Quantification of the Uncertainties for the Ares I A106 Ascent Aerodynamic Database

Heather P. Houlden * and Amber L. Favaregh.†
ViGYAN, Hampton, Virginia, 23666

Michael J. Hemsch‡
NASA Langley Research Center, Hampton, Virginia, 23681

A detailed description of the quantification of uncertainties for the Ares I ascent aero 6-DOF wind tunnel database is presented. The database was constructed from wind tunnel test data and CFD results. The experimental data came from tests conducted in the Boeing Polysonic Wind Tunnel in St. Louis and the Unitary Plan Wind Tunnel at NASA Langley Research Center. The major sources of error for this database were: experimental error (repeatability), database modeling errors, and database interpolation errors.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>BDM</td>
<td>booster deceleration motor</td>
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<tr>
<td>C</td>
<td>force or moment coefficient</td>
</tr>
<tr>
<td>$C_{C1}$</td>
<td>force or moment coefficient for Ares I A106 wind tunnel model configuration C1 (full protuberances)</td>
</tr>
<tr>
<td>$C_{C4}$</td>
<td>force or moment coefficient for Ares I A106 wind tunnel model configuration C1 (no protuberances)</td>
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<tr>
<td>$C_{C4\text{adjusted}}$</td>
<td>force or moment coefficient for $C_{C4}$ with biases removed, 'clean' axisymmetric data</td>
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<tr>
<td>$C_{C7}$</td>
<td>force or moment coefficient for Ares I A106 wind tunnel model configuration C7 (C1 with BDMs shifted forward)</td>
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<tr>
<td>$C_{DB}$</td>
<td>database value of coefficient at nominal breakpoints</td>
</tr>
<tr>
<td>$C_{DB,C1}$</td>
<td>baseline C1 database, derived from wind tunnel data</td>
</tr>
<tr>
<td>$C_{DB,\text{EXP}}$</td>
<td>coefficient value of wind tunnel data-derived database, includes BDM increments</td>
</tr>
<tr>
<td>$C_{\text{Flt.Re},\text{CFD}}$</td>
<td>coefficient value of CFD solutions obtained at flight Reynolds number conditions</td>
</tr>
<tr>
<td>$C_{\text{WT.Re},\text{CFD}}$</td>
<td>coefficient value of CFD solutions obtained at wind tunnel Reynolds number conditions</td>
</tr>
<tr>
<td>$C_{\text{TRUE}}$</td>
<td>the true value of the coefficient C</td>
</tr>
<tr>
<td>$C_{\text{UQDB}}$</td>
<td>alternate version of $C_{DB,C1}$, constructed without smoothing, used to quantify database modeling errors</td>
</tr>
<tr>
<td>$C_{\text{AF}}$</td>
<td>body axis axial force coefficient</td>
</tr>
<tr>
<td>$C_{\text{LLF}}$</td>
<td>body axis rolling moment coefficient</td>
</tr>
<tr>
<td>$C_{\text{LMF}}$</td>
<td>body axis pitching moment coefficient, moment reference center 13.88 diameters forward of nozzle gimbal point</td>
</tr>
<tr>
<td>$C_{\text{LNLF}}$</td>
<td>body axis yawing moment coefficient, moment reference center 13.88 diameters forward of nozzle gimbal point</td>
</tr>
<tr>
<td>$C_{\text{NF}}$</td>
<td>body axis normal force coefficient</td>
</tr>
<tr>
<td>$C_{\text{YF}}$</td>
<td>body axis side force coefficient</td>
</tr>
<tr>
<td>$C_{1}$</td>
<td>1%-scale Ares I A106 model configuration with all protuberances</td>
</tr>
<tr>
<td>$C_{4}$</td>
<td>1%-scale Ares I A106 model configuration with no protuberances, axisymmetric model</td>
</tr>
<tr>
<td>$C_{4\text{adjusted}}$</td>
<td>$C_{4}$ coefficient data with biases removed, 'clean' axisymmetric data</td>
</tr>
<tr>
<td>$C_{7}$</td>
<td>$C_{1}$ configuration with BDMs shifted forward</td>
</tr>
<tr>
<td>$C_{Q}$</td>
<td>coefficient value obtained by querying the database response surface at non-canonical inference space points</td>
</tr>
<tr>
<td>D</td>
<td>prefix for coefficient residuals</td>
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* Aerospace Engineer, Member.
† Aerospace Engineer, Senior Member.
‡ Aerospace Engineer, Configuration Aerodynamics Branch, Mail Stop 499, Associate Fellow.

American Institute of Aeronautics and Astronautics

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A statement of uncertainty is desired for any aerodynamic database, in addition to the best-value response surface for the predictions. The statement should include the effects of all of the significant sources of uncertainty. In this paper, the user-interpolated (queried) value in the database response surface for any of the six aero force and moment coefficients is designated as $C_Q$. Then the true value, $C_{TRUE}$, is defined in Eq. (1), where $U_{C,Q}$ is the estimated error bound for $C_Q$.

$$C_{TRUE} = C_Q \pm U_{C,Q}$$ (1)

This paper presents a description of the uncertainty modeling of the Ares I A106 ascent aerodynamics 6-DOF database. Error bounds were estimated for all processes used to generate the data and to modify these data during database construction. This includes wind tunnel repeatability errors, database modeling errors, and database interpolation errors. The individual uncertainties were combined by the root-sum-square (RSS) method. It was also assumed that the errors are samples from a random variable and that errors from different sources are independent and uncorrelated.

The A106 ascent database was constructed using both experimental and CFD data. The experimental data was obtained during wind tunnel tests conducted in the Boeing Polysonic Wind Tunnel (PSWT) and the NASA Langley Research Center Unitary Plan Wind Tunnel (UPWT) on a 1% scale model. Data from the PSWT test were obtained at Mach numbers from 0.5 to 1.6, and UPWT testing was conducted at Mach numbers ranging from 1.6 to 4.5.

Wind tunnel data from three model configurations were used in the database construction. These configurations were: the model with all protuberances (C1), the C1 configuration with booster deceleration motors (BDM) shifted forward (C7), and a "clean" axisymmetric configuration with no protuberances (C4). At the time of these wind tunnel tests, the location of the BDM's on the vehicle had not been finalized. Therefore data were obtained for both BDM positions (C1 and C7). During these A106 wind tunnel tests roll run data were obtained at total angles of attack ranging from 0° to 10° and pitch runs were obtained at roll angles every 45°.

II. Ares I A106 Ascent Database Construction

The A106 ascent aero database consisted of three components: an experimentally-derived response surface of the C1 configuration, a wind-tunnel-to-flight Reynolds number increment model (constructed from CFD data), and BDM increments (constructed from wind tunnel data). The C1 configuration is not axisymmetric, due to all of the protuberances. In order to minimize the effect of systematic errors from the wind tunnel tests, a strategy was developed during the previous design phase (A103) to remove these errors from the database. With this strategy, the baseline C1 database was constructed by adding C1 increments ($C_{C1} - C_{C4}$) to results from the axisymmetric model (C4), and invoking symmetry or anti-symmetry where appropriate. This approach is consistent with the way the
A wind tunnel database was constructed in the previous design phase (A103). The A106 database is a function of Mach number, pitch angle, and roll angle. The buildup of the final A106 ascent database is defined in Eq. (2).

$$C_{DB} = (C_{C1} - C_{Cd}) + C_{C4,adjusted} + (C_{C7} - C_{C1}) + \Delta C_{Flt Re, CFD}$$

The first three terms in Eq. (2) were constructed using wind tunnel data and the last term was developed from CFD results. The first term represents the increments due to the protuberances. The second term, $C_{C4,adjusted}$, represents the C4 pitch data, which have been adjusted for symmetry, anti-symmetry, and axisymmetry where appropriate. This particular term represents a "clean" baseline to which increments due to protuberances (C1-C4) and BDMs-forward (C7-C1) are added. The adjusted C4 data and the configuration increments are presumed to be free of systematic errors. It is assumed that by subtracting data from two configurations that the systematic errors are naturally subtracted out. The adjustments made to create the $C_{C4,adjusted}$ term include shifting all coefficients (except axial force) to zero at zero degrees total angle of attack.

### III. General Description of Database Error Modeling

For any database using a combination of CFD and experiment, the uncertainty is composed generally of four types of terms:

1. Experimental error associated with the adjusted wind tunnel dataset used to build the database response surface (i.e., wind tunnel data repeatability, $U_{Exp}$)
2. Curve-fit error associated with creating the database response surface from the computational dataset or the adjusted experimental dataset (database modeling error, $U_{DBM}$)
3. Errors associated with the validation models for the CFD dataset
4. Error from linear interpolation in the database response surface by the user (database interpolation error, $U_{DBI}$)

There were not sufficient CFD data to develop a detailed CFD validation model, so the error type defined in #3 above was not expressed in the overall A106 database uncertainty model. A general expression of the error buildup for this database is described in Eq. (3).

$$U_{C,Q}^2 = U_{Exp}^2 + U_{DBM}^2 + U_{DBI}^2$$

The A106 combined baseline database contains experimental errors, database modeling errors for the experimentally-derived C1 response surface, and database modeling errors for the Reynolds number increment model. The database interpolation error was computed from the final database release and combined with the other database error components. A discussion of these errors and how they were combined into a total error for the complete ascent database is presented in the following sections.

All analyses presented in this paper were done in body axis coefficients. Normal force and side force coefficients, as well as pitching and yawing moment coefficients, were combined to perform this error analysis, as they are similar in magnitude and behavior in the body axes.

### IV. Experimental Errors in Database

#### A. Within-Test Repeatability

Wind tunnel tests were conducted on a 1% -scale Ares I A106 CLV model in the Boeing Polysonic Wind Tunnel (PSWT) and the NASA LaRC Unitary Plan Wind Tunnel (UPWT). A quantitative analysis was performed on groups of replicate runs obtained during the 1%-scale A106 CLV wind tunnel tests to quantify the within-test repeatability of the experimental data, $\sigma_{rep}^4$.

There were numerous pairs of replicate runs obtained at a range of Mach numbers during each wind tunnel test. Repeat runs were obtained for both pitch and roll runs. For each set of replicate runs, the data were interpolated to
nominal angles. The wind tunnel facilities were generally very accurate in setting the desired angles, so this interpolation was minimal. For each pitch or roll angle, the absolute value of the difference (i.e., range, $R$) between the coefficients at replicate points was computed. Then the average of the ranges for a given replicate run set was computed, and the standard deviation was estimated using Eq. (4).\(^5\)

$$\sigma_{rep} = \frac{R}{1.128} \quad (4)$$

The results from Eq. (4) were plotted versus Mach number. A final estimate of the repeatability was determined by comparing the A106 repeatability standard deviations to those computed for previous configurations.\(^5,7\) Figures 1 through 4 show the final repeatability standard deviations determined for the A106 configuration in body axis coefficients. The average repeatability from earlier Ares configurations A101 and A103 are shown in red on the figures. The black symbols represent the repeatability sigmas computed with Eq. (4) above. The dashed lines in Figure 1-4 represent the final standard deviations chosen to represent the A106 repeatability. The final A106 repeatability sigmas were determined by drawing a smooth curve roughly between the computed A103 and A106 standard deviations. With just two sets of points at each Mach number, it is not possible to say that one point is more accurate than the other. There is no statistical difference between the black and red symbols at each Mach number. Therefore, the smooth dashed curves were estimated by eye for each aerodynamic coefficient.

B. Application of Repeatability to Database Error

To quantify the total experimental error associated with the A106 ascent aero database, the procedure was to count the number of times wind tunnel-based terms appear in the database buildup, resulting in the experimental uncertainty 3-sigma limits as defined in Eq. (5), where $n$ is the number of experimental-based terms in the database that count towards the total experimental error.

$$U_{exp} = 3\sqrt{n\sigma_{rep}} \quad (5)$$

For the A106 database experimental error, $n=2$ because there are two terms that must be counted towards the total experimental error. The equation buildup for the database at wind tunnel Reynolds numbers is shown in Eq. (6). In this equation there are two $C_{C4}$ terms, denoted in red, which cancel each other out. Additionally $C_{C4,adjusted}$ is not counted towards $U_{exp}$ because it is assumed to be free of biases. This leaves two ($n=2$) terms that contribute to the total experimental error in the A106 ascent database.

$$C_{DB,exp} = (C_{C1} - C_{C4}) + C_{C4,adjusted} + (C_{C7} - C_{C4}) \quad (6)$$

Therefore, the contribution of experimental error to the ascent database is defined as follows:

$$U_{exp} = 3\sqrt{2\sigma_{rep}} \quad (7)$$

V. Database Modeling Errors

Database modeling errors include all errors introduced in constructing a database response surface at canonical inference space points, from an experimental or CFD input dataset. Examples of database modeling errors are:

1. smoothing and/or curve-fitting of the input dataset
2. assumptions used to create reduced-order models from sparse input data, in order to fill out the database inference space.

A. Experimentally-Derived Baseline Response Surface

The database modeling error for the portion of the A106 ascent aero database constructed from experimental data was determined by comparing the official database response surface for the C1 configuration at wind tunnel Reynolds numbers ($C_{DB,C1}$) to an alternate C1 response surface constructed from wind tunnel pitch runs, without using smoothing. This other database will be referred to as the UQ response surface for the remainder of this paper. The UQ response surface was constructed from wind tunnel data that were not used to construct the A106 C1
database response surface. The database modeling errors were estimated by comparing the database values at conditions where data were acquired but not used, with UQ response surface values where the acquired data were used to construct the surface.

The database team constructed the C1 response surface using pitch sweep data at zero degrees roll and increments computed from roll sweep data at pitch angles of 0° to 10°, and adding these to the adjusted C4 data.

\[ C_{DB,C1} = (C_{C1} - C_{C4}) + C_{C4, adjusted} \]  
\[ (8) \]

The UQ response surface was constructed using only pitch sweep data that were obtained at roll angles every 45°. No roll run data were used to create the UQ response surface. Additionally, no smoothing was performed on the wind tunnel data to create the UQ response surface. Figure 5 depicts the difference between the wind tunnel input data set inference space points used to construct the official and UQ version of \( C_{DB,C1} \). The red squares indicate the roll and pitch angle combinations used to construct the official baseline C1 database, and the black circles represent the roll and pitch angles where points from the UQ response surface exist. The database modeling errors were captured by comparing the two databases at common breakpoints (i.e., where the red squares and black circles coincide in Figure 5). Residuals between the two databases at these common pitch and roll angles were computed (Eq. (9)) for each Mach number. The two databases were compared with the moment coefficients computed at the balance moment center, where the forces and moments are uncorrelated.

\[ DC_{DBM} = C_{DB,C1} - C_{UQDB} \]  
\[ (9) \]

Plots of the database modeling errors are shown in Figures 6 through 9, where the residuals computed using Eq. (9) are plotted versus Mach number. Experimental error bounds are included on these plots, to compare the relative magnitude of the database modeling errors to experimental errors. Recall that \( DC_{DBM} \) is computed from just the baseline C1 database component, \( C_{DB,C1} \). The experimental error bounds associated with \( C_{DB,C1} \) are represented by dashed blue lines in Figures 6 through 9 and were computed using Eq. (7).

Note in the figures that many of the residuals lie outside the experimental error bounds. This indicates that the uncertainties associated with smoothing the data and combining smoothed and fitted pitch and roll run data generally exceeded the experimental error (i.e., wind tunnel repeatability). If the database modeling residuals mostly fell below the experimental error bounds, then the database modeling errors would be considered insignificant, and the experimental and database interpolation errors would be the only sources of uncertainty for the wind tunnel-derived database.

However, since the database modeling residuals are generally greater than the experimental error bounds, \( U_{DBM} \) must be included in the quantification of the overall database error. Database modeling error bounds (\( U_{DBM} \)) were estimated by eye from the residual plots shown in Figures 6 through 9, and are represented as red dashed lines in these figures. Since experimental errors are already accounted for with Eq. (7), it is necessary to subtract the experimental error (i.e., wind tunnel data repeatability) from \( U_{DBM} \), so as not to exaggerate the modeling errors by double counting the effect of repeatability. The net database modeling error for the experimentally-derived portion of the ascent database is computed using Eq. (10), where the contribution of experimental error is subtracted from the total database modeling error.

\[ U_{DBMnet} = \sqrt{U_{DBM}^2 - U_{EXP}^2} \]  
\[ (10) \]

Recall that \( U_{EXP} \) represents the error due to wind tunnel data repeatability and is defined in Eq. (7). As such, Eq. (10) can be written as follows, where \( n=2 \).

\[ U_{DBMnet} = \sqrt{U_{DBM}^2 - 2(3\sigma_{rep})^2} \]  
\[ (11) \]

The database modeling residuals lie within the experimental error bounds at the lowest Mach numbers for every coefficient except pitching and yawing moment. At these Mach numbers the database modeling errors are insignificant and the net database modeling error term becomes zero.
B. CFD-Derived Reynolds Number Increments

It was not possible to conduct wind tunnel tests at flight Reynolds numbers because there was no facility available that could attain full flight Reynolds number for the A106 configuration. Therefore, CFD data were used to develop a wind tunnel-to-flight Reynolds number increment model, which is the last term of $C_{DB}$ in Eq. (2). CFD solutions for the full protuberance (C1) configuration were computed at both wind tunnel and flight Reynolds numbers. The complete A106 CFD dataset was searched to find instances where solutions existed for both wind tunnel and flight Reynolds numbers for the same configuration, at the same Mach number, pitch angle, and roll angle combinations. These data pairs were then grouped by Mach number and the flight Reynolds number coefficient values were plotted versus the wind tunnel Reynolds number values. Linear curve fits were computed for each aerodynamic coefficient and Mach number range (see Equation 12).

$$C_{Ft,Re,CFD} = mC_{WT,Re,CFD} + \delta \pm U_0$$

Eq. (12) is re-written to be expressed as an increment as shown in Equation 13. The left-hand side of Eq. (13) is equivalent to the last term in Eq. (2), and is what was included in the database to model the effect of Reynolds number. The offset $\delta$ was found to be essentially zero for all aerodynamic coefficients except CAF. This increment is what was included in the database to model

$$\Delta C_{Ft,Re,CFD} = (m-1)C_{WT,Re,CFD} + \delta \pm U_0$$

There were not sufficient CFD data available to develop a detailed validation error model for CFD increments. Therefore, the scatter in the increment models was used to set the error bounds, $U_{DBM.Re.CFD}$, for the Reynolds number increment term in the database. The increment model scatter and corresponding error bounds are presented in Figures 10 through 13. The Reynolds number increment uncertainty is a function of total angle of attack (ALPHAT) and Mach number for side/normal force and pitching/yawing moment coefficient increments, as shown in Figures 10 and 11, respectively. Axial force and rolling moment increment modeling errors are only a function of Mach number, as seen in Figures 12 and 13, respectively.

VI. Database Interpolation Error

In practice, linear interpolation is used in the simulation environment to obtain values from the 6DOF database at non-canonical inference space points (i.e., between database breakpoints). To estimate the error associated with interpolating within the database, we queried the database at non-canonical points and then used both linear interpolation and a cubic spline to compute the aerodynamic coefficients. Figure 14 illustrates that the midpoints between the canonical database inference space points were used to estimate the database interpolation error. The black circles represent the database breakpoint values, and the red circles represent the midpoints used to compute the interpolation error. The official database release was presented as a function of angle of attack and sideslip, so the database interpolation error was computed in the alpha-beta inference space. The database values computed with a spline interpolation were subtracted from the values computed using linear interpolation. These residuals were plotted versus Mach number and the error bounds ($U_{DBI}$) were estimated by eye from the charts, as shown in Figures 15 through 18. The database interpolation error is quite small compared to the other error terms associated with the database.

VII. Total Error Buildup for the A106 AFMA Aero Ascent Database

The total error for the A106 AFMA aero ascent database is computed by using the root-sum-square method to combine the individual errors defined in the preceding sections, as shown in Eq. (14).

$$U_{C.\sigma} = \sqrt{U_{EXP}^2 + U_{DBMnet}^2 + U_{DBM.Re.CFD}^2 + U_{DBI}^2}$$

Recall that the first two terms under the square are defined in Equations 7 and 11, respectively. The third term on the right hand side of Eq. (14) represents the modeling error associated with the CFD-derived Reynolds number increment model and was computed from the scatter in the increment model. The fourth term represents the interpolation error due to querying the database at non-canonical points.
The error buildup for the A106 ascent aero database is shown in Figures 19 through 24. The total database error is compared with the individual error components in these figures. Figures 19 and 20 depict the errors for side and normal force coefficients at 0° and 4° total angle of attack. (Recall that the uncertainties for the flight Reynolds number increments are a function of both total angle of attack and Mach number.) Notice the net database modeling errors ($U_{DBM_{net}}$) are zero at the lower Mach numbers. This is a result of the database modeling residuals being less than or equal to the experimental error bounds.

VIII. Concluding Remarks

A detailed uncertainty analysis was developed for the Ares I A106 CLV ascent aero 6DOF database. This database was constructed using both experimental and CFD data. The errors for each component of the database were identified and quantified. The total error was obtained by root sum squaring the individual error terms. All of the error terms are expressed as a function of Mach number, and with the exception of the flight Reynolds number increment uncertainties for CNF, CYF, CLMF, and CLNF, none of the database errors were a function of angle of attack. In general, the database uncertainty consists of experimental errors (wind tunnel repeatability), modeling errors, and database interpolation errors.

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References

Figure 1. Wind Tunnel Repeatability Standard Deviations for CNF and CYF.

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