Sensitivity Analysis of Launch Vehicle Debris Risk Model

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Abstract: As part of an analysis of the loss of crew risk associated with an ascent abort system for a manned launch vehicle, a model was developed to predict the impact risk of the debris resulting from an explosion of the launch vehicle on the crew module. The model consisted of a debris catalog describing the number, size and imparted velocity of each piece of debris, a method to compute the trajectories of the debris and a method to calculate the impact risk given the abort trajectory of the crew module. The model provided a point estimate of the strike probability as a function of the debris catalog, the time of abort and the delay time between the abort and destruction of the launch vehicle. A study was conducted to determine the sensitivity of the strike probability to the various model input parameters and to develop a response surface model for use in the sensitivity analysis of the overall ascent abort risk model. The results of the sensitivity analysis and the response surface model are presented in this paper.

Keywords: Space launch vehicles, ascent abort, debris strike probability

1. INTRODUCTION

Manned space launch vehicle systems, such as the Apollo/Saturn V shown in Figure 1, typically have a launch abort system (LAS) designed to quickly separate the crew module from the rest of the vehicle in case of an abort during ascent. The LAS consists of a rocket attached to the crew module that can pull the crew module away from the launch vehicle when activated. The sequence of events for a Mode I abort is shown in Figure 1(b). The LAS is designed to improve the survivability of the crew during ascent. However, there are still risks involved in a launch and ascent abort, even with an available LAS. Such risks arise from the overpressure and debris field resulting from an explosion of the launch vehicle and possible failures of the LAS itself. Understanding and quantifying the risks associated with the LAS and ascent aborts contribute to the overall crew risk assessment of the launch vehicle.

Figure 1: Launch Abort System of a manned space launch vehicle
(a) LAS of Apollo/Saturn V
(b) Sequence of events for Mode I abort

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As part of the assessment of the risk associated with launch failures of a manned launch vehicle, a model was developed to compute the probability of debris striking the crew module after an abort and destruction of the launch vehicle during ascent [1]. The model consisted of a debris catalog, post-abort trajectories for the crew module, launch vehicle and debris field, and a method to calculate the launch vehicle debris strike probability on the crew module along its abort trajectory. The results of the model were used in the overall assessment of the risk associated with ascent launch aborts.

The model computed strike probability as a function of the mission elapsed time (MET) of the abort and the delay time between the abort and the destruction of the launch vehicle. The strike probability was also dependent upon other parameters within the model, including the number of pieces in the debris catalog, the magnitude and direction of the imparted velocity on the debris due to the explosion and the combination of debris size and impact velocity required to penetrate the crew module. A sensitivity analysis was conducted to determine the influence of these parameters on the overall strike probability. The uncertainty derived from this sensitivity analysis was used in the overall risk assessment to determine the uncertainty bounds for the loss-of-crew results for the launch abort system.

Depending upon the size of the debris catalog and time of abort, the debris strike model could require large amounts of computational resources and wall clock time to determine the strike probability. The computational requirements were prohibitive for sensitivity and dispersion analyses (e.g., effects of ascent and abort trajectory dispersions) in which the strike probabilities were required for a large number of trajectories. To reduce the computational resources required to predict the strike probability, a response surface model was created using the sensitivity analysis data. The response surface model was then used to predict the sensitivity of the strike probability due to changes in the input parameters of the model. The debris strike model and response surface model are described in the following section.

2. DEBRIS STRIKE MODEL

The baseline debris strike model consisted of a debris catalog, a method to calculate the debris trajectories and a method to calculate the strike probability given the debris and crew module abort trajectories. The response surface model consisted of the sensitivity data and used Kriging [2] to predict the strike probability for a new set of input parameters.

2.1 Baseline Debris Strike Model

The baseline debris catalog was derived from the Space Shuttle debris catalog [3]. The catalog classified debris according to the source (e.g., hardware or solid propellant) and provided a range of values for the number of pieces arising from each category, debris mass, reference area, drag coefficient and imparted velocity due to the explosion. A description of each piece of debris was generated based on the catalog information. The initial position of the debris could be set at the midpoint of the segment or randomly distributed along the length of the segment. The direction of the imparted velocity could be set to be in a random direction (spherical), directed normal to the vehicle centerline (cylindrical), or a mixture of both (mixed).

A trajectory was computed for each piece of debris using a three-degree-of-freedom trajectory tool [4]. The initial conditions were derived from the state of the launch vehicle at the time of explosion, based on the launch vehicle ascent trajectory. The trajectory tool could also be used to determine the abort trajectory of the crew module. However, the crew module abort trajectories, along with the launch vehicle ascent trajectory, were usually provided as inputs to the model.

The strike probability was computed from the trajectory data. At a given time after the abort and explosion, the position of the debris and crew module were obtained from the trajectory data, as shown in Figure 2. The distance between the crew module and the explosion center was computed. The
same distance measure was computed for each piece of debris. Debris that had traveled the same
distance as the crew module from the explosion center, to within some tolerance, were counted as
being a probable strike threat. The counted debris is shown in red in Figure 2. The debris flux, \( F \), was
computed using the number of counted debris pieces, \( n_{\text{counted}} \), and the circular area, \( A_{\text{avesep}} \), based on
the average separation distance, \( d_{\text{avesep}} \), between the counted debris and the crew module, and the
exposure time, \( dt \)

\[
F = n_{\text{counted}} / (A_{\text{avesep}} dt)
\]  

(1)

The probability of being hit by at least one piece of debris, \( p_{\text{strike}} \), was computed using the Poisson
distribution and the debris flux

\[
p_{\text{strike}} = 1.0 - \exp(-F A_{\text{exposed}} dt)
\]  

(2)

where \( A_{\text{exposed}} \) is the cross-sectional area of the crew module and \( dt \) is the exposure time. The overall
strike probability was computed by integrating in time along the entire crew module abort trajectory.
This approach is similar to one used for computing strike probability of orbital debris for spacecraft in
low earth orbit (e.g., Ref. 5).

**Figure 2: Relative position of debris field and crew module after abort and destruct of launch vehicle**

2.2 Sensitivity Analysis

The debris strike model was used to provide point estimates of the debris risk as a function of mission
elapsed time (MET) and flight termination system (FTS) delay time [1]. However, the input
parameters to the model were best guess estimates based on available data and expert opinion. It was
important to understand how the strike probability varied with changes to the other input parameters.
A sensitivity analysis was conducted to quantify the effects of the input parameters on the strike
probability results.

The parameters and range of values used in the sensitivity analysis are listed in Table 1. The number
of debris pieces and imparted velocities were obtained by scaling the baseline debris catalog or using
other debris catalogs. A “cylindrical” debris pattern assumed the debris imparted velocity was
directed normal to the vehicle centerline. A “mixed” debris pattern assumed the imparted velocity for
hardware debris was oriented normal to the vehicle centerline while the imparted velocity for propellant debris was oriented in a random direction. The launch season corresponded to ascent and abort trajectories computed based on atmospheric models for typical February and August conditions. The penetration criterion used a mass/relative velocity curve to determine if a given piece of debris with a given impact velocity could penetrate the crew module. If the criterion was applied, only those pieces of debris that could penetrate the crew module were used in the strike probability calculation. Without the criterion, all debris pieces within a given tolerance distance of the crew module were used in the strike probability calculation. The abort times were selected based on times of interest during ascent, such as reaching Mach 1 or experiencing maximum dynamic pressure. The FTS delay time (or warning time) was the time between the abort and when the launch vehicle was destroyed by the flight termination system (FTS).

Table 1: Parameters used in sensitivity analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of debris pieces</td>
<td>(0.008 to 1.24)*baseline</td>
</tr>
<tr>
<td>Imparted velocity</td>
<td>(0.15 to 1.42)*baseline</td>
</tr>
<tr>
<td>Debris pattern</td>
<td>(cylindrical, mixed)</td>
</tr>
<tr>
<td>Launch season</td>
<td>(Feb, Aug)</td>
</tr>
<tr>
<td>Penetration criterion</td>
<td>(on, off)</td>
</tr>
<tr>
<td>Abort time</td>
<td>6 METs</td>
</tr>
<tr>
<td>FTS delay time</td>
<td>(0, 1, 2, 5, 10) sec</td>
</tr>
</tbody>
</table>

The overall uncertainty in the strike probability was computed by generating data within the parameter space. A full factorial design was used, resulting in 2762 data points. The debris strike probability uncertainty results were then carried forward to determine the uncertainty in the overall loss-of-crew probability resulting from launch vehicle failures during ascent.

2.3 Response Surface Model

The large number of debris pieces predicted by the baseline debris catalog meant that the model required extensive computational resources and wall clock time to compute a set of strike probabilities as a function of MET and FTS delay time. The need to quickly determine the strike probability for a given set of sensitivity parameters led to the development of a response surface model.

The response surface model used Kriging to predict the strike probability given a set of input values based on the parameter set used in the sensitivity analysis. The computed sensitivity data were used as the training data for the Kriging model. For this application, the Kriging model consisted of a first-order polynomial regression model and an exponential correlation function. A separate testing data set was used to set the correlation parameter, $\theta$, in the Kriging model and to obtain a measure of accuracy of the response surface. Response surfaces were created using the training data and a range of $\theta$ values and then used to predict the strike probability at the test data locations. The root mean square error (RMSE) between the predicted and computed strike probabilities in the test data set was used to determine the best value of $\theta$ of the values tested. The response surface corresponding to this value of $\theta$ was then used to predict strike probabilities at various locations within the parameter space to provide an understanding of how the strike probability changed with variations in the input parameter values.

3.0 RESULTS

The results of the sensitivity analysis and response surface model are presented in the following sections. The sensitivity analysis was conducted by varying one parameter value at a time and computing the strike probability. The overall uncertainty in the strike probability model was determined by computing strike probabilities within the entire input parameter space. The data within
the parameter space were used as the training set for a response surface model. The response surface was then used to predict the change in strike probability due to changes in input parameter values.

### 3.1 Sensitivity Analysis

The effect of number of debris pieces on strike probability is shown in Figure 3. As the number of debris pieces decreased (Figure 3(a)), the strike probability decreased, due to a smaller number of debris pieces being counted in the debris flux calculation. The effect of imparted velocity magnitude on strike probability is shown in Figure 3(b). As the velocity decreased, the strike probability decreased. Lower imparted velocities reduced the ability of the debris to “catch up” to the crew module after the explosion, especially for the longer delay times. The effect of debris pattern (i.e., the direction of the imparted velocity relative to the segment centerline) is listed in Table 2. The spherical and mixed patterns yielded higher strike probabilities due to some of the high-velocity fragments being directed towards the crew module. This caused the debris flux to increase by increasing the number of counted pieces and decreasing the average area used in the flux calculation since the debris tended to be closer to the crew module than in the “cylindrical” pattern case. The baseline model counted all debris that met the distance criterion (e.g., the debris and crew module were at the same distance from the blast center at a given time after launch vehicle destruct). Applying the penetration criterion, only debris that had a sufficient velocity relative to the crew module, as a function of debris mass, were counted. This effectively reduced the debris flux, leading to a reduction in the strike probability.

![Figure 3: Sensitivity of strike probability to input parameters](image)

![Figure 3: Sensitivity of strike probability to input parameters](image)

### Table 2: Effect of debris pattern on strike probability

<table>
<thead>
<tr>
<th>Pattern</th>
<th>0 sec FTS delay</th>
<th>1 sec FTS delay</th>
<th>2 sec FTS delay</th>
<th>3 sec FTS delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylindrical</td>
<td>0.999</td>
<td>0.947</td>
<td>0.0928</td>
<td>0.0212</td>
</tr>
<tr>
<td>Spherical</td>
<td>1</td>
<td>0.999</td>
<td>0.782</td>
<td>0.480</td>
</tr>
<tr>
<td>Mixed</td>
<td>1</td>
<td>0.999</td>
<td>0.760</td>
<td>0.450</td>
</tr>
</tbody>
</table>

The results from the full factorial parameter space data set is plotted as a function of abort time for three different delay times in Figure 4. In the figure, the lines connect the 5th (dotted), 50th (dash-dot) and 95th (dashed) percentile values at each abort time. The wide range of values for the input parameters, especially for the number of debris pieces and imparted velocity, resulted in large...
uncertainties in the strike probability. The proximity of the 50th percentile values to the 95th percentile values for the zero delay time case, shown in Figure 4(a), indicated that half of the cases yielded strike probabilities close to 1.0. Similarly, half the cases for delay times of 1 and 5 seconds yielded strike probabilities closer to 0.0.

Figure 4: Uncertainty in strike probability

(a) FTS delay time = 0 sec
(b) FTS delay time = 1 sec
(c) FTS delay time = 5 sec

Figure 4 showed that another approach was required to better understand the effect of the input parameters on the strike probability. The approach adopted in this analysis was to create a response surface from the sensitivity data and use the response surface to generate trending information.

3.2 Response Surface Model

A response surface model was created using the data from the full factorial parameter space. The values for the input parameters were selected based on the six candidate debris catalogs and ascent and abort trajectories at conditions of interest for the design of the crew and launch vehicle. As such, the parameter space was not optimized for a response surface model. Two different views of the parameter space shown in Figure 5 illustrate the sparseness of the data in some regions. As a consequence, the accuracy of the response surface suffered in the regions of sparse data.
Figure 5: Two views of the input parameter space

(a) All combinations of debris and velocity ratios
(b) All combinations of abort and delay times

Figure 6: Effect of number of debris pieces on response surface predictions and test data

(a) debris ratio = 0.25
(b) debris ratio = 0.50
(c) debris ratio = 1.00
The number of debris pieces and imparted velocity were expressed as ratios of values obtained from the baseline debris catalog. The other input parameters were assigned integer values, ranging from 0 to the total number of options used for each parameter. Kriging was used to perform the interpolation. A first-order polynomial regression model and an exponential correlation function were used in the kriging model. The correlation parameter, $\theta$, was chosen to minimize the root mean square error (RMSE) between the computed and predicted values of strike probability for a set of test cases.

In order to determine the accuracy of the response surface, the strike probabilities predicted using the response surface were compared against a computed set of test cases. The test cases focused on aborts at two conditions of interest during ascent – when the launch vehicle reached Mach 1 and when the launch vehicle experienced maximum dynamic pressure. The other input parameters were varied to obtain a representation of the parameter space. The comparison of the response surface predictions and the test data for aborts at Mach 1 using different values for the number of debris pieces is shown in Figure 6. Some of the training data near the testing data conditions are plotted as open symbols connected by dotted lines. The test data is plotted as red symbols while the response surface predictions at the testing conditions are plotted as black closed symbols. In general, the response surface was able to resolve the trends in the data for both changes in FTS delay time and number of debris pieces. In regions of sparse data, the response surface interpolation was fairly linear. Where additional data were available, as shown in Figure 6(a) for 3 and 4 sec delays, the response surface incorporated the additional data to provide an improved prediction.

Similar results were observed when varying the velocity, as shown in Figure 7. The results plotted in Figure 7 indicated that a reduction in imparted velocity reduced the strike probability, reflecting the results shown in Figure 3(b).

**Figure 7: Effect of imparted velocity on response surface predictions and test data**

(a) velocity ratio = 0.50  
(b) velocity ratio = 1.00

The comparison with test data showed that the response surface was sufficient for the purposes of predicting trends in strike probability due to changes in input parameters. Examples of this use of the response surface are shown in Figure 8. In Figure 8(a) and 8(b), strike probabilities were predicted for aborts with 1 sec of delay time. As the number of debris pieces in the catalog increased, the strike probability increased, as shown in Figure 8(a). A reduction in the imparted velocity reduced the strike probability, as shown in Figure 8(b). The effect of delay time on strike probabilities assuming debris and velocity ratios of 1.0 is shown in Figure 8(c). There was a significant reduction in strike probability predicted for delay times greater than 1 sec, especially for aborts early in the ascent trajectory. As debris catalogs designed for specific launch vehicles are developed, the response surface model can be used to assess the effect of different catalog parameters on debris risk.
4. CONCLUSION

A model was created to predict the probability of debris resulting from a launch vehicle explosion striking the crew module along its abort trajectory. This model was used to compute point estimates of strike probability as a function of debris catalog and abort and delay times for use in the overall assessment of a launch abort system loss-of-crew risk. Because many of the inputs to the model were based on uncertain data and expert opinion, a sensitivity analysis was conducted to determine the uncertainty associated with the strike probability.

A response surface model was created from the sensitivity data as a tool to generate trend information. A comparison of the response surface predictions with a set of test data showed that the accuracy of the predictions ranged from good to poor, depending upon the amount of training data in the region. The response surface model was able to predict the trending information and thus could be used as a guide to assessing new debris catalogs and ascent and abort trajectories.

The wide range of values for the inputs, especially the number of debris pieces and imparted velocity, yielded a wide range of values for strike probability for a given abort and delay time. The 5th, 50th and 95th percentile values provide an indication of the where the majority of values lie and could be used to determine conservative and optimistic values for use in the launch abort system risk assessment.
References


