Microstructural Characterization of Metal Foams:
An Examination of the Applicability of the Theoretical Models for Modeling Foams

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Abstract

Establishing the geometry of foam cells is useful in developing microstructure-based acoustic and structural models. Since experimental data on the geometry of the foam cells are limited, most modeling efforts use an idealized three-dimensional, space-filling Kelvin tetrakaidecahedron. The validity of this assumption is investigated in the present paper. Several FeCrAlY foams with relative densities varying between 3 and 15 percent and cells per mm (c.p.mm.) varying between 0.2 and 3.9 c.p.mm. were microstructurally evaluated. The number of edges per face for each foam specimen was counted by approximating the cell faces by regular polygons, where the number of cell faces measured varied between 207 and 745. The present observations revealed that 50 to 57 percent of the cell faces were pentagonal while 24 to 28 percent were quadrilateral and 15 to 22 percent were hexagonal. The present measurements are shown to be in excellent agreement with literature data. It is demonstrated that the Kelvin model, as well as other proposed theoretical models, cannot accurately describe the FeCrAlY foam cell structure. Instead, it is suggested that the ideal foam cell geometry consists of 11 faces with three quadrilateral, six pentagonal faces and two hexagonal faces consistent with the 3-6-2 Matzke cell. A compilation of 90 years of experimental data reveals that the average number of cell faces decreases linearly with the increasing ratio of quadrilateral to pentagonal faces. It is concluded that the Kelvin model is not supported by these experimental data.

1.0 Introduction

Aircraft engine noise is a major environmental concern especially in regions surrounding an airport during takeoff and landing (Ref. 1). Significant progress has been made since the advent of the first commercial jet engine-powered airplanes with current ultrahigh bypass engines being much quieter than the first generation engines. For example, the effective perceived noise level in decibels (EPNdB) relative to the International Civil Aviation Organization’s (ICAO) Chapter 3 certification standards decreased from about +5 EPNdB for aircraft engines developed in the 1960s to –5 EPNdB for modern engines (Refs. 2 and 3). Despite this large improvement in engine design, there is still a great desire among policy makers and designers to reduce noise much below current levels. For example, the National Aeronautics and Space Administration (NASA) has set ambitious goals to further reduce aircraft noise by –52 db with respect to the newly adapted ICAO’s Chapter 4 certification standards by the year 2020 under its Subsonic Fixed Wing (SFW) project (Ref. 4). It is expected that these noise reduction goals will be achieved through a combination of design changes and development of suitable materials (Refs. 3 and 4).

Polymeric foams have been historically used for sound absorption in several applications (Ref. 5). More recently, metal foams are being investigated for their flow resistance (Refs. 6 and 7) and sound absorption properties (Refs. 8, 9, and 10). Metal foams have been proposed for use in jet engines as acoustic treatment over rotors (Ref. 11), fan blades (Ref. 12) and other applications (Ref. 13). The acoustic and other properties of foams are dependent on their relative density, \( \rho^*/\rho_s \), where \( \rho^* \) and \( \rho_s \) are the densities of the foam and the solid material, respectively, and microstructure (Ref. 5). Simple formulae exist for correlating relative density and some elements of the microstructure, such as, ligament length and thickness (Refs. 5, 6, 7, 8, 9, and 10). However, due to difficulties in controlling process variables, the microstructures of the foams and their properties can vary by large amounts. Although
commercially manufactured foams are specified by pores per inch (p.p.i.) and their relative densities, it is noted that the reported values of p.p.i. are not necessarily identical from one manufacturer to another (Ref. 14). For example, some vendors identify the p.p.i. of their products with that of the precursor polyurethane foam rather than the finished product without accounting for metal shrinkage during the manufacturing process.

In the case of metal foams used as acoustic liners in aircraft engines, it is important to qualitatively and quantitatively understand the role their microstructures play in affecting their acoustic and mechanical properties. Since the complex three-dimensional microstructures of the foams help to dissipate the sound energy, it is evident that a quantitative analysis of the foam microstructures would enable important correlations to be determined between the microstructural features and the gas pressure flow resistance as well as the sound absorption coefficients. These correlations are essential for developing microstructure-based models for designing acoustic liners for aircraft engines. Particularly, establishing the three-dimensional topology of the cell microstructures of foams is important effectively to model fluid flow through them and to understand their mechanical properties.

Modeling activities on foam cell structures fall into two broad categories: (a) idealized topological models based on minimizing the ratio of the surface free energy to volume free energy that can fill three-dimensional (3–D) space; and (b) engineering models based on the actual reconstruction of the 3–D foam microstructures. Among the several possible idealized topological representations of the foam microstructures (Ref. 5), the three-dimensional, space-filling Kelvin tetrakaidecahedron (Refs. 5, 15, and 16) is often favored for modeling the foam cellular network. This cell has 14 faces consisting of six squares and eight hexagonal faces. In other words, about 43 percent of the faces are squares, zero percent faces are pentagonal and 57 percent of the faces are hexagonal. It is worth noting that other topological models have been proposed, where pentagonal faces are incorporated in the cell geometry (Refs. 17 and 18). The Kelvin model assumes that all cells are all of the same size and volume so that the problem becomes one of determining the cell shape that can pack 3–D space resulting in a system with the lowest free energy (Ref. 15). In reality, cells deviate from these ideal conditions, where they may be distorted and their sizes and shapes non-uniform. Alternatively, recent computational models use actual 3–D foam microstructures as an input to the model. However, these models require the availability of high-powered computational capabilities to handle the large megabytes of input data representing the foam microstructures. The input data for these models are expensive to generate, and the models tend to be rather complex. Since foam microstructures are complex, it is necessary to develop both the relatively simple and elegant topological mathematical models, as well as, the complex, but realistic, computational engineering models in order to completely understand the microstructure-property relationships of foams.

Several investigators have tried to evaluate the 3–D shape of fat cells (Ref. 19), soap bubbles (Refs. 20 and 21), grains (Refs. 22, 23, 24, 25, 26, 27, 28, and 29) and foam cells (Refs. 30, 31, 32, and 33). The measurement techniques used in these investigations include conventional microstructural image analysis, serial section metallography, optical and x-ray micro-computerized tomography (µCT), magnetic resonance imaging (MRI), ultrasonic imaging and laser confocal microscopy (Refs. 14 and 34). Most of these procedures have advantages and disadvantages. The well-established quantitative metallography techniques (Refs. 23, 24, 25, 26, 27, 28, 29, 34, and 35) are relatively simple, inexpensive, and provide high resolution images which enable the acquisition of a large amount of statistically relevant data with relative ease. However, these procedures are destructive in nature and the 3–D information of the microstructure can only be inferred from the two-dimensional (2–D) sections using well-developed mathematical and stereological methods (Refs. 24, 25, 34, and 35). The advent of powerful computers and the availability of specialized software with capabilities to reconstruct 3–D images by “stitching” several closely-spaced 2–D images has enabled the recent development of several techniques, such as optical tomography (Ref. 21), MRI (Ref. 30) and µCT (Ref. 32), for accurately reproducing the complex 3–D foam microstructures. The primary advantages of these methods is that the resulting 3–D images along with quantitative information on the foam microstructures can provide a realistic image of the 3–D spatial distribution of the cells. In recent years, µCT is increasingly used to characterize foam microstructures due to the advantages of using 3–D reconstructed images as input to the computational engineering.
models. Despite its relative popularity, it is time consuming, expensive and possesses a lower image resolution than conventional metallographic methods (Ref. 31). Typically, only a few foam cells are sampled so that the statistical sampling size is limited.

The objectives of this investigation were to characterize the microstructures of PORVAIR\(^1\) metal foams. Quantitative information on ligament (or struts) dimensions, cell face dimensions, area fractions of open and closed faces, geometric shapes of the cell faces and distribution of ligament porosity were determined. Specifically, the present paper reports statistical data on the geometrical features of the cells to determine the validity of the Kelvin (Ref. 15) and other theoretical space-filling models (Refs. 16, 17, and 18) in a comprehensive manner. Other quantitative details of the foam microstructure are reported elsewhere (Ref. 36).

2.0 Experimental Procedures

Several FeCrAlY foam panels approximately 210 x 210 mm\(^2\) in cross-sectional area and varying in thicknesses between 3.2 and 25.4 mm were procured from PORVAIR Fuel Cells Technology, Inc., Hendersonville, North Carolina. The foam panels were manufactured from precursor polyurethane foams dipped in metal powder slurries followed by sintering of the powder and burning off the polymer foams. The c.p.mm. varied between 0.2 (5 p.p.i.) and 3.9 (100 p.p.i.), whereas $\rho^b/\rho_S$ varied between 3 and 15 percent. Square specimens ~ 25.4 x 25.4 mm in cross-sectional dimensions or 50 mm in diameter were wire electro-discharge machined from these panels for metallographic analyses. On close examination, it was observed that the microstructures of these foams are extremely complicated and difficult to characterize. The foam microstructures consisted of interconnected cells randomly stacked in a three-dimensional array with the cell boundaries moving in and out of the field of view.

Preliminary attempts to study the shapes of the foam cells using either μCT with resolutions varying between 20 and 100 μm or an automated serial sectioning\(^2\) of a FeCrAlY foam specimen and the subsequent 3–D reconstruction of the 2–D sectioned images proved to be unsatisfactory since the cell outline could not be clearly defined in the images. Instead, macrophotographs were obtained of the as-received foam specimens (Fig. 1(a)). This technique allowed a 3–D visualization of the foam microstructure with several adjacent faces of a cell being clearly demarcated (Fig. 1(b)). It is noted that Figure 1(a) is similar to the 3–D reconstructed image of polymer foams (Ref. 31) except that the present imaging technique is faster and cheaper. Quantitative metallographic measurements were conducted on 6 to 7 randomly selected areas for each foam specimen and a large number of faces were measured to ensure that the measurements were representative and to minimize measurement errors. The number of edges per face was counted by assuming that the faces could be approximated by regular polygons with the number of cell faces measured varying between 207 for foams 0.2 c.p.mm. to 745 for 3.9 c.p.mm. This assumption was not always valid since some faces were either circular or elliptical rather than polygonal and the edges were often curved. In some instances, the edges of a face curved out of the plane of view. In addition, two adjacent edges did not meet always at a relatively sharp point but had a significant curvature, while adjacent faces met at triple surfaces rather than triple points in many instances. These issues complicated the measurements and they are likely to add to the errors in measurements. Nevertheless, by measuring a large number of faces, it was felt that the errors in measurement would be minimized. It is noted that a similar method was used by Montminy et al. (Ref. 31) to analyze 3–D μCT images.

\(^1\)PORVAIR is the trademark of PORVAIR Fuel Cells Technology, Hendersonville, North Carolina.

\(^2\)The automated sectioning of the FeCrAlY foams and the 3–D image reconstruction was conducted by UES, Inc., Dayton, Ohio.
3.0 Results and Discussion

Figures 1(a) shows an optical macrograph of a FeCrAlY foam specimen with a nominal cell density of 0.2 c.p.mm. (5 p.p.i.) and $\rho^*/\rho_S = 3.3$ percent; Figure 1(b) shows the corresponding polygonal representations of the faces. The numbers identify the faces for tracking purposes. The complex nature of the foam microstructures is self evident in these figures. On close examination, it was observed that several neighboring faces were part of the same cell. For example, the faces numbered 1, 2, 3 and 4...
enclosed by the broken circle represent the outer faces a single cell with some of the inner faces of the cell visible in the background (Fig. 1(b)). The volume fractions of the open cells decreased while that of the closed cells increased with increasing relative density. Since it was often difficult to clearly discern the boundaries of closed faces, only the shapes of the open faces were demarcated in these measurements in order to minimize errors in measurement. The cells were generally equiaxed irrespective of c.p.mm. and relative density.

Figures 2(a) to (d) show the frequency histogram and cumulative frequency plots of the number of edges per face for four FeCrAlY foams. An examination of Figures 2(a) to (d) clearly establishes that 97 percent of the faces were either four, $n_4$, five, $n_5$, or six, $n_6$, -sided with over 50 percent of the faces being five-sided. Less than 1 percent of the faces were triangular and less than 2 percent were heptagonal except in the case of foams with 2.4 c.p.mm (60 p.p.i), which had about 4 percent heptagonal faces. The average values of the number of edges per face, $\bar{N}$, were determined to be $4.9 \pm 0.7$, $5.0 \pm 0.8$, $4.9 \pm 0.8$, and $4.9 \pm 0.8$ for the FeCrAlY foams with actual values of $\rho*/\rho_S$ being 3.3 percent (0.2 c.p.mm.), 9.5 percent (2.4 c.p.mm.), 10.1 percent (3.1 c.p.mm.) and 9.3 percent (3.9 c.p.mm.), respectively. The errors represent 95 percent confidence levels. Significantly, these observations were not influenced by either the relative densities of the foams or the lineal cell densities.
Figure 3.—Comparison of the frequency histograms of the distributions
of the number of edges per face for soap bubbles (Refs. 20 and 21),
 polyurethane foams (Refs. 30 and 31), Ni foam (Refs. 32) and a
FeCrAlY foam with 0.2 c.p.mm. (5 p.p.i.) and $\rho_s = 3.3$ percent.
The solid squares and associated legends represent the theoretical
values for the Kelvin tetrakaidecahedron.

Figure 3 compares the present results with similar measurements on soap bubbles (Refs. 20 and 21),
polyurethane foams (Refs. 30 and 31) and Ni foam (Ref. 32). These literature data include measurements
conducted on both surface and internal cells using different measurement techniques. Table 1 compares
the percentages of four, five, and six-sided faces observed on the FeCrAlY foams with those reported for
fat cells (Ref. 19), soap bubbles (Refs. 20, 21, and 22), $\beta$-brass grains (Ref. 22), and foams (Refs. 22, 30,
31, and 32). It is noted that the data compiled in Table 1 were obtained by several different techniques
ranging from simple visual observations to complex NMR and µCT 3–D scans over a 90-year period.
Significantly, in all cases, more than 50 percent of the cell faces had a pentagonal geometry irrespective
of the material and measuring technique used (Table 1 and Fig. 3). The present results fall well within the
range of other observations reported in the literature.

An examination of Figure 3 shows that the Kelvin tetrakaidecahedron model (Ref. 15), which predicts
0 percent five-sided faces, is inconsistent with the experimental observations. The fact that the Kelvin
model fails to be consistent with the experimental results is not surprising. This model is based on a
mathematical conjecture that soap bubbles and foam microstructures can be ideally represented by
dividing three-dimensional space into cells of equal volume in a manner that follows Plateau’s rules for
mechanical equilibrium and minimization of the surface area (Ref. 37). It is noted that the Kelvin model
requires the arrangement of tetrakaidecahedron cells to be topologically ordered and spatially periodic to
fill space. Real foams are far from this ideal configuration since factors, such as residual stresses due to
processing methods, topological disorder (Ref. 37), unequal cell volumes, aperiodic spatial ordering of
the cells (Ref. 20), and thick ligaments and triple points, can influence the cell topology. Matzke (Ref. 20)
studied 400 peripheral soap bubbles and observed that the largest number of them possessed 11-hedra
3This nomenclature of identifying the cells was suggested by Kraynik et al. (Ref. 37).
sided. Therefore, it would be interesting to determine the number of faces for the ideal cell representing the microstructures of the FeCrAlY foams.

<table>
<thead>
<tr>
<th>Description</th>
<th>Measurement technique</th>
<th>Percentage of polyhedral faces, ( n_4, n_5 ) and ( n_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fat cells (Lewis (Ref. 19))</td>
<td>Optical microscopy or visual</td>
<td>( n_4 = 21%; n_5 = 53%; n_6 = 23% )</td>
</tr>
</tbody>
</table>
| Soap bubbles (Matzke (Ref. 20)) | Optical microscopy | Peripheral: \( n_4 = 29\%; n_5 = 53\%; n_6 = 16\% \);
| | | Central: \( n_4 = 11\%; n_5 = 67\%; n_6 = 22\% \) |
| Soap bubbles (Monnereau et al. (Ref. 21)) | Optical tomography | Upper bubbles: \( n_4 = 29\%; n_5 = 52\%; n_6 = 18\% \);
| | | Internal bubbles: \( n_4 = 18\%; n_5 = 58\%; n_6 = 24\% \) |
| β-brass grains | Visual | \( n_4 = 20\%; n_5 = 44\%; n_6 = 28\% \) |
| Soap bubbles | | \( n_4 = 20\%; n_5 = 50\%; n_6 = 22\% \) |
| Ammonium oleate foams | | \( n_4 = 21\%; n_5 = 50\%; n_6 = 25\% \) |
| Gelatin foams (Desch (Ref. 22)) | | \( n_4 = 19\%-38\%; n_5 = 32\%-57\%; n_6 = 10\%-25\% \) |
| Polyurethane foams (Kose (Ref. 30)) (Montminy et al. (Ref. 31)) | NMR | \( n_4 = 9\%; n_5 = 70\%; n_6 = 21\% \) |
| | \( \mu \text{CT} \) | \( n_4 = 24\%; n_5 = 55\%; n_6 = 19\% \) |
| Open cell Ni foam (Dillard et al. (Ref. 32)) | \( \mu \text{CT} \) | \( n_4 = 18\%; n_5 = 57\%; n_6 = 22\% \) |
| FeCrAlY foams (Present investigation) | Optical microscopy | \( n_4 = 25\%; n_5 = 57\%; n_6 = 15\% \)
| | | \((0.2 \text{ c.p.mm.; } \rho*/\rho_s = 3.3\%)\) |
| | | \( n_4 = 24\%; n_5 = 54\%; n_6 = 18\% \)
| | | \((2.4 \text{ c.p.mm.; } \rho*/\rho_s = 9.5\%)\) |
| | | \( n_4 = 28\%; n_5 = 52\%; n_6 = 18\% \)
| | | \((3.1 \text{ c.p.mm.; } \rho*/\rho_s = 10.1\%)\) |
| | | \( n_4 = 26\%; n_5 = 50\%; n_6 = 22\% \)
| | | \((3.9 \text{ c.p.mm.; } \rho*/\rho_s = 9.3\%)\) |

Figure 4.—A 3-6-2 11-hydra cell with three quadrilateral, six pentagonal and two hexagonal faces (Ref. 20). The numbers represent the number of edges enclosing the cell face. The blue solid lines representing the forward faces are identified by the blue lettering, while the red broken lines representing the back faces are identified by the red lettering.
Since quantitative optical metallography gives 2–D information, the 3–D topographical characteristics of the microstructure can be determined from well established stereology equations (Refs. 23, 24, 25, 26, 27, 28, 29, 34, 35, 38, 39, and 40). The number of faces per cell, \( F \), the number of edges per cell, \( E \), and the number of vertices per cell, \( V \), of the 3–D cell are related by the Euler equation (Refs. 5, 25, 34, and 39) and they can be determined from \( \bar{N} \) using the Coxeter equations (Ref. 41)

\[
F = \frac{12}{6 - \bar{N}} \quad (1a)
\]

\[
E = \frac{6\bar{N}}{6 - \bar{N}} \quad (1b)
\]

\[
V = \frac{4\bar{N}}{6 - \bar{N}} \quad (1c)
\]

Table 2 shows the calculated values of \( F, E, V \), and the corresponding experimental values of \( N_4, N_5 \) and \( N_6 \) for the four FeCrAlY foams.\(^4\) Using the measured values of \( \bar{N} \), the corresponding values of \( F \) calculated from equation (1a) are 11.0, 11.7, 11.1 and 11.4 for foams with 0.2 (5 p.p.i.), 2.4 (60 p.p.i.), 3.1 (80 p.p.i.) and 3.9 c.p.mm. (100 p.p.i.), respectively. Based on these results, the topological characteristics of the ideal PORVAIR foam cell are: \( F = 11, E = 27 \) and \( V = 18 \), which satisfy Euler’s theorem (i.e., \( V - E + F = 2 \)) with \( N_4 = 3 \), \( N_5 = 6 \) and \( N_6 = 2 \). These values are independent of relative density.

**Table 2.—Calculated Values of \( F, E, V, N_4, N_5 \) and \( N_6 \) for FeCrAlY Foams**

<table>
<thead>
<tr>
<th>Linear cell density (c.p.mm)</th>
<th>( \rho^*/\rho_s ) (%)</th>
<th>( \rho^* )</th>
<th>( F )</th>
<th>( E )</th>
<th>( V )</th>
<th>( N_4 )</th>
<th>( N_5 )</th>
<th>( N_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 (5 p.p.i.)</td>
<td>3.3</td>
<td>11.0</td>
<td>26.7</td>
<td>17.8</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2.4 (60 p.p.i.)</td>
<td>9.5</td>
<td>11.7</td>
<td>30.0</td>
<td>20.0</td>
<td>3</td>
<td>6 or 7</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3.1 (80 p.p.i.)</td>
<td>10.1</td>
<td>11.1</td>
<td>26.7</td>
<td>17.8</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3.9 (100 p.p.i.)</td>
<td>9.3</td>
<td>11.4</td>
<td>26.7</td>
<td>17.8</td>
<td>3</td>
<td>6</td>
<td>2 or 3</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>11.3</td>
<td>27.5</td>
<td>18.4</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3 compares the topological features of the FeCrAlY foams with several simple cell shapes (Ref. 5), where \( C \) is the number of cells. The topological characteristics of the FeCrAlY foams do not agree with any of these simple geometries. Instead, they appear to be closer to the topological structure of clathrates although more detailed topological modeling needs to be conducted to establish this possibility (Refs. 42 and 43). As noted above, Matzke (Ref. 20) observed that most of the peripheral soap bubbles were eleven-hedra cells with three four-sided, six five-sided and two six-sided faces (3-6-2). Based on the excellent agreement between the present results and Matzke’s data on peripheral soap bubbles (Ref. 20) (Fig. 3) taken together with the fact that the total number of faces for the FeCrAlY foams was determined to be 11 (Table 2), it is reasonable to suggest that the 11-hedra 3-6-2 cell is the most representative of the FeCrAlY foam cellular structure.

Table 4 shows the predicted (Refs. 15, 16, 17, and 18) and the experimental (Refs. 20, 21, and 30) percentage distributions of polyhedral faces and the average number of faces per cell, \( F_{\text{average}} \). As noted earlier, the data were obtained by different methods on several materials over a 90-year period. The average value of \( F = 11.3 \) determined for the FeCrAlY foam cells (Table 2) is in very good agreement with the experimental observations on the peripheral (Ref. 20) or upper (Ref. 21) soap bubbles and gelatin foams (Ref. 22) for which the average number of faces is about 11.

\(^4\)In this paper, \( n_i \) represents the percentage of faces with \( i \) edges, whereas \( N_i \) is number of such faces enclosing the cell.
that current theoretical models agree incorrectly assume that three topological models (Refs. 15, 16, 17, and 18). The Kelvin cell (Ref. 15) does not possess any \( \Delta G \), where,

\[
\Delta G = (\Delta g_v + \Delta g_e) \cdot V_C + \Delta g_s \cdot S_C
\]

(2)

where, \( \Delta G \), \( \Delta g_v \), \( \Delta g_e \) and \( \Delta g_s \) are the changes in the total, volume, residual strain and surface Gibbs free energies, respectively, \( V_C \) is the cell volume and \( S_C \) is the surface area of the cell. It is important to note that current theoretical models agree incorrectly assume that \( \Delta g_s = 0 \) for real foams.

A close examination of Table 4 reveals that the present results do not agree with the predictions of the three topological models (Refs. 15, 16, 17, and 18). The Kelvin cell (Ref. 15) does not possess any pentagonal faces, whereas the Weaire-Phelan model (Refs. 16, 18, and 30) does not have any quadrilateral faces, with the total number of faces being either 14 or 13.4, respectively. The Williams cell (Ref. 17) with 14 faces possesses 14 percent quadrilateral, 57 percent pentagonal and 29 percent hexagonal faces. However, this model also does not agree with the present observations on the FeCrAlY foams. This difference between the experimental results and the theoretical predictions is to be expected since theoretical efforts mainly consider the surface and volume free energy contributions to the total free energy (Ref. 37). As indicated earlier, other factors can influence the final cell topology of real foams. For example, the effects of residual stresses developed in the foam panels during processing have not included in these theoretical derivations. Qualitatively, one can modify the Gibbs free energy equation as follows:

\[
\Delta G = (\Delta g_v + \Delta g_e) \cdot V_C + \Delta g_s \cdot S_C
\]
Table 4 shows that $F_{\text{average}}$ varied between 9.0 and 14.5 (Refs. 20, 21, 22, and 30). On further examination of the data, $F_{\text{average}}$ decreases linearly with the increasing ratio, $n_4/n_5$, (Fig. 5(a))

$$F_{\text{average}} = -5.1\left(\frac{n_4}{n_5}\right) + 14.3 \quad (R_d^2 = 0.461) \quad (3)$$

where $R_d^2$ is the coefficient of determination. In contrast, it is independent of $n_6/n_5$ (Fig. 5(b)). The regression Equation (3) is represented by the solid line in Figure 5(a); the broken horizontal line in Figure 5(b) represents the average value of $F_{\text{average}} = 12.2$ for all the data. Equation (3) predicts a value of $F_{\text{average}} = 14.3$ for $n_4 = 0$, $F_{\text{average}} = 9.2$ for $n_4 = n_5$, and $F_{\text{average}} = \infty$ for $n_5 = 0$. Thus, the Kelvin model, for which $n_5 = 0$, is not supported by the trend in the experimental data.

Figure 5.—Variation of the average number of cell faces against the ratio of (a) quadrilateral to pentagonal, and (b) hexagonal to pentagonal faces. The regression line through the data is represented by the solid line in (a). The present data on FeCrAlY foams are compared with literature data (Refs. 20, 21, 22, 30, 31, and 32).
4.0 Summary and Conclusions

A detailed microstructural analysis of several FeCrAlY metal foams with relative densities varying between 3 and 15 percent, and linear cell densities varying between 0.2 and 3.9 c.p.mm., was conducted to evaluate the topology of the foam cells. The shapes of cell faces were evaluated by approximating the faces by regular polygons. It was observed that between 24 and 28 percent of the cell faces were quadrilateral, 50 to 57 percent pentagonal, and 15 to 22 percent hexagonal in morphology. The present results are in excellent agreement with observations on soap bubbles (Refs. 20 and 21). Based on Matzke’s observations (Ref. 20), it is suggested that the FeCrAlY foam cells had a total of 11 faces with three quadrilateral, six pentagonal and two hexagonal faces. Both sets of results do not agree with the 14-hedra Kelvin tetrakaidecahedron model (Ref. 15), which only has 43 and 57 percent quadrilateral and hexagonal faces, respectively. Neither do the present results agree with the Williams (Ref. 17) and Weaire-Phelan models (Refs. 16, 18, and 30) models. The present calculations show that the 3-6-2 cell, which probably best describes the FeCrAlY foam cells, has 27 edges and 18 vertices. A compilation of 90 years of experimental data reveals that the average number of cell faces decreases linearly with the increasing ratio of quadrilateral to pentagonal faces. It is concluded that the Kelvin model is not supported by these experimental data.

References

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Microstructural Characterization of Metal Foams: An Examination of the Applicability of the Theoretical Models for Modeling Foams

Establishing the geometry of foam cells is useful in developing microstructure-based acoustic and structural models. Since experimental data on the geometry of the foam cells are limited, most modeling efforts use an idealized three-dimensional, space-filling Kelvin tetrakaidecahedron. The validity of this assumption is investigated in the present paper. Several FeCrAlY foams with relative densities varying between 3 and 15 percent and cells per mm (c.p.mm.) varying between 0.2 and 3.9 c.p.mm. were microstructurally evaluated. The number of edges per face for each foam specimen was counted by approximating the cell faces by regular polygons, where the number of cell faces measured varied between 207 and 745. The present observations revealed that 50 to 57 percent of the cell faces were pentagonal while 24 to 28 percent were quadrilateral and 15 to 22 percent were hexagonal. The present measurements are shown to be in excellent agreement with literature data. It is demonstrated that the Kelvin model, as well as other proposed theoretical models, cannot accurately describe the FeCrAlY foam cell structure. Instead, it is suggested that the ideal foam cell geometry consists of 11 faces with three quadrilateral, six pentagonal faces and two hexagonal faces consistent with the 3-6-2 Matzke cell. A compilation of 90 years of experimental data reveals that the average number of cell faces decreases linearly with the increasing ratio of quadrilateral to pentagonal faces. It is concluded that the Kelvin model is not supported by these experimental data.