shuttle entry guidance and as is proposed in Ref 29, could be tried in future studies to assess whether its use results in an improvement of the tracking performance, and to evaluate the design needs when determining the control gains.

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