Ultimate Temperature of Pulse Tube Cryocoolers

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Consultant

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Introduction

- Ideal pulse tube cooler with real gas
  - No losses (entropy generation) except in orifice / inertance tube

- $T_{min}$; result of real gas properties
Regenerator Thermodynamics

Constraint: \( dT = 0 \)

General expression  Ideal gas  Real gas

\[ ds = c_P \frac{dT}{T} - R \frac{dP}{P} \]

\[ ds = (\frac{V}{T}) \ dP \]

\[ ds = -V \beta \ dP \]

\[ dh = c_P \ dT + [1 - T \beta] \ V \ dP \]

\[ dh = 0 \]

\[ dh = [1 - T \beta] \ V \ dP \]

Volume expansivity: \( \beta = \frac{1}{V} \frac{dV}{dT} \bigg|_P \)

Ideal gas: \( T \beta = 1 \)
## Pulse Tube Thermodynamics

**Constraint:** \( ds = 0 \)

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<thead>
<tr>
<th>General expression</th>
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<th>Real gas</th>
</tr>
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<tbody>
<tr>
<td>( ds = c_P , dT/T - R , dP/P )</td>
<td>( ds = 0 )</td>
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<tr>
<td>( dT/dP = V/c_P )</td>
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<td>( dh = c_P , dT + [1 - T , \beta]V , dP )</td>
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Volume expansivity: \( \beta = 1/V \, dV/dT \big|_P \)

Ideal gas: \( T \, \beta = 1 \)
Cooling Power

Change in Enthalpy flow at cold heat exchanger

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<td>( dq = \Delta dh )</td>
<td>( dq = V , dP )</td>
<td>( dq = T , \beta , V , dP )</td>
</tr>
<tr>
<td>minimum ( T )</td>
<td>( T_{min} = 0 )</td>
<td>when ( \beta = 0 )</td>
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<td></td>
<td></td>
<td>( (dV/dT</td>
</tr>
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Volume expansivity: \( \beta = 1/V \, dV/dT|_P \)

Ideal gas: \( T \, \beta = 1 \)
Calculated Loci of $\beta = 0$

ref: HE3PAK v1.2, HEPAK v3.4, and HEPAK v4a
Meaning of $\beta \neq 0$?

- $\beta > 0 \implies$ Conventional PT
  - Can PT operate below 0.5 K?

- $\beta < 0 \implies$ What does this mean?

- Effect of mixing $^3\text{He}$ and $^4\text{He}$?
\( \beta \) of \(^3\)He and \(^4\)He

- **Volume Expansivity vs. Temperature**
  - For \(^3\)He and \(^4\)He at different pressures (0.1 MPa, 0.5 MPa, 1.0 MPa, 2.0 MPa).
  - Graphs show the change in volume expansivity with temperature for each isobaric condition.
\( \beta \) of \(^3\text{He}\) and \(^4\text{He}\)

\[ \begin{align*}
\beta < 0: & \text{ reverse enthalpy flow or } \Delta \Phi \text{ between } \dot{m} \text{ and } P \text{ by } \sim 180^\circ \\
\beta_3 < 0, \ T < 1 \text{ K}: & \beta \text{ is small} \\
\beta_4 > 0, \ T < 1 \text{ K}: & \beta \text{ is very small} \quad \text{to small to be useful}
\end{align*} \]
$\beta = 0$; Mixed $^3\text{He} / ^4\text{He}$

- all data at SVP
- limited data in He-I region
  $\approx$ straight line fit
- more data in He-II region
- lower $\beta > 0$ region
  over estimated

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Esel'son, B.N., et.al.,
*Solutions of He$^3$ - He$^4$ Quantum Liquids*,

Ebner, C. and Edwards, D.O.,
“The Low Temperature Thermodynamic Properties of Superfluid Solutions of $^3\text{He}$ in $^4\text{He}$,”
Physics Reports 2, pp. 77-154 (1971)
Los Alamos superfluid pulse tube

3He in Superfluid 4He

- 2-fluid behavior
  - \( T < 1 \text{ K} \): 3He low-density gas moving in fixed 4He background
  - Los Alamos pulse tube cooler
    - filled with 17% 3He and operated between 1 K and 0.6 K
  - Compressor does not cause pressure oscillations
    - causes the 3He concentration, \( x_3 \), and the osmotic pressure, \( \Pi_3 \), to oscillate.
    - in regenerator, heat exchangers, and orifices
      - \( \nabla P \) replaced by \( \nabla \Pi_3 \)
    - In the pulse tube
      - constraint that \( \nabla P = 0 \) is replaced by \( \nabla \mu_4 = 0 \)
      - \( \mu_4 \) is the chemical potential of the 4He.
Loci of Constant $\mu_4$

- Dashed line: approx operation of Los Alamos cooler

- $T_{\text{min}} = \text{phase separation}$

- Lowest $T \Rightarrow$
  
  $x_3 < 6.4 \% \ @ \text{cold hx}$

  $x_3 < 1 \% \ @ 0.6 \text{ K}$

  - Low density of $^3\text{He}$ limits the mass flow and cooling in practical cooler

Radebaugh, R., “Thermodynamic Properties of He$^3$-He$^4$ Solutions with Applications to the He$^3$-He$^4$ Dilution Refrigerator,” NBS TN 362 (1967)
Summary

- Below \(\approx 1\) K, \(^3\text{He}\) concentration driven pulse tube
  - possible with no known ultimate limiting temperature
  - lack of thermodynamic data at very low \(T\)

- Limit of conventional pulse tube cryocoolers: \(\beta = 0\)
  - \(^4\text{He}\) limit \(T >\approx 2.2\) K
  - \(^3\text{He}\) limit \(T >\approx 1\) K
  - mixture of \(^4\text{He}\) and \(^3\text{He}\):
    - limit adjustable
  - mixing ratio not constant throughout the cooler
    - because \(\mu_4(T, P)\)
  - \(T_{\text{min}}\) depends on the mixing ratio at the cold hx