Advances in Adaptive Control Methods

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Constraint-Based Adaptive Control - Optimal Control Modification

Objective
- Introduces notion of constraint-based adaptive control that combines adaptive control with optimal control to achieve constrained error minimization.
- Develops robust optimal control modification adaptive law that ensures linear quadratic constraints.

Technical Challenges
- Persistent excitation (PE) can adversely affect robustness of adaptive control due to high-frequency input signals.
- Nonlinear input-output mapping of adaptive control can result in unpredictable performance.

Technical Approach
- Minimize LQ cost function $J = \frac{1}{2} \int_{t_0}^{t_f} \left[ \dot{x}(t) - \Delta(t) \right]^T P \left[ \dot{x}(t) - \Delta(t) \right] dt$

subject to error dynamics $\dot{e}(t) = -A_n e(t) + B_T \hat{\phi} e(t) + c(t)$

- Approach based on application of Pontryagin's Minimum Principle
- Optimal Control Modification Adaptive Law
  \[ \hat{\theta}(t) = \gamma(t) \left[ \frac{e(t)}{|e(t)|} P \right] - \gamma(t) \theta(t) B_T P \frac{A_{out}(t)}{B} \]

Lyapunov stability proof shows that the adaptive law is stable and tracking error is UUB.
- Modification term proportional to persistent excitation (PE) to counteract adverse effects of PE

Example
\[ \dot{x} = -x + 2u + 0.1p \]
\[ y = 100u + 100y = 7x \]
\[ u = -0.5x + r - e(t) + u(t) \]

1st-order plant with 2nd-order unmodelled dynamics and input at the same frequency as that of unmodelled dynamics

Robustness to Unmodelled Dynamics

Asymptotic Input-Output Linear Mapping

Adaptive Control of Time-Delay Systems - Time-Delay Margin of MRAC

Objective
- Develops stability analysis for time-delay adaptive system and analytical tool to compute time delay margin (TDM) based on Bounded Linear Stability Analysis

Technical Challenges
- Currently no analytical tool exists to provide non-conservative and practical TDM estimate.

Technical Approach
- Input-delay adaptive system
  \[ \dot{e}(t) = A_n e(t) + B_T \hat{\phi} e(t) + c(t) - \gamma(t) B_T P e(t) \]
  \[ u(t) = -K x(t) - K_x r(t) - u(t) \]

- Asymptotic linearity for linear uncertainty
  \[ \text{Fast adaptation condition } \dot{\hat{\phi}}(t) = \gamma(t) \Phi(t) \Phi(t) = \Phi(t) \Phi(t) > 0 \]

- Asymptotic behavior $\dot{e}(t) = A_n e(t) - \gamma(t) B_T P e(t)$

- Linear tracking error dynamics for linear uncertainty
  \[ \dot{e}(t) = -A_n e(t) - \gamma(t) B_T P e(t) \]

- Bounded Linearity Stability: approximates adaptive system as a locally bounded LTI system using time-window analysis

Matrix Measure Properties
\[ \mu(C) = \max_{\|x\|_2 = 1} \frac{\|C x\|_2}{\|x\|_2} = \lim_{\|x\|_2 \to \infty} \frac{\|C x\|_2}{\|x\|_2} \]

\[ \mu(C) \leq \text{Rn}\{C\} \leq \|C\| \leq \mu(-\mu(C)) \]

Given $\dot{z}(t) = A z(t) + B R e(t - \tau(t))$, $\lambda(A - B K) \in C^+$

System is stable if $\tau(t) \leq \lambda^{-1}(\|\Phi(\bar{\lambda}) + C R\|) + \|B K\|$

Summary
- Optimal Control Modification can provide stable fast adaptation to improve tracking
- Asymptotic linearity with fast adaptation can guarantee linear stability for linear structurally uncertain system
- Pilot-in-the-loop simulations demonstrate effectiveness of the method

Summary
- New analytical method provides non-conservative TDM estimate
- Method can easily be extended to sigma-modification and optimal control modification

10K ft, 250 KIAS
A = 0, B scaled
Doublet to capture flight director task