19.1 Attitude Determination and Control Systems

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In the year 1900, Galveston, Texas, was a bustling community of approximately 40,000 people. The former capital of the Republic of Texas remained a trade center for the state and was one of the largest cotton ports in the United States. On September 8 of that year, however, a powerful hurricane struck Galveston island, tearing the Weather Bureau wind gauge away as the winds exceeded 100 mph and bringing a storm surge that flooded the entire city. The worst natural disaster in United States’ history—even today—the hurricane caused the deaths of between 6000 and 8000 people. Critical in the events that led to such a terrible loss of life was the lack of precise knowledge of the strength of the storm before it hit.

In 2008, Hurricane Ike, the third costliest hurricane ever to hit the United States’ coast, traveled through the Gulf of Mexico. Ike was gigantic, and the devastation in its path included the Turk and Caicos Islands, Haiti, and huge swaths of the coast of the Gulf of Mexico. Once again, Galveston, now a city of nearly 60,000, took the direct hit as Ike came ashore. Almost 200 people in the Caribbean and the United States lost their lives; a tragedy to be sure, but far less deadly than the 1900 storm. This time, people were prepared, having received excellent warning from the GOES satellite network. The Geostationary Operational Environmental Satellites have been a continuous monitor of the world’s weather since 1975, and they have since been joined by other Earth-observing satellites. This weather surveillance to which so many now owe their lives is possible in part because of the ability to point accurately and steadily at the Earth below. The importance of accurately pointing spacecraft to our daily lives is pervasive, yet somehow escapes the notice of most people. But the example of the lives saved from Hurricane Ike as compared to the 1900 storm is something no one should ignore. In this section, we will summarize the processes and technologies used in designing and operating spacecraft pointing (i.e. attitude) systems.

Figure 19-1: Satellite image of Hurricane Ike (NASA image).
**Attitude** is the three-dimensional orientation of a vehicle with respect to a specified reference frame. **Attitude systems** include the sensors, actuators, avionics, algorithms, software, and ground support equipment used to determine and control the attitude of a vehicle. Attitude systems can have a variety of names, such as attitude determination and control system (ADCS), attitude ground system (AGS), attitude and orbit control system (AOCS), guidance, navigation and control (GNC), or whatever other term describes the designers’ focus in achieving the attitude needs of a particular mission. When we use an acronym in this section, we will use ADCS, but any given specialist may be more familiar with other terms.

Spacecraft attitude changes according to the fundamental equations of motion for rotational dynamics, the Euler equations, here expressed in vector form in the spacecraft’s reference frame:

\[
\dot{H} = T - \omega \times H
\]

This vector equation represents the conservation equations for the physical vector quantity of a body or collection of bodies called **angular momentum**, which is denoted by \( H \). Recall that linear momentum is the translational motion of a body that will remain constant unless a force acts to change it, and it is calculated (in Newtonian physics anyway) as mass times velocity. Analogously, angular momentum is the rotational motion of a body that will continue unless changed by a **torque**, and it is calculated as the body’s **moment of inertia** times its angular velocity. The moment of inertia is a 3-by-3 matrix of values that describe the distribution of mass in a body. There is always a coordinate frame, called the **principal axis frame**, for which the moment of inertia matrix is diagonal. The difference between the geometric and principal axis frames is of great interest to ADCS designers, as we will see later in this section.

Note that in the form above, the Euler equation makes it clear that the magnitude of angular momentum in a system can only be changed by applying external torques, \( T \), because the change due to the term \( \omega \times H \) can only change the direction of \( H \), not the magnitude. In fact, a body’s angular momentum is always conserved in the absence of external torques upon it, even when parts of the body can move with respect to other parts, such as a gyroscope spinning in its housing. If part of the body starts to spin in one direction in the absence of external torques, the rest of the body will have to spin in the opposite direction so that total angular momentum is conserved. With this in mind, we can relate the angular velocity of the spacecraft, \( \omega \), to \( H \) by the equation

\[
H = I\omega + h
\]

where \( I \) is the moment of inertia and \( h \) is the angular momentum stored by any rotating objects that are part of the spacecraft, such as momentum wheels or gyroscopes. So, by the product rule of calculus the Euler equations can be rewritten as a matrix equation:

\[
\dot{\omega} + I\omega + h = T - \omega \times H
\]

or, after moving some terms around:

\[
I\dot{\omega} = T - h - I\omega - \omega \times H
\]

The form of Equation 4 allows us to understand how attitude can change due to a variety of causes. The first term on the right-hand side represents external torque’s direct contribution to attitude dynamics; this term includes how some actuators can be used to control spacecraft attitude by creating external torques. The second term gives the relationship between changes in onboard rotating objects’ speeds and changes in the spacecraft’s rotational velocity; this term is where certain other control actuators enter into the dynamics as so-called internal torques. The third term shows how changes in the spacecraft moment of inertia (representing how mass is distributed in the spacecraft), such as by solar array articulation, can affect attitude dynamics; in the absence of changes in mass properties, the third term disappears. The fourth term is called the gyroscopic torque, and it shows how the angular momentum appears to change direction, but not magnitude, in the spacecraft’s frame of reference when the spacecraft is rotating. All these effects combine to determine the rate of change of the angular velocity on the left-hand side.

**Attitude determination** is the process of combining available sensor inputs with knowledge of the spacecraft dynamics to provide an accurate and unique solution for the attitude state as a function of time, either onboard for immediate use, or after the fact (i.e. post-processing). With the powerful microprocessors now available for spaceflight, most attitude algorithms that formerly were performed as post-processing can now be programmed as onboard calculations. Therefore, though there are still good engineering reasons for certain processes to be performed only by ground-based attitude systems, it will be sufficient to focus our attitude determination discussions in this chapter on the design and implementation of onboard systems.
The product of attitude determination, the attitude estimate or solution, is attained by using sensors to relate information about external references, such as the stars, the Sun, the Earth, or other celestial bodies, to the orientation of the spacecraft. Frequently, any single sensor has a noise level or other drawback that prevents it from providing a fully satisfactory attitude solution at all times. Therefore, more than one sensor is often required to meet all mission requirements for a given mission.

The combination of information from multiple sensors is a complex field of study. The possibilities for any given mission range from simple logical combination of sensors, depending on mode, to modern information filtering methods, such as Kalman filtering. Many methods require some prediction of a future attitude from current conditions. Because all spacecraft sensors must use the spacecraft’s reference frame as a basis for attitude determination, the development of angular momentum according to the spacecraft’s frame of reference can be important for some attitude determination algorithms. This is the reason for the spacecraft-referenced aspect of the Euler equations.

Attitude control is the combination of the prediction of and reaction to a vehicle’s rotational dynamics. Because spacecraft exist in an environment of small and often highly predictable disturbances, they may in certain cases be passively controlled. That is, a spacecraft may be designed in such a way that the environmental disturbances cause the spacecraft attitude to stabilize in the orientation needed to meet mission goals. Alternately, a spacecraft may include actuators that can be used to actively control the spacecraft orientation. These two general types of attitude control are not mutually exclusive. A spacecraft may be mostly or usually passively controlled and yet include actuators to adjust the attitude in small ways or to make attitude maneuvers (i.e. slews) to meet other objectives, such as targets of opportunity or communication needs.

So, external torques change the total angular momentum, and internal torques exchange momentum between different rotating parts of the spacecraft. In this way reaction wheels or control moment gyroscopes may be used to change spacecraft pointing without affecting total angular momentum. Because environmental disturbances create external torques on the spacecraft, they also create angular momentum that must be either stored or removed by the attitude system. Small external torques that vary over the course of an orbit but have a mean of zero may be managed just through storage, but those torques that have a non-zero mean (secular torques) will cause a gradual increase in angular momentum, and this momentum build-up must eventually be removed with actuators that create external torques. Thrusters, magnetic torquers, or even solar trim tabs can be used to create controlled external torques on the spacecraft, thus controlling the total angular momentum.

Attitude system design is an iterative process. Table 19-1 lists typical steps in a design process and what inputs and outputs would be expected for each step. Figure 19-2 presents all the processes involved in attitude systems. The FireSat II spacecraft, shown in Figure 19-3, and the Supplemental Communications System (SCS) constellation of spacecraft, shown in Figure 19-4, will be used to illustrate this process.

<table>
<thead>
<tr>
<th>Step</th>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a) Define control modes</td>
<td>Mission requirements, mission profile, type of insertion for launch vehicle</td>
<td>List of different control modes during mission.</td>
</tr>
<tr>
<td>1b) Define or derive system-level requirements by control mode</td>
<td>Requirements and constraints.</td>
<td></td>
</tr>
<tr>
<td>2) Quantify disturbance environment</td>
<td>Spacecraft geometry, orbit, solar/magnetic models, mission profile</td>
<td>Values for torques from external and internal sources</td>
</tr>
<tr>
<td>3) Select type of spacecraft control by attitude control mode</td>
<td>Payload, thermal &amp; power needs Orbit, pointing direction Disturbance environment Accuracy requirements</td>
<td>Method for stabilization &amp; control: three-axis, spinning, gravity gradient, etc.</td>
</tr>
<tr>
<td>4) Select and size ADCS hardware</td>
<td>Spacecraft geometry and mass properties, required accuracy, orbit geometry, mission lifetime, space environment, pointing direction, slew rates.</td>
<td>Sensor suite: Earth, Sun, inertial, or other sensing devices. Control actuators: reaction wheels, thrusters, magnetic torquers, etc. Data processing avionics, if any, or processing requirements for other</td>
</tr>
</tbody>
</table>
5) Define determination and control algorithms

| Performance considerations (stabilization method(s), attitude knowledge & control accuracy, slew rates) balanced against system-level limitations (power and thermal needs, lifetime, jitter sensitivity, spacecraft processor capability) | Algorithms and parameters for each determination and control mode, and logic for changing from one mode to another. |

6) Iterate and document

| All of above | Refined mission and subsystem requirements. More detailed ADCS design. Subsystem and component specifications. |

Figure 19-2. Diagram of a Complete Attitude Determination and Control System. Definitive attitude determination usually occurs in ground processing of telemetry, whereas onboard, real-time determination design focuses on being extremely reliable and deterministic in its operation.
Figure 19-3. Hypothetical FireSat II Spacecraft. We use this simplified example of a low-Earth orbiting satellite to discuss key concepts throughout the section.

Figure 19-4. One Member of the Hypothetical Supplemental Communications System (SCS) Constellation. We will also use this collection of three spacecraft in medium Earth orbit to illustrate attitude system design practices.

19.1.1 Control Modes and Requirements
The first step of the attitude system design process is the definition of guiding requirements based on mission goals. Since mission goals often require more than one mode of operating a spacecraft, the guiding requirements generally begin with a description of the control modes the ADCS is expected to execute to meet those goals. Tables 19-2 and 19-3 describe typical spacecraft control modes and requirements.

The final form of ADCS requirements and control modes will be the result of iteration; control modes are designed to achieve certain sets of requirements, and better understanding of the actual needs of the mission often results from having these modes of controlling the spacecraft well defined. This iteration takes place in a trade space where a single set of ADCS hardware must be used in different ways to meet different sets of requirements. The ADCS will also be dependent on certain other subsystems, such as the power and structural subsystems. Attitude needs will also impose requirements on other subsystems, such as propulsion, thermal control, and structural stability. Figure 19-5 shows many of the complex interactions needed to bring the ADCS design in line with the needs of the whole mission.

Table 19-2: Typical attitude control modes. Performance requirements are frequently tailored to these different control operation modes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbit Insertion</td>
<td>Period during and after boost while spacecraft is brought to final orbit. Options include no spacecraft control, simple spin stabilization of solid rocket motor, and full spacecraft control using liquid propulsion system. May drive certain aspects of ADCS design.</td>
</tr>
<tr>
<td>Acquisition</td>
<td>Initial determination of attitude and stabilization of vehicle for communication with ground and power generation. Also may be used to recover from power upsets or emergencies.</td>
</tr>
<tr>
<td>Normal Mission, On-Station</td>
<td>Used for the vast majority of the mission. Requirements for this mode should drive system design.</td>
</tr>
<tr>
<td>Slew</td>
<td>Reorienting the vehicle when required.</td>
</tr>
<tr>
<td>Contingency or Safe</td>
<td>Used in emergencies if regular mode fails or is disabled. Will generally use less power or fewer components to meet minimal power and thermal needs.</td>
</tr>
<tr>
<td>Special</td>
<td>Requirements may be different for special targets or time periods, such as when the satellite passes through a celestial body’s shadow, or umbra.</td>
</tr>
</tbody>
</table>
Table 19-3: Typical attitude determination and control performance requirements. Requirements need to be specified for each mode. The following lists the performance criteria frequently specified.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Definition</th>
<th>Examples/Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy</td>
<td>Knowledge of and control over a vehicle’s attitude relative to a target attitude as defined relative to an absolute reference</td>
<td>0.25 deg, 3σ, often includes determination errors along with control errors, or there may be separate requirements for determination &amp; control, and even for different axes</td>
</tr>
<tr>
<td>Range</td>
<td>Range of angular motion over which determination &amp; control performance must be met</td>
<td>Any attitude within 30 deg of nadir. Whenever rotational rates are less than 2 deg/sec.</td>
</tr>
<tr>
<td>Jitter</td>
<td>Specified bound on high-frequency angular motion</td>
<td>0.1 deg over 60 sec, 1 deg/s, 1 to 20 Hz; prevents excessive blurring of sensor data</td>
</tr>
<tr>
<td>Drift</td>
<td>Limit on slow, low-frequency angular motion</td>
<td>0.01 deg over 20 min, 0.05 deg max; used when vehicle may drift off target with infrequent command inputs</td>
</tr>
<tr>
<td>Transient Response</td>
<td>Allowed settling time or max attitude overshoot when acquiring new targets or recover from upsets</td>
<td>10% max overshoot, decaying to &lt;0.1 deg in 1 min; may also limit excursions from a set path between targets</td>
</tr>
</tbody>
</table>

*Definitions vary with procuring and designing agencies, especially in details (e.g. 1σ or 3σ, amount of averaging or filtering allowed). It is always best to define exactly what is required.

Figure 19-5: The Impact of Mission Requirements and Other Subsystems on the ADCS. Direction of arrows shows requirements flow from one subsystem to another.

For many spacecraft the ADCS must control vehicle attitude during the firing of large liquid or solid rocket motors for orbit insertion or management. Large motors can create large disturbance torques, which can drive the design to larger actuators than may be needed for the rest of the mission.

Once the spacecraft is on station, the payload pointing requirements usually dominate. These may require planet-relative or inertial attitudes and fixed or spinning fields of view. There is usually also a need for attitude slew maneuvers, and the frequency and speed of those maneuvers must be defined. Reasons for slews can include:
- Acquiring the desired spacecraft attitude initially or after a failure
- Repointing the payload's sensing systems to targets of opportunity or for calibration purposes
- Tracking stationary or moving targets, including communication stations
- Directing the vehicle's strongest motor(s) to the proper direction relative to orbital motion.

19.1.2 Quantify the Disturbance Environment.

The environment in which the spacecraft will operate constrains what types of control methods will be effective. For example, the relatively strong magnetic fields that occur in low Earth orbit (LEO) can create disturbance torques that need to be managed, but they also allow the use of magnetic torquers, a means of attitude control not available at much higher altitudes like geosynchronous orbit (GEO). Here, we will focus on the torque disturbance environment as the primary driver for control mode and hardware selection, but the sensitivity of the ADCS designer must be to more than just the external torque disturbances of the operational orbit. For example, some attitude sensors, such as star cameras that use charge-coupled devices (CCDs) for imaging, can be highly sensitive to the intense radiation in the Van Allen belts of the Earth's magnetosphere. Depending on the specific model, the star camera may underperform or even provide no information at all when the spacecraft occupies these regions.

Only three or four sources of torque matter for the typical Earth-orbiting spacecraft: gravity-gradient effects, magnetic field torques on a magnetized vehicle (as most spacecraft will be), impingement by solar-radiation, and aerodynamic torques for LEO satellites. Figure 19-6 summarizes the relative effects of these disturbances for different flight regimes. Chapter 7 describes the Earth environment in detail, and Hughes [2004] provides a thorough treatment of disturbances.

Figure 19-6. Effects of major environmental disturbances on spacecraft attitude system design. The diagram has a roughly logarithmic scale of altitude. The columns represent the four major disturbance sources, with the intensity of color for each column indicating the strength of that disturbance in the various flight regimes.
Other external disturbances to the spacecraft are either small relative to the four main external disturbances, such as infrared emission pressure, or they are limited in time, such as outgassing. Occasionally, what is normally negligible can become surprisingly large, even exceeding the usual disturbance torque sources, but this is one of the reasons for maintenance of healthy engineering margins and operational plans that are adaptable to unforeseen events.

**Centroids.** Estimation of environmental torques often requires the use of geometrical averaging. Anyone with a technical education will be familiar with the centroid of an area, but it may have been some time since the reader encountered this concept. The centroid is the point in an area through which any line drawn in any direction will evenly divide moments about the line (or any point along the line). To express it another way, the sum of all area elements multiplied by their distances from a line will be zero for any line passing through the centroid. In a sense, it is the average point for the area. If a source of pressure were applied evenly over the area, the solar pressure force could be represented as being applied entirely at the centroid for the purposes of determining moments, and therefore disturbance torques. A solid body can also have centroids. The center of mass (cm) is the point (usually inside) the body through which any plane will divide the mass moment evenly. By applying a force at or along the center of mass, no torques are created. This is why freely rotating bodies rotate about their centers of mass.

As a practical example, the point that may be regarded as the location of a body for purposes of gravitational forces is called the center of gravity (cg); i.e. all effects of gravity on the body can be considered to act at the cg. In the essentially uniform gravity that we humans occupy, the center of mass is usually indistinguishable from the center of gravity, but in the free-fall of a space orbit, the absence of direct gravitational forces and torques means that the change, or gradient, of gravity over the extent of a body can be important. For elongated or flattened objects in orbit, the cm may be offset from the cg, so that the gravitational force is effectively applied with an offset from the cm, creating a torque—this is the gravity gradient torque. Note that the cg is a function of the current attitude of the spacecraft, not just its mass configuration, which is critical in attitude analysis.

Other environmental effects can be understood in terms of offsets between centroids of different effects on a body. When the aerodynamic force centroid, which is at the centroid of the ram area (the area presented to the velocity direction), is not aligned with the cm, a torque is created. Solar radiation pressure is more intense on certain surfaces (reflective) than others (absorptive). The total pressure force over the Sun-pointing surface of a spacecraft can be considered to act through a center of pressure (cp) with an average reflectance, and the offset of that point from the cm results in solar radiation pressure torque. The location of this cp is a function of attitude as well as surface properties. Some modern surfaces can have their reflectance change with a change in applied voltage, usually for thermal reasons, but which results in a controlled change in cp location. So, in detailed modeling of spacecraft, the determination of the weighted averages of various forces is important to a good understanding of the torque environment.

**Modeling Major Disturbances.** Now, we will present the equations used to model major disturbances with some explanation and demonstration of they can be used to design attitude systems. After the explanations, Table 19-4 will show disturbance calculations for the FireSat II and SCS examples.

**Solar Radiation Pressure.** Sunlight has momentum, and therefore it exerts pressure on those objects it strikes. If an object absorbs all the sunlight falling on it, then it absorbs all of its momentum and experiences a certain pressure force because of it. If the sunlight is instead reflected exactly back along its path, such as by a mirror, the pressure force felt is twice as much. If a sunlit flat plate were mirrored on one half and painted black on the other, the pressure distribution across the plate would be uneven and a torque would result. Alternately, if the plate were all black, but a weight were attached to one end in the plate’s shadow, a torque would also result because the center of pressure would be in the center of the plate, but the center of mass would be closer to the weighted end. These phenomena are called solar radiation pressure (SRP) torques.

Now imagine a spacecraft like FireSat II in sunlight. Some parts of the spacecraft stick out further from the center of mass than others. Some surfaces are more reflective than others; solar arrays would absorb more light than reflective metallic surfaces would. Also, surfaces that are angled with respect to the Sun would have less pressure on them than similar surfaces directly facing the Sun. All this goes to demonstrate that accurately predicting SRP torques is very tricky. That said, a good starting estimate can be gleaned by assuming a uniform reflectance and using the following equation:
\[
T_s = \frac{\Phi}{c} A_s (1 + q) (c p_a - c m) \cos \phi
\]

where \(T_s\) is the SRP torque, \(\Phi\) is the solar constant adjusted for actual distance from the Sun (average value: 1367 W/m²), \(c\) is the speed of light (\(3 \times 10^8\) m/s), \(A_s\) is the sunlit surface area in m², \(q\) is the unitless reflectance factor (ranging from 0 for perfect absorption to 1 for perfect reflection), \(\phi\) is the angle of incidence of the Sun, and \(c p_a\) and \(c m\) are the centers of solar radiation pressure and mass.

Atmospheric Drag. In much the same way photons striking a spacecraft can exert pressure, so too can the rarified atmosphere that clings to Earth (and certain other planets) at the edge of space. The atmospheric density is roughly an exponentially decaying function of altitude, so that generally only spacecraft in low Earth orbit (LEO) encounter enough particles to cause noticeable disturbances. Those that do experience a pressure force known as atmospheric (or aerodynamic) drag. The atmospheric drag force itself is an important consideration for orbit planning (Chapter 9) and orbit prediction and tracking (Section 19.2). When the center of atmospheric pressure, determined by the spacecraft area exposed to the atmosphere in the direction of the orbital velocity (i.e. ram direction), is not aligned with the center of mass, a torque results. The atmospheric (or aerodynamic) torque can be estimated as

\[
T_a = \frac{1}{2} \rho C_d A_a V^2 (c p_a - c m)
\]

where \(T_a\) is the atmospheric drag torque, \(\rho\) is the atmospheric density in kg/m³, \(C_d\) is the drag coefficient (usually between 2.0 and 2.5 for spacecraft), \(A_a\) is the ram area in m², \(V\) is the spacecraft's orbital velocity in m/s, and \(c p_a\) and \(c m\) are the centers of aerodynamic pressure and mass in m. Average atmospheric density and orbital velocity as functions of altitude are tabulated in the Appendices of this text.

Magnetic Field. The Earth's liquid core is a dynamo that generates a magnetic field powerful enough to have important effects on the space surrounding the planet. Most spacecraft have some level of residual magnetic moment, meaning they have a weak magnetic field of their own. These residual moments can range anywhere from 0.1-20 A·m², or even more depending on the spacecraft's size and whether any onboard compensation is provided.

When a spacecraft's residual moment is not aligned with a local magnetic field, it experiences a magnetic torque that attempts to align the magnet to the local field, much like a compass needle. The Earth's magnetic field is complex, asymmetric, not aligned with the Earth's spin axis, and varies with both geographical movement of the dipole and changes in solar particle flux. However, for use in the ADCS design process, it is usually sufficient to model the Earth's magnetic field as a dipole and to determine the maximum possible value of the magnetic torque for a spacecraft's altitude. The following equation yields this maximum torque:

\[
T_m = DB \left( \frac{M}{R^3 \lambda} \right)
\]

where \(T_m\) is the magnetic torque, \(D\) is the spacecraft's residual dipole moment in A·m², and \(B\) is the magnetic field strength in tesla. The magnetic field strength in turn is calculated from \(M\), the magnetic moment of the Earth multiplied by the magnetic constant (\(M = 7.8 \times 10^{15}\) tesla·m³); \(R\), the distance between the spacecraft and the Earth's center in m, and \(\lambda\), which is a unitless function of the magnetic latitude that ranges from 1 at the magnetic equator to 2 at the magnetic poles. So, a polar orbit will see roughly twice the maximum magnetic torque of an equatorial orbit.

Gravity Gradient. As described in the earlier subsection on centroids, gravity gradient torques are caused when a spacecraft's center of gravity is not aligned with its center of mass with respect to the local vertical. Without getting into the math of the matter, the center of gravity of a spacecraft in orbit is dependent on its attitude relative to Earth (or whatever body the spacecraft is orbiting), and that cg is not, in general, the same as the center of mass. However, when one of the spacecraft's principal axes is aligned with the local vertical, the cg is always on that principal axis, and therefore there is no gravity gradient torque. The gravity gradient torque increases with the angle between the local vertical and the spacecraft's principal axes, always trying to align the minimum principal axis with the local vertical.

A simplified expression for the gravity gradient torque for a spacecraft with the minimum principal axis in its Z direction is

\[
T_g = \frac{3 \mu}{2 R^3} |I_z - I_y| \sin(2\theta)
\]

Where \(T_g\) is the gravity gradient torque about the X principal axis, \(\mu\) is the Earth's gravitational constant \((3.986 \times 10^{14}\) m³/s²), \(R\) is the distance from the center of the Earth in m, \(\theta\) is the angle between the local
vertical and the Z principal axis, and $I_x$ and $I_z$ are the moments of inertia about Y and Z in kg m$^2$.

Table 19-4. Disturbance Torque Summary and Sample Calculations. See text for detailed discussion and definition of symbols. FireSat II is mainly affected by magnetic torques, with the 30-degree offset attitude also affected by gravity gradient torques. SCS satellites are mainly affected by solar radiation pressure torques.

<table>
<thead>
<tr>
<th>Disturbance Type</th>
<th>FireSat II</th>
<th>SCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar radiation</td>
<td>Solar Cyclic for FireSat II is very small and Earth-oriented (though not Earth-pointing), and has body-mounted arrays. Therefore, its surface area is small and its $c_p$ is close to its $c_m$.</td>
<td>SCS is small and Earth-pointing, so the surface area will be fairly small. However, the need to balance the masses of three stowed satellites on the launch vehicle may reduce our control over mass placement and may cause the center of pressure to be considerably offset from the $c_m$ with respect to the Sun.</td>
</tr>
<tr>
<td></td>
<td>$A_y = 1.2 m \times 1.1 m = 1.3 m^2$; $q = 0.6$ $\varphi = 0$ deg; $c_p - c_m = 0.1 m$</td>
<td>$A_y = 2 m \times 1.5 m = 3 m^2$; $q = 0.6$ $\varphi = 0$ deg; $c_p - c_m = 0.3 m$</td>
</tr>
<tr>
<td></td>
<td>$T_y = (1367)(1.3)(1+0.6)(0.1)(3 \times 10^6)$ $= 9.6 \times 10^{-7} N \cdot m$</td>
<td>$T_y = (1367)(3.0)(1+0.6)(0.3)(3 \times 10^6)$ $= 6.6 \times 10^{-6} N \cdot m$</td>
</tr>
<tr>
<td>Atmospheric drag</td>
<td>Similar to SRP. The ram face will always be the same, so the $c_p$ and $c_m$ locations can be designed a bit better in this direction.</td>
<td>Similar assumptions as for SRP, except that being Earth-pointing, the same face will be presented to the ram direction all the time, so we can expect a little more control over the $c_p$ and $c_m$ locations.</td>
</tr>
<tr>
<td></td>
<td>$A_y = 1.3 m^2$; $c_p - c_m = 0.05 m$; $C_d = 2.0$ For 700 km orbit: $\rho \approx 10^{-13} kg/m^3$; $V = 7504 m/s$</td>
<td>$A_y = 3 m^2$; $c_p - c_m = 0.2 m$; $C_d = 2.0$ For 21,000 km orbit: $\rho \approx 10^{-18} kg/m^3$; $V = 3816 m/s$</td>
</tr>
<tr>
<td></td>
<td>$T_e = (0.5)(10^{-13})(2.0)(1.3)(7504)^2(0.05)$ $= 3.7 \times 10^{-7} N \cdot m$</td>
<td>$T_e = (0.5)(10^{-18})(2.0)(3)(3816)^2(0.2)$ $= 8.7 \times 10^{-12} N \cdot m$</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>55 deg inclination orbit; assume 0.5 A-m$^2$ for a very small, uncompensated vehicle.</td>
<td>Equatorial orbit; assume 1 A-m$^2$ for a small, uncompensated vehicle.</td>
</tr>
<tr>
<td></td>
<td>$R = (6,378 + 700) km = 7,078 km$ $D = 0.5 A-m^2$; $\lambda = 1.9$ for 55 deg $B = (7.8 \times 10^{-6})(1.9)(7.078 \times 10^6)^3$ $= 4.2 \times 10^{-5} N \cdot m$ $T_m = (0.5)B = 2.1 \times 10^{-5} N \cdot m$</td>
<td>$R = (6,378 + 21,000) km = 27,378 km$ $D = 1 A-m^2$; $\lambda = 1.2$ for equatorial orbit $B = (1)(7.8 \times 10^{-12})(1.2)(2.7378 \times 10^6)^3$ $= 4.6 \times 10^{-7} N \cdot m$ $T_m = 1(B) = 4.6 \times 10^{-7} N \cdot m$</td>
</tr>
<tr>
<td>Gravity gradient</td>
<td>With body-mounted arrays, FireSat II is fairly symmetric and small, so the moment of inertia can be balanced very well: We'll set $\theta = 1$ deg.</td>
<td>Deployed solar arrays dominate moment of inertia, so $I_x = I_z &gt; I_y$. However, the ability to balance mass may be limited by the need to fit 3 satellites on the same launch vehicle, so we'll assume a large difference between the geometric and principal axes: $\theta = 10$ deg.</td>
</tr>
<tr>
<td></td>
<td>$R = 7,078 km$ $I = 25 kg \cdot m^2$; $I_y = 50 kg \cdot m^2$</td>
<td>$R = 27,378 km$ $I_z = 90 kg \cdot m^2$; $I_y = 60 kg \cdot m^2$</td>
</tr>
<tr>
<td></td>
<td>Normal mode: $\theta = 1$ deg $T_y = (3)(3.986 \times 10^{14})(25-50)\sin(2 \deg)$ $= 1.5 \times 10^6 N \cdot m$</td>
<td>$T_y = (3)(3.986 \times 10^{14})(90-60)\sin(20 \deg)$</td>
</tr>
</tbody>
</table>
Target-of-opportunity: θ = 30 deg
\[ T_x = 3.7 \times 10^{-5} \text{ N-m} \]

(2)(2.7378 \times 10^7)^3 = 3.0 \times 10^7 \text{ N-m}

Remaining significant disturbances on the control system are internal to the spacecraft. Fortunately, we have some control over them. If we find that one is much larger than the rest, we can specify tighter values for that item. This change would reduce its significance but most likely add to its cost or weight. Table 19-5 summarizes the common internal disturbances. Misalignments in the center of gravity and in thrusters will show up during thrusting only and are corrected in a closed-loop control system and through on-orbit calibration of the thrusters.

Likewise, momentum wheel friction torques can be compensated in either a closed-loop or a compensatory fashion; some reaction wheels are designed with friction compensation included in some commanding modes. Liquid slosh and operating machinery torques are of greater concern but depend on specific hardware. If a spacecraft component has fluid tanks or rotating machinery, the system designer should investigate disturbance effects and ways to compensate for the disturbance, if required. Standard techniques include propellant management devices (e.g. slosh baffles) or counter-rotating elements.

### Table 19-5 Principal internal disturbance torques

Spacecraft designers can minimize internal disturbances through careful planning and precise manufacturing, which may increase cost.

<table>
<thead>
<tr>
<th>Disturbances</th>
<th>Effect on Vehicle</th>
<th>Typical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty in Center of Gravity (cg)</td>
<td>Unbalanced torques during firing of couples thrusters</td>
<td>1-3 cm</td>
</tr>
<tr>
<td></td>
<td>Unwanted torques during translation thrusting</td>
<td></td>
</tr>
<tr>
<td>Thruster Misalignment</td>
<td>Same as cg uncertainty</td>
<td>0.1-0.5 deg</td>
</tr>
<tr>
<td>Mismatch of Thruster Outputs</td>
<td>Similar to cg uncertainty</td>
<td>+/- 5%</td>
</tr>
<tr>
<td>Reaction Wheel Friction and Electromotive Force (i.e. back EMF)</td>
<td>Resistance that opposes control torque effort. These torques are the limiting mechanism for wheels speed.</td>
<td>Roughly proportional to wheel speed, depending on model. At top speed, 100% of control torque (i.e. saturation)</td>
</tr>
<tr>
<td>Rotating Machinery (pumps, filter wheels)</td>
<td>Torques that perturb both stability and accuracy</td>
<td>Dependent on spacecraft design; may be compensated by counter-rotating elements</td>
</tr>
<tr>
<td>Liquid Slosh</td>
<td>Torques due to liquid dynamic pressure on tank walls, as well as changes in cg location.</td>
<td>Dependent on specific design; may be mitigated by bladders or baffles</td>
</tr>
<tr>
<td>Dynamics of Flexible Bodies</td>
<td>Oscillatory resonance at bending/twisting frequencies, limiting control bandwidth</td>
<td>Depends on spacecraft structure; flexible frequencies within the control bandwidth must be phase-stabilized, which may be undesirable.</td>
</tr>
<tr>
<td>Thermal Shocks (&quot;snap&quot;) on Flexible Appendages</td>
<td>Attitude disturbances when entering/leaving umbra</td>
<td>Depends on spacecraft structure. Long inertia booms and large solar arrays can cause large disturbances.</td>
</tr>
</tbody>
</table>

19.1.3 Selection of Spacecraft Control Methods

Now that we understand the requirements on the control system and the environment in which it will operate, we can select one or more methods of controlling the spacecraft. Multiple methods may be indicated when different modes of operating the spacecraft have significantly different requirements or result in different disturbance profiles (as we will see in our FireSat II example). Table 19-6 lists several methods of control, along with typical characteristics of each.
**Passive Control Techniques.** *Gravity-gradient control* uses the inertial properties of a vehicle to keep it pointed toward the Earth. This relies on the fact that an elongated object in a gravity field tends to align its longitudinal axis through the Earth's center. The torques that cause this alignment decrease with the cube of the orbit radius and are symmetric around the nadir vector. Thus, the yaw of a spacecraft around the nadir vector is not controllable by this method. This technique is used on simple spacecraft in near-Earth orbits without yaw orientation requirements, often with deployable booms to achieve the desired inertias.

Frequently, we add dampers to gravity-gradient satellites to reduce libration—small oscillations off of the nadir vector caused by other environmental disturbances. For example, long deployed booms are particularly susceptible to thermal shocks when entering or leaving umbra. These spacecraft also need a method of ensuring attitude capture with the correct end pointed at nadir—the gravity-gradient torques stabilize either end of the minimum inertia axis equally.

In the simplest gravity-gradient spacecraft, only two orientation axes are controlled; the orientation around the nadir vector is unconstrained. To control this third degree of freedom, a small constant-speed momentum wheel is sometimes added along the intended pitch axis. The momentum-biased wheel will be most stable when perpendicular to the nadir and velocity vectors, and therefore parallel to the orbital momentum vector. The stable state of the gravity-gradient plus momentum bias wheel establishes the desired attitude through small energy dissipations onboard without the need for active control.

A third type of purely passive control uses permanent magnets onboard to force alignment along the Earth's magnetic field. This is most effective in near-equatorial orbits where the North-South field orientation is reasonably constant for an Earth-referenced satellite.

**Spin Control Techniques.** *Spin stabilization* is a passive control technique in which the entire spacecraft rotates so that its angular momentum vector remains approximately fixed in inertial space. Spin-stabilized spacecraft (or *spinners*), employ the gyroscopic stability discussed earlier to passively resist disturbance torques about two axes. Additionally, spinners are generally designed to be either insensitive to disturbances around the third axis (the spin axis) or else have active means of correcting these disturbances.

The vehicle is stable (in its minimum energy state) if it is spinning about the principal axis with the largest moment of inertia. Energy dissipation mechanisms onboard, such as propellant slosh, structural damping, or electrical harness movement, will cause any vehicle to progress toward this state if uncontrolled. So, disk-shaped vehicles are passively stable whereas pencil-shaped vehicles are not. Spinners can be simple, survive for long periods without attention, provide a thermally benign environment for components (because of even heating), and provide a scanning or sweeping motion for sensors. The principal disadvantages of spin stabilization are that the vehicle mass properties must be carefully managed during vehicle design and assembly to ensure the desired spin direction and stability, and that the angular momentum vector that provides stability also limits maneuverability. More fuel is required to reorient the vehicle than a vehicle with no net angular momentum, making this technique less useful for angular momentum, making this technique less useful for payloads that must be repointed frequently.

A spinner requires extra fuel to reorient because of the gyroscopic stiffness, which also helps it resist disturbances. In reorienting a spinning body with angular momentum, \( H \), a constant torque, \( T \), will produce an angular velocity, \( \omega \), perpendicular to the applied torque and angular momentum vector, of magnitude \( \omega = T/H \). (This follows from the earlier-introduced Euler equations.) Thus, the greater the stored momentum, the more torque must be applied for a given \( \omega \). For a maneuver through an angle \( \theta \), the torque-time product—an indication of fuel required for the maneuver—is a constant equal to \( H \theta \). Alternately, for a vehicle with no initial angular momentum, a small torque can be applied to start it rotating, with an opposite small torque to stop it when it has reached its new target. The fuel used for any angle maneuver can be arbitrarily small if a slow maneuver is acceptable. (Note that the spinner can only be maneuvered relatively slowly; a fast slew is usually not an option.)

### Table 19-6. Attitude control methods and their capabilities.

<table>
<thead>
<tr>
<th>Type</th>
<th>Pointing Options</th>
<th>Attitude</th>
<th>Typical Accuracy</th>
<th>Lifetime Limits</th>
</tr>
</thead>
</table>

As requirements become tighter, more complex control systems become necessary.
<table>
<thead>
<tr>
<th>System</th>
<th>Maneuverability</th>
<th>Thrusters</th>
<th>Rate-Damping</th>
<th>Target Vector (power) or Earth (communication)</th>
<th>Dual-Spin Stabilization</th>
<th>Active Magnetic with Filtering</th>
<th>Zero Momentum (thruster only)</th>
<th>Zero momentum (3 wheels)</th>
<th>Zero Momentum (CMG)</th>
<th>Propellant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravity-Gradient</td>
<td>Very limited</td>
<td>None</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Earth local vertical only</td>
<td>±5 deg (2 axes)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gravity-Gradient + Momentum Bias</td>
<td>Very limited</td>
<td>±5 deg (3 axes)</td>
<td>Life of wheel bearings</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Earth local vertical only</td>
<td>±5 deg (3 axes)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Passive Magnetic</td>
<td>Very limited</td>
<td>None</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>North/South only</td>
<td>±5 deg (2 axes)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Rate-Damping + Target Vector Acquisition</td>
<td>Generally used as robust safe mode.</td>
<td>±5-15 deg (2 axes)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>None</td>
</tr>
<tr>
<td>Pure Spin Stabilization</td>
<td>Repoint with precession maneuvers; very slow with torquers, faster with thrusters</td>
<td>±0.1 deg to ±1 deg in 2 axes (proportional to spin rate)</td>
<td>Thruster propellant (if applies)*</td>
<td>Thruster propellant (if applies)*</td>
<td>Despun section bearings</td>
<td>Life of sensors and wheel bearings</td>
<td>Propellant (if applies)*</td>
<td>Life of sensors and wheel bearings</td>
<td>Propellant (if applies)*</td>
<td>Life sensors and CMG bearings</td>
</tr>
<tr>
<td>Dual-Spin Stabilization</td>
<td>Same as above</td>
<td>Same as above for spin section. Despun dictated by payload reference and pointing</td>
<td>Thruster propellant (if applies)*</td>
<td>Life of sensors and wheel bearings</td>
<td>Propellant (if applies)*</td>
<td>Life of sensors and wheel bearings</td>
<td>Propellant (if applies)*</td>
<td>Life of sensors and wheel bearings</td>
<td>Propellant (if applies)*</td>
<td>Life sensors and CMG bearings</td>
</tr>
<tr>
<td>Bias Momentum (1 wheel)</td>
<td>Fast maneuvers possible around momentum vector Repoint of momentum vector as with spin stabilized</td>
<td>±0.1 deg to ±1 deg</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Local vertical pointing or inertial targets</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Active Magnetic with Filtering</td>
<td>Any, but may drift over short periods</td>
<td>Slow (several orbits to slew); faster at lower altitudes</td>
<td>±1 deg to ±5 deg (depends on sensors)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zero Momentum (thruster only)</td>
<td>No constraints</td>
<td>No constraints</td>
<td>High rates possible</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zero momentum (3 wheels)</td>
<td>No constraints</td>
<td>No constraints</td>
<td>High rates possible</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zero Momentum (CMG)</td>
<td>No constraints</td>
<td>No constraints</td>
<td>High rates possible</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Thrusters may be used for slewing and momentum dumping at all altitudes, but propellant usage may be high. Magnetic torquers may be used from LEO to GEO.

A useful variation on spin control is called dual-spin stabilization, in which the spacecraft has two sections spinning at different rates about the same axis; this kind of spinner is also known as a gyrostabilizer. Normally one section, the rotor, spins rapidly to provide gyroscopic stiffness, while the second section, the stator or platform, is despun to keep one axis pointed toward the Earth or Sun. By combining inertially fixed and rotating sections on the same vehicle, dual spinners can accommodate a variety of payloads in a simple vehicle.

Also, by adding energy dissipation devices to the platform, a dual spinner can be passively stable spinning about the axis with the smallest moment of inertia, as long as the rotor is spinning about its own maximum moment of inertia. This permits more pencil-shaped spacecraft, which fit better in launch vehicle fairings and which would not normally be stable spinning about their long axes. The disadvantage of dual-spin stabilization is the added complexity of the platform bearing and slip rings between the sections.
(Slip rings permit power and electrical signals to flow between the two sections.) This complexity can increase cost and reduce reliability compared to simple spin stabilization.

Spinning spacecraft, both simple and dual, exhibit several distinct types of motion that are often confused. **Precession** is the motion of the angular momentum vector caused by external torques, including thruster firings used to correct environmental disturbances. **Coning** (or wobbling) is the apparent motion of the body when it is spinning about a principal axis of inertia that is not aligned with a body reference axis or axis of symmetry—for example, the intended spin axis. Coning looks like motion of the intended spin axis around the angular momentum vector at the spin rate. Figure 19-7 shows various natural rotations.

**Figure 19-7. Types of Rotational Motion.** $H = \text{angular momentum vector}; \ P = \text{principal axis}; \ \omega = \text{instantaneous rotation axis}; \ Z = \text{geometrical axis}.$

**Nutation** is the torque-free motion of a simple rigid body when the angular momentum vector is not perfectly aligned with a principal axis of inertia. For rod-shaped objects, this motion is a slow rotation (compared to spin rate) of the spin axis around the angular momentum vector. For these objects spinning about a minimum inertia axis, additional energy dissipation will cause increased nutation. For disk-shaped objects, spinning around a maximum inertia axis, nutation appears as a tumbling rotation faster than spin rate. Energy dissipation for these objects reduces nutation, resulting eventually in a clean spin. For these reasons, minimum-axis (or minor-axis) spinners are often concerned with minimizing energy dissipation, whereas maximum-axis (or major-axis) may actually include mechanisms, such as a passive nutation damper, to dissipate energy quickly.

Nutation is caused by disturbances such as thruster impulses, and can be seen as varying signals in body-mounted inertial and external sensors. Wobble is caused by imbalances and appears as constant offsets in body-mounted sensors. Such constant offsets are rarely discernible unless multiple sensors are available.

Spin stability normally requires active control, such as cold-gas thrusters or magnetic torquers, to periodically adjust the spacecraft’s angular momentum vector to counteract disturbance torques. In addition, we may need to damp the nutation caused by disturbances or precession commands. Aggravating this nutation is the effect of energy dissipating phenomena like structural flexure and flexible harness or fluid motion, which are present in any spacecraft to some degree. Once the excitation stops, nutation decreases as these same factors dissipate the kinetic energy added by the control effort. However, this natural damping can take hours. We can neutralize this error source in minutes with nutation dampers. We can also reduce the amount of nutation from these sources by increasing the spin rate and thus the stiffness of the spinning vehicle. If the spin rate is 20 rpm and the nutation angle from a given disturbance is 3 deg, then nutation from the same disturbance would be reduced to 1 deg if the spin rate were 60 rpm. We seldom use spin rates above 90 rpm because of the large centripetal forces demanded of the structure and the consequent effects on design and weight. In thrusting and pointing applications, spin rates under 20 rpm are generally not used as they may allow excessive nutation. However, applications unrelated to attitude control, such as thermal control or specialized payload requirements, are frequently less sensitive to nutation and may employ very low spin rates.

**Three-axis Control Techniques.** Spacecraft stabilized in all three axes are much more common today than those using spin or gravity gradient stabilization. They can maneuver relatively easily and can be more stable and accurate, depending on their sensors and actuators, than more passive stabilization techniques. They are, however, more expensive. They are also often more complex, but processor and reliability improvements have allowed comparable or better total reliability than some more passive systems. For critical space applications, there is no replacement for thorough risk and reliability assessment (see Chapter 24).

The control torques about the axes of three-axis systems come from combinations of momentum wheels, reaction wheels, control moment gyros, thrusters, solar or aerodynamic control surfaces (e.g. tabs), or magnetic torquers. Broadly, however, these systems take two forms: one uses momentum bias by placing a momentum wheel along the pitch axis; the other is called zero momentum and does not use momentum bias at all—any momentum bias effects are generally regarded as disturbances. Either option usually needs some method of angular momentum management, such as thrusters or magnetic torquers, in addition to the primary attitude actuators.
In a zero-momentum system, actuators such as reaction wheels or thrusters respond to disturbances on the vehicle. For example, an attitude error in the vehicle results in a control signal that torques the wheel, creating a reaction torque in the vehicle that corrects the error. The torque on the wheel either speeds it up or slows it down; the aggregate effect is that all disturbance torques are absorbed over time by the reaction wheels, sometimes requiring the collected angular momentum to be removed. This momentum removal—called desaturation, momentum dumping, or momentum management—can be accomplished by thrusters or magnetic torquers acting automatically or by command from the ground.

When high torque is required for large vehicles or fast slews, a variation of three-axis control using control moment gyros, or CMGs, is available. These devices work like momentum wheels on gimbals. The control of CMGs is complex and their lifespan is limited, but their available torque for a given weight and power can make them attractive.

A very specialized form of zero momentum control, here called active magnetic control, can be attained from a combination of a magnetometer, a Global Positioning System (GPS) receiver, and computationally intensive software filtering. The GPS feeds the spacecraft location to the onboard processor, which then determines the local magnetic field based on onboard models. The magnetometer data is filtered using the Euler equations to determine the attitude, and magnetic torquers make corrections in the two available directions at any given moment—corrections about the magnetic field vector are not possible. Active magnetic control can be an inexpensive backup control mode for a LEO satellite, or it can be a primary control mode for a satellite in a highly inclined orbit. (The highly inclined orbit has large changes in magnetic field direction, allowing the filtering algorithm to better determine a three-axis attitude solution.) This attitude knowledge can also be combined with other sensors, such as Sun sensors, for more accuracy. While this is not a common control method, we include it here as an example of how increased onboard computational power and the presence of new resources, such as the GPS constellation, can allow completely new methods of attitude determination and control.

As a final demonstration of zero momentum three-axis control, simple all-thruster systems are used for short durations when high torque is needed, such as during orbit insertion maneuvers or other orbit adjustments (delta-V) from large motors. These thrusters then may be used for different purposes such as momentum dumping or provision of small delta-Vs during other mission phases.

Momentum bias systems often have just one wheel with its spin axis mounted along the pitch axis, ideally normal to the orbit plane. The wheel is run at a nearly constant, high speed to provide gyroscopic stiffness to the vehicle, just as in spin stabilization, with similar nutation dynamics. Around the pitch axis, however, the spacecraft can control attitude by torquing the wheel, slightly increasing or decreasing its speed. Periodically, the momentum in the pitch wheel must be managed (i.e. brought back to its nominal speed), as in zero-momentum systems, using thrusters, magnets, or other means.

The dynamics of nadir-oriented momentum-bias vehicles exhibit a phenomenon known as roll-yaw coupling. To understand this coupling, consider an inertially fixed angular momentum vector at some error angle with respect to the orbit plane. If the angle is initially a positive roll error, then a quarter-orbit later it appears as a negative yaw error with no roll component remaining. As the vehicle continues around the orbit, the angle goes through negative roll and positive yaw before regaining its positive roll character. This coupling (or commutation) is due to the apparent motion of the Earth-fixed coordinate frame as seen from the spacecraft, and it can be exploited to control roll and yaw over a quarter orbit using only a roll (or only a yaw) sensor, instead of needing one sensor for each of the roll and yaw axes.

Effects of Requirements on Control Type. With the above knowledge of control techniques, we can proceed to select a control type that will best meet mission requirements in the expected operational environment. Tables 19-7 and 19-8 describe the effects of orbit insertion and payload slew requirements on the selection process. It is also useful here to once again reference Figure 19-6 for information on how altitude can affect the space environment. Certain control types, such as gravity-gradient stabilization or active magnetic damping, are better in some orbits than in others.

A common control approach during orbit insertion is to use the short-term spin stability of the combination of spacecraft and orbit-insertion motor. Once on station the motor may be jettisoned, the spacecraft despun using thrusters or a yo-yo device, and a different control technique used from that point on.

Payload pointing will influence the attitude control method, the class of sensors, and the number and kind of actuation devices. Occasionally, pointing accuracies are so stringent that a separate, articulated platform is
necessary. An articulated platform can perform scanning operations much more easily than the host vehicle and with better accuracy and stability. Trade studies on pointing requirements must consider accuracy in attitude determination and control, and the most stringent requirements will ultimately drive ADCS component selection. Table 19-9 summarizes the effects of accuracy requirements on the ADCS approach for the spacecraft. Section 14.5 discusses how to develop pointing budgets.

Table 19-7. Orbit Transition Maneuvers and Their Effects. Using thrusters to change orbits creates special challenges for the ADCS.

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Effect on Spacecraft</th>
<th>Effect on ADCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large impulse to complete orbit insertion (thousands of m/s)</td>
<td>Solid motor or large bipropellant stage. Large thrusters or a gimbaled engine or spin stabilization for attitude control during burns.</td>
<td>Inertial measurement unit for accurate reference and velocity measurement. Different actuators, sensors, and control laws for burn vs. coasting phases. Need for navigation or guidance.</td>
</tr>
<tr>
<td>On-orbit plane changes to meet payload needs or vehicle operations (hundreds of m/s)</td>
<td>Large thrusters needed, but these thrusters may be needed for other reasons also, such as orbit insertion, coasting phase, or stationkeeping.</td>
<td>Separate control law for thrusting. Actuators sized for thrusting disturbances (possibly two sizes of thruster). Onboard attitude reference for thrusting phase.</td>
</tr>
<tr>
<td>Orbit maintenance/trim maneuvers (&lt;100 m/s)</td>
<td>One set of thrusters</td>
<td>Thrusting control law. Onboard attitude reference.</td>
</tr>
</tbody>
</table>

Table 19-8. Slewing Requirements That Affect Control Actuator Selection. Spacecraft slew agility can demand larger actuators for intermittent use.

<table>
<thead>
<tr>
<th>Slewing</th>
<th>Effect on Spacecraft</th>
<th>Effect on ADCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>None or Time-Unconstrained</td>
<td>Spacecraft constrained to one attitude (highly improbable), or reorientations can take many hours.</td>
<td>- Reaction wheels, if planned, can be smaller - If magnetic torquers can dump momentum, reaction control thrusters may not be needed</td>
</tr>
<tr>
<td>Low Rates from 0.05 deg/s (orbital rate) to 0.5 deg/s</td>
<td>Minimal</td>
<td>- Depending on spacecraft size, reaction wheels may be fully capable for slews - If reaction wheels not capable, thrusters will be necessary - Thrusters may be needed for other reasons; i.e. stationkeeping</td>
</tr>
<tr>
<td>High Rates &gt;0.5 deg/s</td>
<td>- Structural impact on appendages - Weight and cost increase</td>
<td>- Control moment gyros or thrusters needed. If thrusters needed for other reasons, two thrust levels may be needed.</td>
</tr>
</tbody>
</table>

Table 19-9. Effects of Control Accuracy Requirements on Sensor Selection and ADCS Design. More accurate pointing requires better and more expensive sensors and actuators.

<table>
<thead>
<tr>
<th>Required Accuracy (3σ)</th>
<th>Effect on Spacecraft</th>
<th>Effect on ADCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;5 deg</td>
<td>• Permits major cost savings</td>
<td>Without attitude determination • No sensors required for GG stabilization • Boom motor, GG damper, and a bias momentum wheel are only required actuators</td>
</tr>
<tr>
<td></td>
<td>• Permits gravity-gradient (GG) stabilization</td>
<td>With attitude determination • Sun sensors &amp; magnetometer adequate for attitude determination at ≥ 2 deg</td>
</tr>
</tbody>
</table>
| 1 deg to 5 deg | • GG not feasible  
• Spin stabilization feasible if stiff, inertially fixed attitude is acceptable  
• Payload needs may require despun platform on spinner  
• 3-axis stabilization will work | • Higher accuracies may require star trackers or horizon sensors  
• Sun sensors and horizon sensors may be adequate for sensors, especially a spinner  
• Accuracy for 3-axis stabilization can be met with RCS deadband control but reaction wheels will save propellant for long missions  
• Thrusters and damper adequate for spinner actuators  
• Magnetic torquers (and magnetometer) useful |
| --- | --- | --- |
| 0.1 deg to 1 deg | • 3-axis and momentum-bias stabilization feasible  
• Dual-spin stabilization also feasible | • Need for accurate attitude reference leads to star tracker or horizon sensors & possibly gyro  
• Reaction wheels typical with thrusters for momentum unloading and coarse control  
• Magnetic torquers feasible on light vehicles (magnetometer also required) |
| < 0.1 deg | • 3-axis stabilization is necessary  
• May require articulated & vibration-isolated payload platform with separate sensors | • Same as above for 0.1 deg to 1 deg but needs star sensor and better class of gyro  
• Control laws and computational needs are more complex  
• Flexible body performance very important |

**FireSat II Control Selection.** For FireSat II, we consider two options for orbit insertion control. First, the launch vehicle may directly inject the spacecraft into its mission orbit. This common option simplifies the spacecraft design since no special insertion mode is needed. An alternate approach, useful for small spacecraft such as FireSat II, is to use a monopropellant propulsion system onboard the spacecraft to fly itself up from a low parking orbit to its final altitude. For small insertion motors, reaction wheel torque or momentum bias stabilization may be sufficient to control the vehicle during this burn. For larger motors, delta-V thruster modulation or dedicated attitude control thrusters become attractive.

Once on station the spacecraft must point its sensors at nadir most of the time and slightly off-nadir for brief periods. Since the payload needs to be despun and the spacecraft frequently reoriented, spin stabilization is not the best choice. Gravity-gradient and passive magnetic control cannot meet the 0.1-deg pointing requirement or the 30 deg slews. This leaves three-axis control and momentum-bias stabilization as viable options for the on-station control.

The periodic 180-deg yaw slews (i.e. slews around the nadir vector) would have to overcome any momentum bias in the Y axis (perpendicular to the orbit normal). When we do the example momentum bias sizing in Table 19-11 later, we will see that a bias of approximately 20 N⋅m/s would be required to maintain the required 0.1 deg accuracy. The magnetic torquers could not perform this slew in 45 minutes because of the bias to be overcome. Instead, we will use the propulsion system to perform the yaw slew.

For the optional off-nadir pointing requirement, three-axis control with reaction wheels might be more appropriate. Mass and power will be especially precious for this very small satellite, so carrying and running 3 reaction wheels simultaneously may be more than the system budgets can handle. Since the SCS will provide an example of three-axis control with reaction wheels, we will use the magnetic torque and momentum wheel combination for FireSat II, and for actuator sizing (table 19-11) we will assume the 30 degree offset pointing is done about the pitch axis, so that the momentum wheel performs that slew also. If offset pointing is needed about roll, thrusters will have to be used; the thruster force sizing example in Table 19-12 shows how this might work with the momentum wheel stopped.

**SCS Control Selection.** For the Supplemental Communication System, we will focus on taking advantage of the gentle disturbance environment and 1 deg accuracy requirement to design a light, inexpensive attitude system that can be installed in all 3 satellites. At the SCS altitude of 21,000 km, with the configuration we've assumed, there are no good passive stabilization methods available to us. Reaction wheels will be needed to reject the disturbances, which are dominated by the solar radiation pressure torques (see Table 19-4). So, as long as the reaction wheels are there anyway, we might as well use them for three-axis stabilization. Also, three-axis control often can be exploited to simplify the solar array design by using one of the unconstrained payload axes (yaw, in this case) to replace a solar array drive axis. Thus, the reduced array size possible with 2 degrees of freedom can be achieved with one array axis drive and one spacecraft rotation.
While the greater part of the SRP torques will be cyclic, some small part will be secular. Therefore, the momentum stored in the reaction wheels will gradually increase and will need to be removed periodically by thrusters. The use of 100 A-m$^2$ magnetic torquers for momentum removal is feasible from the point of view of the available magnetic field. However, the propulsion system has to be included anyway to separate the 3 satellites and establish the constellation, and constantly running torquers would be an additional power drain. Since power is a major challenge for this mission, we will use the thrusters for momentum unloading.

19.1.4 Selection and Sizing of ADCS Hardware

We are now ready to evaluate and select the individual ADCS components. For all ADCS hardware, we will determine the minimum performance level needed to meet requirements. Then, standard components available from manufacturers will be selected if possible, sometimes resulting in better performance than the minimum required. If standard components are not available, specialized components may be designed and built, but this can often be prohibitively costly for most agencies, and so a revision of the requirements is more likely to be more in line with available hardware.

Actuators. Options for actuator selection are summarized in Table 19-10. First, we will discuss momentum-exchange devices, which conserve angular momentum in the spacecraft: reaction wheels, momentum wheels, and control moment gyros. Then, we will move on to external torque actuators, which change the angular momentum of the spacecraft when they are activated: magnetic torquers & thrusters (cold-gas, hot-gas and electric) are the most commonly used in this category.

Wheel control provides smooth changes in torque, allowing very accurate pointing of spacecraft. Some wheels can cause vibrations, or jitter, at high speeds, but this can often be mitigated with vibration isolators or changes in structural design. Reaction wheels are essentially torque motors with high-inertia rotors. They can spin in either direction and provide one axis of control for each wheel. Momentum wheels are reaction wheels with a nominal spin rate above zero to provide a nearly constant angular momentum. This momentum provides gyroscopic stiffness to two axes, and the motor torque may be controlled to change pointing around the spin axis. In sizing wheels we must always consider two performance quantities: angular momentum capacity, and torque authority.

To determine the necessary momentum capacity, we must distinguish between cyclic and secular disturbances in the spacecraft’s environment. We typically size reaction wheels to be able to store the full cyclic component of momentum without the need for frequent momentum dumping. Therefore, the average disturbance torque for ¼ or ½ an orbit determines the minimum capacity of the wheels. The secular component of momentum will also need to be stored for the amount of time the spacecraft must be operational without a momentum dump being performed. This time may be determined by requirements on payload observation continuity, or it may be the amount of time the spacecraft must survive without ground intervention.

<table>
<thead>
<tr>
<th>Actuator</th>
<th>Typical Performance Range</th>
<th>Mass (kg)</th>
<th>Power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thrusters Hot Gas (Hydrazine) Cold Gas</td>
<td>Thrusters produce force; multiply by moment arm for torque 0.5 to 9000 N &lt; 5 N</td>
<td>Mass and power vary (See Ch. 18 for details on propulsion systems)</td>
<td></td>
</tr>
<tr>
<td>Reaction and Momentum Wheels</td>
<td>Maximum torques: 0.01 to 1 N·m Practical momentum storage capacity: 0.4 to 3000 N·m·s</td>
<td>2 to 20</td>
<td>Varies with speed: 10 to 100</td>
</tr>
<tr>
<td>Control Moment Gyros (CMG)</td>
<td>Max. torques: 25 to 500 N·m</td>
<td>&gt; 10</td>
<td>90 to 150</td>
</tr>
<tr>
<td>Magnetic Torquers</td>
<td>1 to 4,000 A·m$^2$ (See Table 19-4 for torque examples)</td>
<td>0.4 to 50</td>
<td>0.6 to 16</td>
</tr>
</tbody>
</table>

The necessary torque authority of the reaction wheels is defined either by slew requirements or the need for control authority to be well above the peak disturbance torque. If the peak disturbance is too close to the torque authority available, pointing accuracy may suffer. For three-axis control, at least three wheels with non-coplanar spin axes are required. Often, a fourth wheel is carried in case one of the primary wheels fails. If the wheels are not orthogonal (and a redundant wheel never is), additional torque authority or momentum capacity may be necessary to compensate for non-optimal
geometry. However, the redundancy of the fourth wheel can have additional benefits, such as being able to avoid any wheel speed passing through zero (which can cause attitude error transients) or even as power storage, as driving a spinning wheel toward zero speed will provide power to the spacecraft.

For spin-stabilized or momentum-bias systems, the cyclic torques will cause cyclic variations of the attitude, whereas the secular torques cause gradual divergence of the attitude away from the ideal target. We typically design the stored angular momentum, determined by spin rate and inertia of the spinning body, to be large enough to keep the cyclic motion within accuracy requirements without active control. Periodic torquing will still be needed to counteract secular disturbances, and the cost of this torquing in propellant, if performed by thrusters, will constrain the maximum stored momentum.

For high-torque applications in which fine control is still needed, control moment gyros (CMG) may be used instead of reaction wheels. CMGs are single- or double-gimbaled wheels spinning at constant speed (and therefore providing momentum bias stiffness when not actuating). By turning the gimbal axis, we can obtain a high-output torque whose size depends on the speed of the rotor and the gimbal rate of rotation. Control systems with two or more CMGs can produce large torques about all three of the spacecraft's orthogonal axes, so we most often use them where agile (i.e. high angular rate) maneuvers are required. The use of CMGs requires complex control laws and careful momentum management to avoid wheel saturation. Also, because the CMG torque is created by twisting what is essentially a stiff gyroscope perpendicular to its spin axis, the bearings of the wheel suffer a great deal of wear and tear, causing most CMGs to have shorter lifetimes than other actuators. Because CMGs combine a short life with high cost, weight, and power needs, they are generally used only on very large spacecraft and only when absolutely necessary to achieve the mission goals.

Spacecraft also use magnetic torquers as actuation devices. These torquers are magnetic coils (electromagnets) designed to generate magnetic dipole moments of commanded magnitude. When three orthogonal torquers are mounted to a spacecraft, they can create a magnetic dipole of any direction and magnitude up to the strength of the torquers. Magnetic torquers can compensate for a spacecraft’s residual magnetic field or attitude drift from minor disturbance torques. They also can be used to desaturate momentum-exchange systems, though they usually require much more time than thrusters. A magnetic torquer produces a torque proportional and perpendicular to the Earth’s varying magnetic field; therefore, at any given moment, no torque can be provided about the Earth’s magnetic field vector. However, as a spacecraft changes in latitude or altitude while following its orbit, different directions of the field become available.

Electromagnets have the advantage of no moving parts, requiring only a magnetometer for sensing and wire coiled around a metallic rod in each axis. (Some torquers are even internally redundant, as they have two coils around the same metallic core.) Because they use the Earth’s natural magnetic field, and this field reduces in strength with the cube of distance from the Earth’s center, magnetic torquers are less effective at higher orbits. We can specify the torquer's dipole strength in A·m² and tailor it to any application. Table 19-11 describes sizing rules of thumb for wheels and magnetic torquers, and works through those rules for the FireSat II and SCS examples. SCS has no slewing requirements, so we size its reaction wheels based only on momentum storage needs.

Thrusters (i.e. rocket engines) are possibly the most frequently flown attitude actuator because of their dual use in adjusting orbital parameters. Almost every spacecraft that needs to perform orbital maneuvers will use thrusters to achieve that goal, and in many cases, some subset of the thrusters used is for attitude control. Thrusters produce a force on the spacecraft by expelling material, called propellant, at high velocity from their exit nozzles. Hot-gas propulsion systems include thrusters that chemically alter the propellant to extract the energy needed for rapid mass expulsion. These systems may be monopropellant, in which the propellant is catalyzed to break down chemically, or bipropellant, in which a fuel is mixed with an oxidizer to achieve combustion just prior to expulsion. Cold-gas systems include thrusters whose propellant is not altered chemically during propulsion. In a cold-gas system, the energy may come from phase change of the propellant, or simply from pre-pressurizing the propellant in its tank. A third type of thruster is electrical, which, due to the usually small forces involved, is only used for attitude control in special circumstances. Electrical propulsion is accomplished by using magnetic or electrostatic fields to eject plasma or magnetic fluid to achieve a reaction force on the spacecraft.

Thrusters provide torque proportional to their moment arm, which is the amount that their line of force is offset from the vehicle’s center of mass. When thrusters apply their force along a line that intersects the center of mass of the vehicle, there is no torque. So, while the
amount of force available from a thruster may be large, the torque available is limited by the physical extent of the vehicle and how the thrusters are mounted on the vehicle. When mounted to maximize torque authority, thrusters have the advantage of being able to provide large, instantaneous control torques at any time in the orbit. The disadvantages of thrusters are that they use expendable propellant, can disrupt orbit determination activities, and that the plumes of expelled matter can impinge on the surface of the spacecraft, possibly heating or contaminating surfaces.

Attitude functions to which thrusters can be applied include controlling attitude, controlling nutation and spin rate, performing large, rapid slews, and managing angular momentum. In all of these functions, large torque authority can be helpful, but also necessary to some extent is the ability to change angular momentum by relatively small amounts. So, another consideration in selection of thrusters is how fine and how precise the delivered torque impulse needs to be. Torque applied over a period time creates a change in angular momentum, which has a corresponding change in the rotational rate of the spacecraft. As attitude changes with non-zero rate, attitude control accuracy is directly determined by the minimum thruster impulse available. If a 50 N thruster with a 1 meter moment arm must provide a change in angular momentum accurate to within 1 N·m·s, then it will be necessary to fire the thruster for a period of time no greater than 20 msec. The thruster valves must be capable of opening and closing that rapidly and precisely, and of propellant actually flowing into the thruster and out the nozzle, which takes a finite amount of time. This need for speed and precision in engine valve control also has implications in the design of the avionics that drive the thruster valves; often, special electronics cards must be developed for a given mission to meet thruster-based attitude control requirements.

FireSat II Actuators. The baseline FireSat II spacecraft will use magnetic torquers for momentum dumping, but thrusters will be needed for the 180 degree slews that must be done periodically to keep the Sun off the radiator. Because the slew must be about the yaw axis, the momentum bias in the wheel would resist the thruster torque, complicating the control design. Instead, we will use the thrusters to remove the momentum bias, rotate the spacecraft, and then re-establish the momentum bias. One way to remove the wheel momentum is to use thrusters for three-axis control while driving the wheel to zero speed. As the wheel slows down, it causes the spacecraft to rotate, and the thrusters are fired to counteract the rotation; the same thing happens when the wheel is once again sped up to re-establish the momentum bias. The thrusters must provide much more torque than the wheel, but generally the attitude control accuracy requirements are loosened for this kind of maneuver to avoid firing the thrusters more than is necessary. Good accuracy is less important here because science data would not be collected anyway during the slew. The total fuel mass needed for the momentum changes and the slew depends on the specific impulse, $I_{sp}$, of the selected thruster/propellant combination; we will assume the $I_{sp}$ is 218 seconds.

SCS Actuators. The SCS satellites will need to use thrusters for orbit maintenance and momentum dumping. We will want to minimize the number and size of thrusters, since the spacecraft are so small. However, the lower limit will most likely come from the orbital maneuvering needs, and not the attitude control needs. We will suppose 4 1-lb thrusters are needed for orbital maneuvering and steering, and in Table 19-12, we will present and work through procedures and simplified equations for sizing thrusters using SCS as an example. SCS has no stated slew requirement, but for the purposes of the example, a slew requirement of 30 deg in 60 seconds will be supposed. A thorough discussion on the topic of propulsion systems, including estimating propellant needs, can be found in Chapter 18.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simplified Equations</th>
<th>Application to FireSat II and SCS Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque from Reaction Wheel for Disturbance</td>
<td>Control torque, whether from reaction wheels or magnetic torquers, must equal worst-case anticipated disturbance torque plus some margin: $T_d \approx T_m = 2.1 \times 10^{-5}$ N·m (see Table 19.4).</td>
<td>For the FireSat II spacecraft, magnetic torque dominates during regular observations:</td>
</tr>
</tbody>
</table>

TABLE 19-11. Simplified Equations for Sizing Reaction Wheels, Momentum Wheels, and Magnetic Torquers. The FireSat II momentum wheel is sized for the baseline pointing requirements and for the optional design with 30-deg slew requirements. SCS reaction wheels are sized for momentum storage capacity.
**Rejection**

| Rejection | \( T_C = (T_D)(\text{Margin Factor}) \) | This torque is manageable by most reaction wheels and magnetic torquers. Momentum requirements or slew torque, not disturbance rejection, will be used for actuator sizing. |

**Slew Torque for Reaction Wheels**

| Slew Torque for Reaction Wheels | For max-acceleration slews with no resisting momentum (1/2 distance in 1/2 time): \( \theta/2 = \frac{1}{2} (T/I)(t/2)^2 \) \( T = 4\theta I/t^2 \) | FireSat II needs to make a 30-deg slew about the Y axis (50 kg-m²) in 10 minutes using its momentum wheel:

\[
T = (4)(30 \text{ deg})(\pi/180 \text{ deg})(50 \text{ kg-m}^2)/(600 \text{ sec})^2
\]

\[
= 2.9 \times 10^{-4} \text{ N-m} \quad \text{(This is a small value.)}
\]

The total momentum change for the wheel during the slew is \( T \cdot t/2 = 0.087 \text{ N-m s} \). |

**Momentum Storage in Reaction Wheel**

| Momentum Storage in Reaction Wheel | One approach to estimating wheel momentum, \( h \), is to integrate the worst-case disturbance torque, \( T_D \), over a full orbit, with period \( P \). For gravity gradient, the maximum momentum accumulates in 1/4 of an orbit. A simplified expression for such a sinusoidal disturbance is:

\[
h = T_D P (0.707)/4
\]

where 0.707 is the rms average of a sinusoidal function.

For the SCS solar pressure torques, the torque buildup depends heavily on attitude and orbit geometry, but to keep this simple we will assume the maximum also accumulates in ¼ orbit. | For FireSat II, the maximum gravity gradient torque (during 30-deg off-pointing) is estimated at:

\[
T_D = 3.7 \times 10^{-5} \text{ N-m (Table 19-4)}
\]

and the 700-km orbital period is 5926 sec:

\[
h = (3.7 \times 10^{-5} \text{ N-m})(5926 \text{ sec})(0.707/4)
\]

\[
= 0.039 \text{ N-m s}
\]

If the magnetic torquers were not capable of handling this momentum, a small reaction wheel with storage of 0.4 N-m-s would be sufficient even if 30-deg off-pointing were held for an entire quarter-orbit.

For SCS, \( T_D = 6.6 \times 10^{-6} \text{ N-m s} \).

SCS storage: \( h = (6.6 \times 10^{-6})(45083)(0.707/4) = 0.06 \text{ N-m s} \)

We will size 1.0 N-m-s wheels to minimize thruster burns and potential jitter from high wheel speeds. |

**Momentum Storage in Momentum Wheel**

| Momentum Storage in Momentum Wheel | For momentum-wheel stabilization, roll and yaw accuracy depend on the wheel’s momentum and the external disturbance torque. A simplified expression for the required momentum storage is:

\[
h = (T \theta_a)/(P/4)
\]

where \( \theta_a \) = allowable motion. | The value of \( h \) for the FireSat II yaw accuracy requirement of 0.1 deg (0.0017 rad) would be:

\[
h = (2.1 \times 10^{-5} \text{ N-m/0.0017 rad})(5926/4 \text{ sec})
\]

\[
= 18.4 \text{ N-m s}
\]

For 1 deg accuracy, we would need only 1.8 N-m-s. The same results would hold for a spinner.

Note that this capacity would need to be added to the momentum change needed for the 30 deg slews, which for FireSat II is less than 0.1 N-m-s. |

**Torque from Magnetic Torquers**

| Torque from Magnetic Torquers | Magnetic torquers use electrical current through the torquer to create a magnetic dipole (D). The Earth’s magnetic field, B, acts on D, resulting in torque (T) on the vehicle:

\[
D = T/B
\]

Magnets used for momentum dumping must equal the peak disturbance + margin to compensate for the lack of complete directional control. (Note that if magnetic torque is the dominant disturbance, the torquers need mainly to cancel the spacecraft’s residual

Table 19-4 estimates the maximum Earth field, B, for FireSat II to be \( 4.2 \times 10^{-5} \text{ tesla} \). We calculate the torque rod’s magnetic torquing ability (dipole) to counteract the worst-case gravity gradient disturbance, \( T_D \), of 2.1 \times 10^{-5} \text{ N-m s} as:

\[
D = (2.1 \times 10^{-5} \text{ N-m})(4.2 \times 10^{-5} \text{ tesla}) = 0.5 \text{ A-m}^2
\]

which is a small actuator. The Earth’s field is cyclic at twice orbital frequency; thus, maximum torque is available only twice per orbit. A torquer of 3 to 10 A-m² capability should provide sufficient margin, which also gives enough capacity to remove increased gravity gradient torques during offset pointing. |
Note: For actuator sizing, the magnitude and direction of the disturbance torques must be considered. In particular, momentum accumulation in inertial coordinates must be mapped to body-fixed actuator axes.

### Table 19-12. Simplified Equations for Preliminary Sizing of Thruster Systems

<table>
<thead>
<tr>
<th>Simplified Equations</th>
<th>Application to Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Thruster force level sizing for external disturbances:</strong></td>
<td>For the SCS worst case $T_D$ of $6.6 \times 10^{-6}$ N·m (Table 19-4) and a thruster moment arm of 0.5 m</td>
</tr>
<tr>
<td>$F = \frac{T_D}{L}$</td>
<td>$F = \frac{(6.6 \times 10^{-6} \text{ N·m})}{(0.5 \text{ m})} = 3.3 \times 10^{-6}$ N</td>
</tr>
<tr>
<td>$F$ is thruster force, $T_D$ is worst-case disturbance torque, and $L$ is the thruster’s moment arm</td>
<td>This small value indicates orbit maintenance and momentum dumping requirements, not disturbance torques, will determine thruster size. Also, using thrusters to fight cyclic disturbances uses precious propellant; it is generally better to store the momentum in wheels.</td>
</tr>
</tbody>
</table>

**Sizing force level to meet slew rates:**

Determine fastest slew rate, $\omega$, required in the mission profile. Develop a slew profile that accelerates the vehicle quickly, coasts at that rate, and then decelerates quickly. The acceleration required, $\alpha$, comes from equating these two torque definitions:

$$T = F \cdot L = I \cdot \alpha$$

And therefore the on-time for both starting and stopping the slew of:

$$t = \frac{h}{T} = \frac{h}{(F \cdot L)}$$

**Determining fuel needed to complete FireSat II yaw slew:**

Thrusters will need to remove the momentum bias from the wheel. Rotate the satellite 180 deg, and then re-establish the momentum. Fuel mass, $m$, is estimated as:

$$m = F \cdot t / (g \cdot I_{sp})$$

where $t$ is the on-time of one thruster.
\[
t_{\text{accel}} = t_{\text{decel}} = \frac{(0.7 \text{ N}\cdot\text{m}\cdot\text{s})}{(5 \text{ N} \cdot 0.5 \text{ m})} = 0.3 \text{ sec}
\]

Total thruster on-time per slew: \( t_{\text{total}} = 10 + 0.3 + 0.3 + 10 = 21 \text{ seconds} \)

Fuel required per 180-degree slew:
\[
m = \frac{(5 \text{ N})(21 \text{ s})}{(9.8 \text{ m/s}^2 \cdot 218 \text{ s})} = 49 \text{ g fuel}
\]

**Sizing force level for momentum dumping:**

\[
F = \frac{h}{Lt}
\]

where

- \( h \) = stored wheel momentum
- \( L \) = thruster moment arm
- \( t \) = burn time

For SCS with 1.0 N\cdot m\cdot s wheels and 1-sec burns,

\[
F = \frac{(1.0 \text{ N}\cdot\text{m}\cdot\text{s})}{(0.5 \text{ m} \cdot 1 \text{ sec})} = 2.0 \text{ N}
\]

This is still well within the range of 1-lb thrusters, which are commonly used for orbit maintenance on small spacecraft. Reaction wheels with even larger capacity might be desirable if it would further reduce the number of times the thrusters must be used.

**Sensors.** We complete this hardware unit by selecting the sensors needed for attitude determination. Consult Table 19-13 for a summary of typical devices as well as their performance and physical characteristics. Note, however, that sensor technology is changing rapidly, promising ever more accurate and lighter-weight sensors for future missions.

*Sun sensors* are visible-light or infrared detectors that measure one or two angles between their mounting base and incident sunlight. They are popular, accurate, and very reliable, but they require clear fields of view. They can be used as part of the normal attitude determination system, part of the initial acquisition or failure recovery system, or part of an independent solar array orientation system. Since most low-Earth orbits include eclipse periods, Sun-sensor-based attitude determination systems must provide some way of tolerating the regular loss of this data without violating pointing constraints.

Sun sensors can be quite accurate (<0.01 deg), but it is not always possible to take advantage of that feature. We usually mount Sun sensors near the ends of vehicles to obtain an unobstructed field view, so the Sun sensor accuracy can be limited by structural bending on large spacecraft. Spinning satellites use specially designed Sun sensors that measure the angle of the Sun with respect to the spin axis of the vehicle, and they issue a pulse correlated to the time the Sun crosses the sensor to provide spin-phase information. Also popular are coarse Sun sensors, which are simply small solar cells that issue a current roughly proportional to the cosine of the Sun angle. These sensors are so small and inexpensive that it is often feasible to put several in many directions on a spacecraft, and then to estimate the Sun direction by solving the linear system equations that results. Because coarse Sun sensors use no power

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Typical Performance Range</th>
<th>Mass (kg)</th>
<th>Power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gyroscopes</td>
<td>Drift Rate = 0.003 deg/hr to 1 deg/hr</td>
<td>&lt; 0.1 to 15</td>
<td>&lt; 1 to 200</td>
</tr>
<tr>
<td></td>
<td>Drift rate stability varies widely</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sun Sensors</td>
<td>Accuracy = 0.005 deg to 3 deg</td>
<td>0.1 to 2</td>
<td>0 to 3</td>
</tr>
<tr>
<td>Star Sensors (Scanners &amp; Cameras)</td>
<td>Accuracy = 1 arcsecond to 1 arcminute</td>
<td>2 to 5</td>
<td>5 to 20</td>
</tr>
<tr>
<td></td>
<td>= 0.0003 deg to 0.01 deg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizon Sensors</td>
<td>Accuracy: 0.05 deg to 1 deg (0.1 deg is best for LEO)</td>
<td>1 to 4</td>
<td>5 to 10</td>
</tr>
<tr>
<td>- Scanner/Pipper</td>
<td>&lt; 0.1 deg to 0.25 deg</td>
<td>0.5 to 3.5</td>
<td>0.3 to 5</td>
</tr>
<tr>
<td>- Fixed Head (Static)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Magnetometer</td>
<td>Accuracy = 0.5 deg to 3 deg</td>
<td>0.3 to 1.2</td>
<td>&lt; 1</td>
</tr>
</tbody>
</table>
Horizon sensors can be accommodated in their application. Where the star trackers are replaced (or augmented) by the spinning spacecraft body. Scanning horizon sensors used on spinning spacecraft to measure Earth phase and types (called star cameras, rather than trackers), allowing starlight to fall on a CCD to create an image of the star field. Then, internal processing determines a three-axis attitude based on a star catalog. Many units are able to determine a very accurate attitude within seconds of being turned on.

While star sensors excel in accuracy, care is required in their specification and use. The most accurate star cameras are unable to determine attitude at all if the spacecraft is rotating too fast, and other star sensors must know roughly where they are pointing to make their data useful. Therefore, the vehicle must be stabilized to some extent before the trackers can operate effectively. This stabilization may require alternate sensors, which can increase total system cost. Also, star sensors are susceptible to being blinded by the Sun, Moon, planets, or even high radiation levels, such as in the Van Allen belts, which is a disadvantage that must be accommodated in their application. Where the mission requires the highest accuracy and justifies a high cost, we often use a combination of star trackers and gyroscopes. The combination of these sensors is very effective: the gyro can be used for initial stabilization and during periods of inference in the star trackers, while the trackers can be used to provide a high-accuracy external reference unavailable to the gyro.

**Horizon sensors** (also known as Earth sensors) are infrared devices that detect the contrast between the cold of deep space and the heat of the Earth’s atmosphere (about 40 km above the surface in the sensed band). Simple narrow field-of-view fixed-head types (called pippers or horizon crossing indicators) are used on spinning spacecraft to measure Earth phase and chord angles, which, together with orbit and mounting geometry, define two angles to the Earth (nadir) vector. **Scanning horizon sensors** use a rotating mirror or lens to replace (or augment) the spinning spacecraft body. They are often used in pairs for improved performance and redundancy. Some nadir-pointing spacecraft use **staring sensors**, which view the entire Earth disk (from GEO) or a portion of the limb (from LEO). The sensor fields of view stay fixed with respect to the spacecraft. This type works best for circular orbits, as they are often tuned for a tight range of altitudes.

Horizon sensors provide Earth-relative information directly for Earth-pointing spacecraft, which may simplify onboard processing. The scanning types require clear fields of view for their scan cones (typically 45, 60, or 90 deg half-angle). Typical accuracies for systems using horizon sensors are 0.1 to 0.25 deg, with some applications approaching 0.03 deg. For the highest accuracy in low-Earth orbit, it is necessary to correct the data for Earth oblateness and seasonal horizon variations.

**Magnetometers** are simple, reliable, lightweight sensors that measure both the direction and magnitude of the Earth’s magnetic field. Magnetometer output helps us establish the spacecraft’s attitude relative to the local magnetic field, which information can be combined with magnetic field models and orbit information to determine attitude relative to the Earth and inertial reference frames. However, their accuracy is not as good as that of star or horizon sensors. The Earth's magnetic field can shift with time and is not known precisely in the first place. To improve accuracy, we often combine magnetometer data with data from Sun or horizon sensors. As a vehicle using magnetic torquers passes through changing magnetic fields during each orbit, we use a magnetometer to control the magnitude and direction of the torquers’ output relative to the present magnetic field. In earlier spacecraft the torquers usually needed to be inactive while the magnetometer was sampled to avoid corrupting the measurement. However, improvements in onboard computing capability mean that coupling matrices can be used to extract the torquer inputs from the field measurement, allowing constant sampling even while torquing. Finally, good spacecraft ephemeris and magnetic field models can be used in place of magnetometers for some missions, but magnetometers will generally be more accurate.

**GPS receivers** are well known as high-accuracy navigation devices, but they can also be used for attitude determination. If a spacecraft is large enough to place multiple antennas with sufficient separation, attitude can be determined by employing the differential signals from the separate antennas. Such sensors offer the promise of low cost and weight for LEO missions. They can provide attitude knowledge accurate to 0.25 – 0.5 deg for antenna baselines on the order of 1 meter [Cohen 1996], and so are being used in low accuracy applications or as back-up sensors.
Development continues to improve their accuracy, which is limited by the separation of the antennas, the ability to resolve small phase differences, the relatively long wavelength of the GPS signal, and multipath effects due to reflections off spacecraft components.

Gyrosopes are inertial sensors that measure the speed or angle of rotation from an initial reference, but without any knowledge of an external, absolute reference. We use gyros in spacecraft for precision attitude determination when combined with external references such as star or Sun sensors, or, for brief periods, for nutation damping or attitude control during thruster firing. Manufacturers use a variety of physical phenomena, from simple spinning rotors to ring lasers, hemispherical resonating surfaces, and laser fiber optic bundles. Gyros based on spinning rotors are called mechanical gyros, and they may be large iron gyros using ball or gas bearings, or may reach very small proportions in so-called MEMS gyros. (MEMS stands for microelectromechanical systems.) The gyro manufacturers, driven largely by aircraft markets, steadily improve accuracy while reducing size and mass.

Error models for gyroscopes vary with the technology, but characterize the deterioration of attitude knowledge with time. Some examples of model parameters are drift bias, which is simply an additional, false rate the sensor effectively adds to all rate measurements, and drift bias stability, which is a measure of how quickly the drift bias changes. When used with an accurate external reference, such as a star tracker, gyros can provide smoothing (filling in the gaps between tracker measurements) and higher frequency information (tens to hundreds of hertz), while the tracker provides lower frequency, absolutely referenced information whenever its field of view is clear. Individual gyros provide one or two axes of information and are often grouped together as an inertial reference unit (IRU) for three full axes and, sometimes, full redundancy. IRUs with accelerometers added for position and velocity sensing are called inertial measurement units (IMU).

Sensor Selection. Sensor selection is most directly influenced by the required orientation of the spacecraft (e.g. Earth-, Sun- or inertial-pointing) and its accuracy. Other influences include redundancy, fault tolerance, field of view requirements, and available data rates. Typically, we identify candidate sensor suites and conduct a trade study to determine the most cost-effective approach that meets the needs of the mission. In such studies the existence of off-the-shelf components and software can strongly affect the outcome. In this section we will only briefly describe some selection guidelines.

Full three-axis knowledge requires at least two external, non-parallel vector measurements, although we use IRUs or spacecraft angular momentum (in spinners or momentum-biased systems) to hold or propagate the attitude between external measurements. In some cases, if attitude knowledge can be held for a fraction of an orbit, the external vectors (e.g. Earth or magnetic) will have moved enough to provide the necessary information. In three-axis star trackers, each identified star acts as a reference vector, which allows a single piece of hardware to generate a full three-axis attitude solution.

For Earth-pointing spacecraft, horizon sensors provide a direct measurement of pitch and roll axes, but require augmentation for yaw measurements. Depending on the accuracy required, we use Sun sensors, magnetometers, or momentum-bias control with its roll-yaw coupling for the third degree of freedom. For inertially pointing spacecraft, star and Sun sensors provide the most direct measurements, and IRUs are ideally suited. Frequently, only one measurement is made in the ideal coordinate frame (Earth or inertial), and the spacecraft orbit parameters are required to convert a second measurement or as an input to a magnetic field model. Either the orbit parameters are uplinked to the spacecraft from ground tracking and propagated by onboard processing, or they are obtained from onboard GPS antennas.

FireSat II sensors. The external sensors for FireSat II could consist of any of the types identified. For the 0.1 deg Earth pointing requirement, however, horizon sensors are the most obvious choice since they directly measure the two axes we most need to control. The accuracy requirement makes a star sensor a strong candidate as well; its information would need to be transformed, probably using an onboard orbit ephemeris calculation, to Earth-relative for our use. The 0.1 deg accuracy is at the low end of horizon sensors’ typical performance, so we need to be careful to get the most out of their data. We assume we also need a yaw sensor capable of 0.1 deg, and this choice is less obvious. Often, it is useful to question a tight yaw requirement. Many payloads, e.g. antennas, some cameras, and radars, are not sensitive to rotations around their pointing axis. For this discussion, we will assume this requirement is firm. We could use Sun sensors, but their data needs to be replaced during eclipses. Magnetometers don’t have the necessary accuracy alone, but with our momentum-bias system, roll-yaw coupling, and some yaw data filtering, a magnetometer-Sun sensor system could work for normal operations. The magnetometers would also
improve the control effectiveness of the magnetic torquers.

At this point we consider the value of an inertial reference package. Such packages, although heavy and expensive for high-accuracy equipment, provide a short-term attitude reference that would permit the Earth vector data to be used for full three-axis knowledge over an orbit. A gyro package would also reduce the single measurement accuracy required of the horizon sensors, simplifying their selection and processing. Such packages are also useful to the control system if fast slews are required, and here is where FireSat II demands a gyro. We need to perform the 180 degree slews as quickly as we can, to avoid losing too much data. So, we will include a MEMS-based inertial package for FireSat II. They will be arranged perpendicular to one another, in a pyramid around the Z axis. This arrangement gives 3 axes of information with maximal redundancy around the slewing axis. Now that we have decided to include gyros, a careful trade study should be done to determine whether an inexpensive MEMS gyro package combined with just the Earth sensors eliminates our need to include Sun sensors and magnetometers. Leaving out some of these other sensors could give better reliability or lower total cost. We may also choose a slightly different arrangement of the sensors to improve accuracy in one direction at the expense of accuracy in another direction. We would need to do these kinds of detailed trade studies in later iterations of the design process.

Finally, we will want a simple, coarse control mode to initially point the arrays at the Sun and to protect the spacecraft in the event of an anomaly. By using 6 coarse Sun sensors pointing along the positive and negative of each axis, the spacecraft can derive the location of the Sun from any attitude and use magnetic torquers to rotate the spacecraft so the arrays are lit. Then, since the attitude relative to nadir will change as FireSat II follows its polar orbit, we can be sure to get a good communication signal at some point, so that we can receive telemetry and send commands.

**SCS sensors.** For SCS attitude determination, low-power gyros can provide rate information. Accurate gyros can be heavy and often use a lot of power; we have neither high accuracy needs nor an excess of power or mass in our budgets. Therefore, we will use light and inexpensive MEMS gyros. We need a minimum of 3 MEMS gyros—one for each axis—but by employing 4-6 gyros we can cross-compare the gyro data and remove the larger bias errors that MEMS gyros normally have.

With rate information onboard, we only need an occasional update from an attitude sensor. If the magnetic field were stronger, we might be able to filter magnetometer data to get to 1 deg of accuracy, but it is doubtful at this high altitude. Star cameras are small and very accurate, but they are expensive. One useful rule of thumb is: If at all possible, sense the thing you need to point at. Sun pointers should have Sun sensors and Earth pointers should have Earth sensors. Because the satellites will have the same direction pointing toward the Earth throughout their mission, the best option appears to be an Earth sensor. We would need to select a sensor designed for high altitudes. However, we still have no yaw data, and since the satellites must point accurately at **multiple targets simultaneously,** yaw accuracy is critical.

Because the satellites will be communicating with each other, it is conceivable that the communication signal strength could be used as an attitude determination data source for yaw control. That is, a feedback loop would close around the communication system’s own measure of its link margin; maximizing the link margin would provide the attitude goal we want. For this exercise we will not assume such an option is available. Instead, we will choose the star camera after all; a simple onboard ephemeris calculation will tell the spacecraft where its target satellite is in inertial space. There may be clever tricks that we could use with ground-based methods to avoid using star cameras, such as combining orbit tracking and attitude data. However, the complexity of operating three separate satellites that have to work together will likely prove more expensive in software development and operating costs than just buying three star cameras. At least we save money by not including horizon sensors. As for our rule of thumb of sensing what you’re pointing at, we now see there can be situations for which this rule cannot be followed. Still, it’s always a good place to start.

We will propose the same plan for an initial acquisition and safe control mode as for FireSat II. However, SCS will use reaction wheels for control, and will have the benefit of accurate rate sensors to improve performance. This is a good thing, since SCS satellites need more power than FireSat II, and so will probably not have as much time to acquire the Sun (i.e. get the solar arrays lit). Table 19-14 summarizes our FireSat II and SCS hardware selections.
Table 19-14. FireSat II and SCS Spacecraft Control Component Selection.

<table>
<thead>
<tr>
<th>Mission &amp; Type</th>
<th>Components</th>
<th>Rationale</th>
</tr>
</thead>
<tbody>
<tr>
<td>FireSat II Actuators</td>
<td>Momentum Wheel (1)</td>
<td>- Pitch axis torquing</td>
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<tr>
<td></td>
<td>Magnetic Torquers (3)</td>
<td>- Roll and yaw axis passive stability</td>
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<tr>
<td></td>
<td>Thrusters (4)</td>
<td>- Roll and yaw control</td>
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<tr>
<td></td>
<td></td>
<td>- Pitch wheel desaturation</td>
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<td></td>
<td></td>
<td>- Rapid changes in pitch wheel momentum</td>
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<tr>
<td></td>
<td></td>
<td>- 180-deg yaw slew maneuvers</td>
</tr>
<tr>
<td></td>
<td>Horizon Sensor (1)</td>
<td>- Provide basic pitch and roll reference</td>
</tr>
<tr>
<td></td>
<td>Sun Sensors (6)</td>
<td>- Can meet 0.1 deg accuracy</td>
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<tr>
<td></td>
<td>Magnetometer (1)</td>
<td>- Lower mass and cost than star sensors</td>
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<td></td>
<td>MEMS Gyros (3)</td>
<td>- Initially acquire vehicle attitude from unknown orientation</td>
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<tr>
<td></td>
<td></td>
<td>- Coarse attitude data</td>
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<tr>
<td></td>
<td></td>
<td>- Fine data for yaw</td>
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<td></td>
<td></td>
<td>- Coarse yaw data</td>
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<td></td>
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<td>- Needed for more precise magnetic torquing</td>
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<td></td>
<td></td>
<td>- Needed for rotational rate data during thruster operations</td>
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<tr>
<td>SCS Actuators</td>
<td>Reaction Wheels (3)</td>
<td>- Three-axis stabilization</td>
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<tr>
<td>(for each satellite)</td>
<td>Thrusters (4)</td>
<td>- Storing momentum from solar radiation torques</td>
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<td>- Thrusting and steering during orbit maneuvers</td>
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<tr>
<td></td>
<td></td>
<td>- Removing momentum from reaction wheels</td>
</tr>
<tr>
<td>SCS Sensors</td>
<td>Star Camera (1)</td>
<td>- Determining absolute attitude</td>
</tr>
<tr>
<td>(for each satellite)</td>
<td>MEMS Gyros (4)</td>
<td>- Determining rotational rates; having 4 gyros allows</td>
</tr>
<tr>
<td></td>
<td>Sun Sensors (6)</td>
<td>large biases to be canceled out</td>
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<tr>
<td></td>
<td></td>
<td>- Propagating attitude solution when star sensor data</td>
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<tr>
<td></td>
<td></td>
<td>initially unavailable</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Initially acquire vehicle attitude relative to Sun to</td>
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<tr>
<td></td>
<td></td>
<td>get power quickly</td>
</tr>
</tbody>
</table>

Once the hardware selection is complete, it must be documented for use by other system and subsystem designers as follows.
- Specify the power levels and weights required for each assembly
- Establish the electrical interface to the rest of the spacecraft
- Describe requirements for mounting, alignment, or thermal control
- Determine what telemetry data we must process
- Document how much software we need to develop or purchase to support onboard calculation of attitude solutions

Specific numbers depend on the vendors selected. A typical list for FireSat II might look like Table 19-15, but the numbers could vary considerably with only slight changes in subsystem accuracies or slewing requirements.

Table 19-15. FireSat II Spacecraft Control Subsystem Summary. The baseline ADCS components satisfy all mission requirements, with thrusters required for orbit injection and 180 degree slews.

<table>
<thead>
<tr>
<th>Components</th>
<th>Type</th>
<th>Weight (kg)</th>
<th>Power (W)</th>
<th>Mounting Considerations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum Wheel</td>
<td>Mid-size; 20 N•m total</td>
<td>&lt; 5</td>
<td>10 to 20</td>
<td>Momentum vector on pitch axis</td>
</tr>
<tr>
<td>Electromagnets</td>
<td>3, 10 A • m²</td>
<td>2, including current drive electronics</td>
<td>5 to 10</td>
<td>Orthogonal configuration best to reduce cross-coupling</td>
</tr>
<tr>
<td>Sun Sensors</td>
<td>6 wide-angle coarse Sun sensors providing 4π steradian coverage; 5-10 deg accuracy</td>
<td>&lt; 1 total</td>
<td>0.0</td>
<td>Free of viewing obstructions and reflections</td>
</tr>
<tr>
<td>Horizon Sensors</td>
<td>Scanning type (2) plus electronics; 0.1 deg accuracy</td>
<td>5 total</td>
<td>10</td>
<td>Unobstructed view of Earth's horizon</td>
</tr>
<tr>
<td>Gyroscopes</td>
<td>MEMS (3)</td>
<td>&lt; 1 total</td>
<td>0.1</td>
<td>Mounted</td>
</tr>
</tbody>
</table>
**19.1.5 Define the Determination and Control Algorithms**

Finally, we must tie all of the control components together into a cohesive system. Generally, we begin with a single-axis control system design (See Figure 19-8). As we refine the design, we add or modify feedback loops for rate and attitude data, define gains and constants, and enhance our representation of the system to include three axes of motion (though we may still treat these as decoupled for early design iterations). To confirm that our design will meet requirements, we need good mathematical simulations of the entire system, including sensor error models and internal and external disturbances. Usually, linear differential equations with constant coefficients describe the dynamics of a control system well enough to allow us to analyze its performance with the highly developed tools of linear servomechanism theory. With these same tools, we can easily design linear compensation to satisfy specifications for performance.

**Figure 19-8. Block diagram of a Typical Attitude Control System with Control along a Single Axis.**

Control algorithms are usually implemented in an onboard processor and analyzed with detailed simulations.

We typically apply linear theory only to preliminary analysis and design. We also maintain engineering margin against performance targets when using linear theory because, as the design matures, so does our understanding of the nonlinear effects in the system. Nonlinear effects may be inherent or intentionally introduced to improve the system’s performance.

Another reason to maintain margin, especially in actuator sizing, is that while systems engineers always carefully budget mass, they cannot usually track the moment of inertia matrix as well. Moment of inertia is the most important quantity to ADCS designers, and a given mass may have a wide range of moment of inertia values depending on how the mass is distributed.

Feedback control systems are of two kinds based on the flow of their control signals. They are *continuous-data* systems when sensor data is electrically transformed directly into continuously flowing, uninterrupted control signals to the actuators. By contrast, *sampled-data* systems have sensor sampling at set intervals, and control signals are issued or updated at those intervals. Most modern spacecraft process data through digital computers and therefore use sampled-data control systems.

Although it is beyond the scope of this book to provide detailed design guidance on feedback control systems, the system designer should recognize the interacting effects of attitude control system loop gain, capability of the attitude control system to compensate for disturbances, accuracy of attitude control, and control system bandwidth.

**Three-axis stabilization.** Different types of active control systems have different key parameters and algorithms. For systems in which spacecraft rates will be kept small, three-axis control can frequently be decoupled into three independent axes. In this simplest form, each axis of a spacecraft attitude control system can be represented as a double-integrator plant (i.e. $1/Is^2$ in the s-domain) and may be controlled by a *proportional-derivative (PD) controller*, which has control torque $T_C = K_P\theta_E + K_D\dot{\theta}_E$. Here, $\theta_E$ is the attitude error angle, and $\dot{\theta}_E$ is the attitude rate error.

The most basic design parameter in each axis is its *proportional gain*, $K_P$. ($K_P$ is also often called a position gain.) This gain represents the amount of control torque we want to result from a unit of attitude.
error, and so it has units of torque divided by angle, e.g. newton-meters per radian. The proportional gain is selected by the designer and must be high enough to provide the required control accuracy in the presence of disturbances, which can be guaranteed by setting $K_P > T_D/\theta_{E,max}$, where $T_D$ is the peak disturbance torque, and $\theta_{E,max}$ is the allowable attitude error. Note that this error will remain for as long as the disturbance torque remains; the error that remains in the presence of disturbances is called steady-state error.

The value of $K_P$ also largely determines the attitude control system bandwidth, which is a measure of its speed of response. The mathematical definition of bandwidth requires a bit more explanation than we want to go into here, but it can be approximated as $\omega_n = (K_P/I)^{1/2}$, where $I$ is the spacecraft moment of inertia. The bandwidth defines the frequency of disturbances and of commanded motions at which control authority begins to diminish. Attitude control and disturbance rejection are effective from 0 frequency (i.e. constant or d.c. inputs) up to the bandwidth. Speed of response is

**Table 19-16 ADCS Vendors.** Typical suppliers for ADCS components. An up-to-date version of this table can be found at the SMAD web site.

<table>
<thead>
<tr>
<th>Company</th>
<th>Sun Sensors</th>
<th>Earth Sensors</th>
<th>Magnetometers</th>
<th>Star Sensors</th>
<th>Gyro</th>
<th>GPS</th>
<th>Mom./CMG</th>
<th>Magnetic Torquers</th>
<th>Thrusters</th>
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<td>Adcole Corporation</td>
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<td>Aerojet</td>
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<td>Ball Aerospace and Technologies Corp.</td>
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<td>Billingsley Aerospace &amp; Defense</td>
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<td>Bradford Engineering</td>
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<td>Comtech AeroAstro</td>
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<td>Finmeccanica (incl. SELEX Galileo)</td>
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<td>L-3 Space &amp; Navigation</td>
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<td>Kearfott Guidance &amp; Navigation Corp.</td>
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<td>NASA Goddard Space Flight Center</td>
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<td>Northrop Grumman (incl. Litton)</td>
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<td>Optical Energy Technologies</td>
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<td>Servo Corp. of America</td>
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<td>StarVision Technologies</td>
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approximately the reciprocal of bandwidth. Note that
proportional gain is inversely proportional to allowable
error, and bandwidth is proportional to the square root
of the proportional gain. Therefore, high accuracy
implies high \( K_P \) and high bandwidth. However, there
are practical limits on bandwidth. If the bandwidth is
high enough that it includes bending frequencies for the
spacecraft structure, then control-structure interactions
must be carefully considered. If the bandwidth is higher
than the maximum sampling rate of critical attitude
sensors, some care must be taken to avoid excessive
attitude responses to the slow updates. Finally, high
bandwidth control systems may be more likely to have
adverse reactions to inaccurate or lagging system
inputs; this condition is referred to as having
insufficient stability margin.

There are design techniques to address these concerns.
One such technique is to modify the system response to
increase stability while keeping the same attitude
accuracy by increasing attitude rate damping, which is
controlled by the derivative gain, or rate gain, \( K_D \). For
a given value of \( K_P \), increasing \( K_D \) will slow down the
response of the control system, but it will not diminish
the accuracy in the end, provided the disturbance
environment or slewing requirements do not require
very high speed of response. This technique is often
used to improve stability margins in systems where the
final accuracy is more important than the amount of
time it takes to achieve that accuracy. Note that, though
there are multiple methods for selecting \( K_P \) and \( K_D \),
such as linear-quadratic regulation (LQR) and other
state-space methods, the system response generally
must still be understood in terms of bandwidth and
damping to achieve confidence that the control gains
will provide both stability and adequate performance.

Sometimes the problem facing a designer is that
stability margin requirements are not difficult to meet,
but accuracy requirements are very difficult. A common
technique for improving on the performance of a PD is
to add an integrator to the feedback loop; this is called a
proportional-integral-derivative controller (PID).
Recall that in a regular PD controller, a steady-state
error of some kind will remain as long as there are
disturbance torques to create it. The integrator causes
the correcting control torque to gradually increase over
time to remove that steady-state error completely. If the
disturbance torque changes magnitude or direction, the
PID controller will take some time to adjust, but it will
track with no steady-state error. This improved
performance generally comes at the price of reduced
stability margins, so the stability and performance
requirements must always be carefully balanced.

In addressing control-structure interactions, the control
designer must keep in mind that, if bending causes
apparent attitude errors, the control system may
aggravate the bending motion, eventually affecting
control stability and perhaps damaging the bending
structures. Though there are control design techniques
deal with these potential problems, the control
designer will often start by recommending a minimum
frequency for structural bending modes that would
allow flexible effects to be neglected. This minimum
frequency is generally an order of magnitude greater
than the bandwidth needed to achieve required
accuracy. Then, structural engineers will attempt to
honor this restriction in their design. For further
discussion of how structural flexibility interacts with
feedback control systems, see section 3.12 of Agrawal
[1986].

Spin stabilization and momentum bias. The
fundamental concept in spin stabilization is the nutation
frequency of the vehicle. For a spinning body, the
inertial nutation frequency \( (\omega_{ni}) \) is equal to
\( \omega_S I_y/I_T \) where \( \omega_S \) is the spin frequency, \( I_S \) is the spin axis of
inertia, and \( I_T \) is the transverse axis inertia. For a
momentum-bias vehicle with a non-rotating body and a
momentum wheel (or a dual-spin vehicle with a non-
rotating platform), the nutation frequency is
\( \omega_{ni}=h/I_T \) where \( h \) is the angular momentum of the spinning body
(or bodies). (Note that this is really the same equation
as for a spinning body, for which \( h = \omega_S I_S \).) Thus,
spacecraft with large inertias and small wheels have
small nutation frequencies (i.e. long periods).

Attempting to control the vehicle with a bandwidth
faster than the nutation frequency causes it to act more
like a three-axis stabilized vehicle, that is, the stiffness of the spin is effectively reduced. In general, we attempt to control near the nutation frequency or slower, with correspondingly small torques for given attitude errors. Because we generally wish to avoid changing the spin rate with control torques, the torque is applied perpendicularly to the spin. We can then relate the achieved angular rate, \(\omega\), to the applied torque by simplifying Euler’s equations to only the gyroscopic torque component: \(T = \omega \times H = \omega H\) where \(H\) is the system angular momentum. A lower limit on control bandwidth is usually provided by the orbit rate \(\omega_0\), which for a circular orbit is \(\omega_0 = \sqrt{\mu / r^3}\) where \(\mu = 3.986 \times 10^{14}\) m\(^3\)/s\(^2\) for Earth orbits and \(r\) is the orbit radius.

**Example of Magnetic Momentum Management.** Managing the long-term buildup of angular momentum in the orbiting spacecraft often drives the ADCS requirements. As would be expected for an element so central to ADCS design, there are many different methods for keeping momentum to levels that do not degrade data or equipment. We present here a simple, common, and relatively inexpensive algorithm for commanding magnetic torquers using only magnetometer data. It functions as an excellent active rate damping controller, and after the spacecraft’s rate is very low, it keeps the angular momentum at manageable levels by canceling it out on an orbit-averaged basis. Often called B-dot, this algorithm is based on back-differencing magnetometer data to estimate the time derivative of the magnetic field (or “Bdot”). For rate damping purposes, we assume the B-field is changing in the body due only to spacecraft rotation, which is true for all but rotation rates on the order of orbital rate. Specifically, \(B_{dot} = B_{k+1} - B_k = -\omega \times B\). The spacecraft dipole vector, \(D\), is set equal to a gain multiplied by the magnetic field: \(D = -k B_{dot} = k\omega \times B\). Then, when the magnetic field acts on that vector, it produces a control torque \(T_c = D \times B\) that is always in a direction that slows down the spacecraft rotation except when \(\omega\) is aligned with \(B\) (in vector terminology, \(\omega \cdot T_c \leq 0\)). Once the spacecraft is slowed down, the algorithm will attempt to follow the magnetic field vector, its success determined by the strength of the gain, \(k\), that is chosen. However, whether it follows the field doesn’t matter: The goal of slowing the spacecraft and then maintaining a slow momentum has been achieved. Flatley, Morgenstern, Reth and Bauer [1997] did not invent the B-dot controller, but their paper nicely explains in clear English and equations how something so simple works so well.

**Algorithms for attitude determination.** A full discussion of attitude determination algorithms requires a dedicated reference such as Wertz [1978]. We will highlight only some of the basic concepts.

The basic algorithms for determination depend on the coordinate frames of interest and the geometry and parameterization of the measurements. Useful coordinate frames can include the individual sensor frames, the local vertical frame, and an inertial frame, such as Earth-centered inertial (ECI). The geometry of the measurements is different for different sensors, as discussed in Sec. 19.1.4, and they are generally parameterized as sequential rotations about the axes of a frame, called Euler angles (e.g. roll, pitch, yaw), or as attitude quaternions, which are unit vectors with four elements that define a rotational axis in a frame, called the eigenaxis, and the amount of rotation about the eigenaxis. Inertial reference units (with their supporting software) and star sensors are well suited to quaternions, whereas Earth-pointing vehicles often use a local-vertical set of Euler angles, much like aircraft.

Simple spacecraft may use the sensor output directly for control, whereas more complex vehicles or those with higher accuracy requirements employ some form of averaging, smoothing or Kalman filtering of the data. The exact algorithms depend on the vehicle properties, orbit, and sensors used.

**FireSat II algorithms.** For our momentum-bias FireSat II example, control separates into pitch-axis control using torque commands to the momentum wheel and roll-yaw control using current commands to the magnetic torquers. The pitch-wheel desaturation commands must also be fed (at a slower rate) to the torquers. The pitch-wheel control is straightforward, using PD control and, optionally, integral control, in which commands are augmented by a small torque proportional to the integral of the attitude error. The roll-yaw control design starts by using the linearized nutation dynamics of the system, and is complicated by the directional limitations of electromagnetic torque (the achievable torque is perpendicular to the instantaneous Earth magnetic field).

The nadir-oriented control system may use an Earth-referenced, aircraft-like Euler angle set (i.e. roll, pitch & yaw), although quaternions may simplify calculations during off-nominal pointing; quaternion calculations generally require fewer arithmetic steps, they simplify tracking commands, and they have no singularities. The horizon sensors directly read two of the angles of interest, pitch and roll. Yaw needs to be measured directly from Sun position (during orbit day) or from the magnetometer readings (using a stored model of the Earth’s field plus an orbit ephemeris), or inferred from the roll-yaw coupling described earlier.
MEMS gyro data can be used to enhance the attitude estimate, but they can have large biases that change with temperature, so it may be best not to have to rely on them for our accuracy requirements. The magnetic field and Sun information require an uplinked set of orbit parameters, and increase the computational requirements of the subsystem. Overall, meeting the 0.1 deg yaw requirement when the Sun is not visible will be the toughest challenge facing the ADCS designer, and planning to coast through the dark periods without direct yaw control may be most appropriate.

**SCS Algorithms.** A 3-axis PD controller will be sufficient for our control accuracy needs. In this control algorithm, an error angle and error rate is calculated for each axis, and a control torque calculated based on those. That calculation done for the 3 axes gives a control torque vector, which is then distributed among the reaction wheels. If one of the wheels is asked to issue a control torque that exceeds its capability, then the spacecraft would have a total torque vector that is not in the same direction as the requested vector. For our mission, that is not too much of a danger, since we don't have any elements that would be damaged by Sun exposure or the like. Still, it is a simple matter to scale all 3 vector components by the same factor that would make the largest of them equal to its limit while preserving the direction of the torque vector. Note that this is safe for a PD controller, but that stability problems can result if the same tactic is used for a PID controller without taking some care to prevent problems.

The data from the star camera and the rate gyros in each SCS satellite can be combined in any of a number of ways. We will simply trust the star camera attitude as accurate whenever it gives us data it indicates is valid, and we will use the gyros to provide direct rate measurements and to propagate the attitude solution by integrating the rate over the sampling time if the star camera fails to provide a valid attitude. This method is not the most accurate, but generally, more accuracy comes at the expense of more complexity, which then costs money and time in flight software development and testing. This is a good lesson to wrap up this section: Though it can be tempting to always reach for your best, most expensive tool, it is better engineering to try to get by with the cheapest system that will meet your requirements with appropriate margin.

**References**

Keep all previous references and also include the following (I have other texts and papers to add to this list, but I wanted to get this draft out. I'll send the reference list later.):


Interplanetary spacecraft can get very cold, have little power. May want thrusters only as reaction wheels may seize.

**Gravity Gradient** are both strong in LEO.

Magnetic Field & Gravity Gradient are both strong in LEO.

Gravity gradient too weak at high altitude for good passive stabilization.

Magnetic field too weak at GEO for magnetic torquers.

Geosynchronous Orbit (GEO) 35,786 km

Medium Earth Orbit (MEO) ~2000 km–GEO

Low Earth Orbit (LEO)

Solar Radiation Pressure

Atmospheric Drag

Drag is largest disturbance in very low orbits.

Increasing launch costs. May need thrusters or solid rocket booster to reach orbit.

SRP is only important disturbance far from planets or moons.

Radiation environment varies with altitude.

SRP increases as spacecraft approaches the Sun.