Using Doppler Shifts of GPS Signals To Measure Angular Speed

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A method has been proposed for extracting information on the rate of rotation of an aircraft, spacecraft, or other body from differential Doppler shifts of Global Positioning System (GPS) signals received by antennas mounted on the body. In principle, the method should be capable of yielding low-noise estimates of rates of rotation. The method could eliminate the need for gyroscopes to measure rates of rotation.

The method is based on the fact that for a given signal of frequency \( f_t \) transmitted by a given GPS satellite, the differential Doppler shift is attributable to the difference between those components of the instantaneous translational velocities of the antennas that lie along the line of sight from the antennas to the GPS satellite. On the basis of straightforward geometric considerations (see figure), it can be readily shown that the differential Doppler shift is related to the angular velocity \( \omega \) of the rotating body by

\[
 f_1 - f_2 = 2f_t \omega \cdot r / c,
\]

where \( f_1 \) and \( f_2 \) are the instantaneous Doppler-shifted frequencies of the replicas of the ft signal received by the two antennas, \( r \) is half of the baseline vector between the two antennas, \( a \) is a unit vector along the line of sight from the antennas to the GPS satellite, and \( c \) is the speed of light.

It must be noted that the equation above can be solved to obtain only partial information about \( \omega \). However, if there are three or more antennas and if signals can be received from two or more GPS satellites, then one can form simultaneous independent equations for different pairs of antennas and different unit vectors that can be solved to obtain all of the components of \( \omega \).