Advances in Adaptive Control Methods

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Constraint-Based Adaptive Control - Optimal Control Modification

Objective
- Introduces notion of constraint-based adaptive control that combines adaptive control with optimal control to achieve constrained error minimization.
- Develops robust optimal control modification adaptive law that enforces linear quadratic constraints.

Technical Challenges
- Persistent excitation (PE) can adversely affect robustness of adaptive control due to high-frequency input signals.
- Nonlinear input-output mapping of adaptive control can result in unpredictable performance.

Technical Approach
- Minimize LQ cost function $J = \lim_{t \to \infty} \frac{1}{2} \int_{0}^{t} [e(t) - A_{w}(t) + B(\theta(t)) \Phi(t) \psi(t)]^T P [e(t) - A_{w}(t)] dt$
  subject to error dynamics $\dot{e}(t) = A_{w}(t) + B(\theta(t)) \Phi(t) \psi(t)$.
- Approach based on application of Pontryagin's Minimum Principle.
- Optimal Control Modification Adaptive Law
  $$\hat{\theta}(t) = -\Phi^T(t) \psi(t) [P + \gamma(t)\Phi^T(t) \psi(t)]^{-1} \gamma(t) \Phi(t) e(t)$$

Asymptotic Linearity for Linear Uncertainty
- Fast adaptation condition $\Phi^T(t) \Phi(t) \psi(t) > \|A_{w}\|^2 > 0$.
- Asymptotic behavior $\gamma(t) \Phi^T(t) \Phi(t) \psi(t) = 1, \text{ as } t \to \infty$.
- Linear tracking error dynamics for linear uncertainty
  $$\dot{e}(t) = -P^{-1} \left( \frac{1}{2} \gamma(t) \Phi(t) \psi(t) - \frac{1}{2} \gamma(t) \Phi(t) \psi(t) \right) \dot{e}(t) - R \Phi \psi(t)$$
- Adaptive law can be designed to guarantee stability for given bound on $R \Phi \psi(t)$ using projection operator such that $A_{w} = P^{-1} \left( \frac{1}{2} \gamma(t) \Phi(t) \psi(t) - \frac{1}{2} \gamma(t) \Phi(t) \psi(t) \right)$ is Hurwitz.
- Note: modification term proportional to persistent excitation (PE) to counter adverse effects of PE.

Example
- 1st-order plant with 2nd-order unmodeled dynamics and input at the same frequency as that of unmodeled dynamics.

Robustness to Unmodeled Dynamics

Asymptotic Input-Output Linear Mapping

Adaptive Control of Time-Delay Systems - Time Delay Margin of MRAC

Objective
- Develops stability analysis for time-delay adaptive system and analytical tool to compute time delay margin (TDM) based on Bounded Linear Stability Analysis.

Technical Challenges
- Currently no analytical tool exists to provide non-conservative and practical TDM estimate.

Technical Approach
- Input-delay adaptive system
  $$\dot{e}(t) = A_{w}(t) + B(t - t_d) + \Phi(t) \psi(t)$$
  $$\psi(t) = -\Phi(t) \psi(t) e(t)$$
- Bounded Linear Stability approximates adaptive system as a locally bounded LTI system using time-window analysis
  $$\tilde{\delta}(t) = -\gamma(t) \Phi(t) \psi(t)$$
  $$\gamma(t) = \frac{1}{2} R \Phi \psi(t)$$
- Locally LTI approximation of tracking error dynamics
  $$\dot{e}(t) = C(t) e(t)$$
- TDM estimation by matrix measure approach - system is locally stable if time delay is less than TDM
  $$\omega < \mu(-J^T C)$$
  $$\omega = \max_{\alpha} \lambda \left( \frac{\gamma^T \alpha}{2} \right) = \lim_{\alpha \to \infty} \frac{\|f - x(0)\|}{\|e(0)\|}$$

Matrix Measure Properties
- $\mu(C) = \max_{\alpha} \lambda \left( \frac{C^T \alpha}{2} \right)$
- $\mu(C) \leq \text{Im} \lambda(C) \leq \mu(C)$
- $\text{Im} \lambda(C) \leq \mu(-J^T C)$

Example
- $\tilde{z}(t) = A_{w}(t) - B \Phi e(t)$
- System is stable if $\tilde{t} < \frac{1}{\omega} \text{cos}^{-1} \left( \frac{\|f\| + \|e(0)\|}{\|e(0)\|} \right)$

Summary
- New analytical method provides non-conservative TDM estimate.
- Method can be easily extended to sigma-modification and optimal control modification.