An Improved Wake Vortex Tracking Algorithm for Multiple Aircraft

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The accurate tracking of vortex evolution from Large Eddy Simulation (LES) data is a complex and computationally intensive problem. The vortex tracking requires the analysis of very large three-dimensional and time-varying datasets. The complexity of the problem is further compounded by the fact that these vortices are embedded in a background turbulence field, and they may interact with the ground surface. Another level of complication can arise, if vortices from multiple aircrafts are simulated. This paper presents a new technique for post-processing LES data to obtain wake vortex tracks and wake intensities. The new approach isolates vortices by defining "regions of interest" (ROI) around each vortex and has the ability to identify vortex pairs from multiple aircraft. The paper describes the new methodology for tracking wake vortices and presents application of the technique for single and multiple aircraft.

Nomenclature

- $b_0$ = initial vortex separation
- $B$ = wing span
- $N$ = Brunt-Väisälä frequency
- $\Gamma_\infty$ = Circulation
- $N^*$ = non-dimensional Brunt-Väisälä frequency = $2\pi N b_0^2 / \Gamma_\infty$
- $r$ = radius from vortex center
- $t$ = time ($t = 0$ at vortex generation)
- $T^*$ = non-dimensional time = $tv_0 / b_0$
- $V_0$ = initial vortex descent velocity = $\Gamma_\infty / 2\pi b_0$
- $x$ = direction along the aircraft flight path (longitudinal coordinate)
- $y$ = direction perpendicular to the aircraft flight path (lateral coordinate)
- $z$ = vertical coordinate
- $\Delta \rho$ = pressure deviation from environmental pressure
- $\rho_s$ = two-dimensional $yz$-plane
- $\varepsilon$ = eddy dissipation rate
- $\xi_x$ = vorticity component in the direction of the flight path

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I. Introduction

In recent years considerable research efforts have focused on increasing the capacity of the National Airspace System (NAS). The demand already exceeds the capacity at several major U.S. airports and the future projections predict more airports facing similar problems in the coming years. One bottleneck to increased airport capacity is the wake separation standard, which requires mandatory spacing between aircraft to avoid wake-related hazards. A less restrictive wake-separation standard that maintains current levels of safety could increase throughput, thereby alleviating congestion at crowded airports. Any such re-evaluation of wake separation standard requires a clear understanding of wake vortex dynamics as well as an understanding of flight operations.

The quantification of wake vortex behavior relies on both field observations and numerical simulations. Since observations are generally sparse, in both time and space, wake vortex research relies heavily upon numerical simulations to generate detailed wake vortex data under varying conditions of atmospheric stability and turbulence. The resulting numerical datasets can be very large and require computationally efficient and accurate analysis techniques for post-processing. These vortices are embedded in a background turbulence field and can interact with the ground surface, which increases the complexity of the problem. The interactions of vortices from multiple aircraft add yet another layer of complication to the problem. The vortex track and intensity data obtained from the numerical simulations, coupled with observations form the basis for developing fast-time empirical models. These empirical models, in turn can then help to define standards for safe operating conditions in a given airspace. The ability to extract accurate vortex related information from the numerical datasets is therefore of paramount importance.

This paper presents a methodology for tracking multiple vortex systems (Figure 1). The novel method described in the paper builds upon an existing tracking algorithm and enables analysis of more complex operational scenarios. The new approach isolates vortices from multiple aircrafts by defining a region-of-interest (ROI) around each vortex and then tracking the individual ROI throughout the evolution of vortices. The only limitation of the presented methodology is that it assumes the existence of a single vortex of the same rotation in a given search region. In the following sections, a brief overview of the large eddy simulation model is given, the new ROI-based wake tracking technique is described in detail, and the results of its application for single and multiple aircraft are presented.

Figure 1. Multiple vortex systems can arise in certain operational scenarios such as Closely Spaced Parallel Runways (CSPR). The figure shows a schematic of wake vortices generated from closely spaced aircraft. The Federal Aviation Administration (FAA) defines closely spaced parallel runways as those having a separation of less than 2500ft (762 m).
II. Terminal Area Simulation System (TASS)

The Terminal Area Simulation System (TASS) is a state-of-the-art, cloud resolving, large eddy simulation model. The TASS model has been extensively validated in the past against observational data and applied to a diverse set of problems ranging from the simulation of shallow cumulus to supercell storms, including convective phenomena such as downbursts, gust fronts and hailstorms. TASS has also been used to define hazard metrics for aircrafts flying in regions of high turbulence and wind shear.\(^6,7\) In recent years, TASS has been employed to study the decay and transport of wake vortices generated by aircrafts under different conditions of atmospheric stability and turbulence. The objective of these efforts has been to support research projects geared towards increasing the airport capacity.\(^8,9\) The wake vortex simulations from TASS simulation data is used for developing empirical wake prediction algorithms that can be used in an operational setting. The TASS model can also simulate wakes from multiple aircraft embedded either at the same time or with a time delay. This unique simulation capability provides a numerical test bed for analyzing a wide variety of operational scenarios.

III. Analysis Technique

Our previous method for determining wake vortex position and circulation is described in Proctor et al.\(^10\). In this section limitations of the previous method are briefly described and a new method that overcomes these weaknesses is proposed.

In the previous method, vortex track data is extracted from the three-dimensional computational domain by first analyzing each cross track plane \((p_x)\) for all \(x\) and then synthesizing the results to a domain-averaged value. Location of the vortex positions of a counter-rotating vortex pair is determined for each plane, \(p_x\), assuming a vorticity-pressure deviation function:

\[
\phi(y, z) = \xi^3 \Delta p^2. \tag{1}
\]

The centroid from Eq. (1) defines the vortex pair coordinates in the plane as follows:

\[
L(y, z) = \left[ \frac{\int \int \phi y \Delta \phi dy dz}{\int \int \phi \Delta \phi dy dz}, \frac{\int \int \phi z \Delta \phi dy dz}{\int \int \phi \Delta \phi dy dz} \right], \tag{2}
\]

where \(\delta\) differentiates the starboard (positive) and port (negative) vortices by the sign of the vorticity:

\[
\delta = \max(\beta \frac{\phi}{|\phi|}, 0). \tag{3}
\]

The value of \(\beta\) assumes is either 1 or -1, in order to isolate the starboard or port vortex contribution to the position. A weakness of this technique occurs if more than one vortex of similar strength and same sign of vorticity exists in any given plane. In this case, Eq. (2) may return an incorrect position and circulation.

The new method based on a region of interest (ROI) takes advantage of knowing the vortex initial position by restricting the analysis region to insure tracking of a single vortex. The following modification to Eq. (2) demonstrates the application of this idea:

\[
L(y, z) = \left[ \frac{\int \int_{ROI} \phi y \Delta \phi dy dz}{\int \int_{ROI} \phi \Delta \phi dy dz}, \frac{\int \int_{ROI} \phi z \Delta \phi dy dz}{\int \int_{ROI} \phi \Delta \phi dy dz} \right]. \tag{4}
\]

The ROI is a subset of \(p_x\), and is unique for the starboard (positive) and port (negative) vortices. As in the original technique, the ROI method also can only return two vortex positions. This technique also allows for the tracking of wake vortices from multiple aircraft, since a ROI can be uniquely defined for each vortex.
In this paper, the ROI assumes a box with the size of the initial vortex spacing, \( b_0 \), for two reasons: 1) the box has an area that isolates each vortex, and 2) the size is large enough to insure that a vortex will be within the ROI at the next time sequence. Figure 2 is from a wake vortex simulation of two closely-spaced aircraft. The black boxes depict the ROI. The vortex circulation is calculated after determining the position of the vortices from Eq. (2) or Eq. (4). The circulation based on the longitudinal vorticity assumes the following form:

\[
\Gamma_z(x) = \int_{-r}^{r} \zeta_x \, dydz .
\]  

(5)

In Eq. (5), \( r \) represents the radius of integration for the longitudinal vorticity in each \( p_x \) plane. The average circulation uses the following relation:

\[
\Gamma_{r,a,b} = \frac{\int_{a}^{b} \Gamma_z(x) \, dr}{b-a},
\]

(6)

where, the values of \( a \) and \( b \) depend upon the type of averaged circulation. As in Ref. [10], the values of \( a \) and \( b \) were set to 9 m and 15 m, respectively.

The following sections will evaluate the ROI technique (Eq. 4) with the original method (Eq. 2) for identification of vortex locations in single and multiple aircraft scenarios. The purpose of the single aircraft example is to demonstrate that the ROI method performs as well as the original method. A second example examines the ROI method for wake vortices from multiple aircraft.

Figure 2. Orthographic views at \( T^* = 2.16 \) showing the wake vortex dynamics for a pair of Boeing 747s. The figures shown clockwise from the lower right are end, side, and top views, respectively. The black boxes represent sample ROI boxes for port and starboard vortex positioning for the left aircraft, respectively. Isosurfaces shown in the figures represent the eigenvalue of the velocity gradient tensor.\(^\text{11}\) The blue and red Isosurfaces represent the port and starboard vortices, respectively.
IV. Single Aircraft Analysis

In this section, the ROI-based wake-tracking algorithm is compared with the original (non-ROI) method. The LES wake vortex dataset for a Boeing 747 in-ground-effect (IGE) was used for this test. The ambient background turbulence intensity was characterized by an eddy dissipation rate of $10^{-2} \text{ m}^2 \text{s}^{-3}$. This case should be challenging for two reasons: 1) secondary vortices are generated when the wake vortices interact with the ground and 2) the environment contains thermals and turbulent eddies characteristic of the atmospheric boundary layer. Therefore, the tracking algorithms must differentiate the port and starboard wake vortices from turbulence and IGE induced secondary vortices.

The specifications of the LES dataset are given in Tables 1 and 2. The initial lateral positions of the port and starboard vortices were specified at $y = 25 \text{ m}$ and $y = 75 \text{ m}$, respectively. The flight path coordinate is defined at $y = 50 \text{ m}$. Figure 3 shows the comparison of the ROI and non-ROI techniques. The lateral wake vortex trajectory for the ROI shows identical agreement with the non-ROI approach until approximately 100 s, after which only minor deviation occurs. The vertical wake positions are very similar, as well, with the lower (starboard) vortex track deviating slightly earlier at around $t = 70 \text{ s}$.

Table 1. Initial vortex parameters.

<table>
<thead>
<tr>
<th>Initial Vortex System Parameters</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circulation</td>
<td>$565 \text{ m}^2 \text{s}^{-1}$</td>
</tr>
<tr>
<td>Span</td>
<td>$63.66 \text{ m}$</td>
</tr>
<tr>
<td>Lateral separation</td>
<td>$50 \text{ m}$</td>
</tr>
<tr>
<td>Descent speed</td>
<td>$1.8 \text{ m} \text{s}^{-1}$</td>
</tr>
<tr>
<td>Vortex core radius</td>
<td>$3.75 \text{ m}$</td>
</tr>
<tr>
<td>Aircraft flight track</td>
<td>$50 \text{ m}$</td>
</tr>
<tr>
<td>Vortex initial height</td>
<td>$70 \text{ m}$</td>
</tr>
</tbody>
</table>

Table 2. Domain size and resolution.

<table>
<thead>
<tr>
<th>Domain Parameters</th>
<th>Physical Dimension</th>
<th>Vortex Spacing Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal dimension</td>
<td>$651 \text{ m}$</td>
<td>$13 b_0$</td>
</tr>
<tr>
<td>Lateral and vertical dimensions</td>
<td>$450 \text{ m}$</td>
<td>$9 b_0$</td>
</tr>
<tr>
<td>Longitudinal grid spacing</td>
<td>$1.5 \text{ m}$</td>
<td>$\approx1/33 b_0$</td>
</tr>
<tr>
<td>Lateral and vertical grid spacing</td>
<td>$1.25 \text{ m}$</td>
<td>$\approx1/40 b_0$</td>
</tr>
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</table>

Figure 4 shows the comparison of wake vortex circulation obtained using the ROI and original methods. The ROI approach indicates a decay rate very similar to the original approach. The example demonstrates the ability of the ROI approach to perform well, even in a turbulent atmospheric boundary layer.
Figure 3. Comparison of lateral and vertical position for the ROI and original method wake analysis from a single Boeing 747 IGE wake vortex pair. The original method is shown as a solid line, and the ROI method as a dashed line.

Figure 4. As in Figure 3, but for circulation comparisons of ROI versus original analysis method.

V. Multiple Aircraft Analysis

Having shown the ROI method to perform well for a single aircraft, we now proceed to demonstrate the ability of the ROI method to analyze a multiple aircraft case. In this scenario, two sets of port and starboard vortices are generated IGE. This case is challenging since, as discussed earlier, our original method is unable to distinguish between the port vortex of the left aircraft and the port vortex of the right aircraft (blue isosurfaces in Figure 2).

The case scenario assumes a pair of Boeing 747 aircraft flying abreast and in close proximity to the ground surface. Table 3 shows the initial specifications assumed for each aircraft in the LES simulation of this scenario. Specifications for the LES domain size are shown in Table 4. Again the ROI size is based on $b_0$. 

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Table 3. Initial vortex parameters.

<table>
<thead>
<tr>
<th>Initial Vortex System Parameters</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circulation</td>
<td>$565 , m^2 , s^{-1}$</td>
</tr>
<tr>
<td>Span</td>
<td>$63.66 , m$</td>
</tr>
<tr>
<td>Lateral separation</td>
<td>$50 , m$</td>
</tr>
<tr>
<td>Descent speed</td>
<td>$1.8 , m , s^{-1}$</td>
</tr>
<tr>
<td>Vortex core radius</td>
<td>$5.2 , m$</td>
</tr>
<tr>
<td>Right Aircraft flight track</td>
<td>$61 , m$</td>
</tr>
<tr>
<td>Left Aircraft flight track</td>
<td>$-61 , m$</td>
</tr>
<tr>
<td>Vortex initial height</td>
<td>$50 , m$</td>
</tr>
</tbody>
</table>

Table 4. Domain size and resolution.

<table>
<thead>
<tr>
<th>Domain Parameters</th>
<th>Physical Dimension</th>
<th>Vortex Spacing Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal dimension</td>
<td>$1248 , m$</td>
<td>$25 , b_0$</td>
</tr>
<tr>
<td>Lateral dimension</td>
<td>$920 , m$</td>
<td>$18.4 , b_0$</td>
</tr>
<tr>
<td>Vertical dimension</td>
<td>$375 , m$</td>
<td>$7.5 , b_0$</td>
</tr>
<tr>
<td>Longitudinal grid spacing</td>
<td>$3.0 , m$</td>
<td>$\approx 1/17 , b_0$</td>
</tr>
<tr>
<td>Lateral and vertical grid spacing</td>
<td>$2.5 , m$</td>
<td>$1/20 , b_0$</td>
</tr>
</tbody>
</table>

The ROI approach isolates and tracks each vortex. Figure 5 shows the vortex position for both the right and the left aircraft using the ROI approach. Figure 6 shows a top-down perspective of the wake vortices at $t = 117 \, s$. The dashed line in Figure 6 indicates the mean lateral positions extracted from the ROI approach. The tracks obtained from the ROI method are good agreement with the location of the isosurfaces in Figure 6. Figure 7 shows the same information as Figure 6, but from an along flight path view. The red circles in Figure 7 mark the mean vortex positions calculated using the ROI method and again are in good agreement with the isosurfaces.

Figure 5. Lateral and vertical position history for the wake trajectories of both right and left aircraft using the ROI wake-tracking algorithm. The dashed and solid lines refer to the right and left aircraft, respectively. The dotted vertical line represents the time of comparison with three-dimensional views of the eigenvalue of the velocity gradient tensor.
Figure 6. Top view of the computational domain showing isosurfaces of the eigenvalue of the velocity gradient tensor at a time of 117 s. The red dashed lines indicate the position extracted from the ROI approach.

Figure 7. The end view of the isosurfaces shown in Figure 6. The red circles show the mean positions extracted by the ROI approach.

The circulation history of wakes from both starboard and port aircraft is shown in Figure 8. The ROI method captures the more rapid rate of decay of the outer vortices and the weaker rate of decay associated with the inner vortices.
VI. Summary

A new method is proposed for tracking wake vortices and determining their circulation from LES data. The new method should be able to perform for a single aircraft as well as multiple aircraft scenarios. This method is also applicable for IGE simulations and for turbulent environments. We have shown the ROI method for tracking wake vortices performs as well as the previous method. This new method can be a valuable tool for analyzing vortex dynamics of multiple aircraft wakes.

Acknowledgments

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References