Ultimate Temperature of Pulse Tube Cryocoolers

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Consultant

This work was funded through University Affiliated Research Center (UARC) Subcontract S0181769. UARC is managed by the University of California, Santa Cruz under NASA Ames Research Center Contract NAS2-03144
Introduction

- Ideal pulse tube cooler with real gas
  - No losses (entropy generation) except in orifice / inertance tube

- $T_{\text{min}}$; result of real gas properties
## Regenerator Thermodynamics

**Constraint:** \( dT = 0 \)

<table>
<thead>
<tr>
<th>General expression</th>
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<th>Real gas</th>
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<tr>
<td>( ds = c_P \frac{dT}{T} - R \frac{dP}{P} )</td>
<td>( ds = \left( \frac{V}{T} \right) dP )</td>
<td>( ds = -V \beta dP )</td>
</tr>
<tr>
<td>( dh = c_P \frac{dT}{T} + \left[ 1 - T \beta \right] V \frac{dP}{T} )</td>
<td>( dh = 0 )</td>
<td>( dh = \left[ 1 - T \beta \right] V dP )</td>
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**Volume expansivity:** \( \beta = \frac{1}{V} \frac{dV}{dT} \bigg|_P \)

**Ideal gas:** \( T \beta = 1 \)
# Pulse Tube Thermodynamics

**Constraint:** \( ds = 0 \)

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<td>( dT/dP = V/c_P )</td>
<td>( dT/dP = T \beta \frac{V}{c_P} )</td>
<td></td>
</tr>
<tr>
<td>( dh = c_P dT + [1 - T \beta]V dP )</td>
<td>( dh = V , dP )</td>
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**Volume expansivity:** \( \beta = 1/V \frac{dV}{dT} \bigg|_P \)

**Ideal gas:** \( T \beta = 1 \)
## Cooling Power

Change in Enthalpy flow at cold heat exchanger

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<td>( dq = \Delta dh )</td>
<td>( dq = V , dP )</td>
<td>( dq = T \beta , V , dP )</td>
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<tr>
<td>minimum ( T )</td>
<td>( T_{\text{min}} = 0 )</td>
<td>when ( \beta = 0 ) (( \frac{dV}{dT}</td>
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Volume expansivity: \( \beta = \frac{1}{V} \frac{dV}{dT}|_P \)

Ideal gas: \( T \beta = 1 \)
Calculated Loci of $\beta = 0$

ref: HE3PAK v1.2, HEPAK v3.4, and HEPAK v4a
? Meaning of $\beta \neq 0$ ?

- $\beta > 0 \Rightarrow$ Conventional PT
  - Can PT operate below 0.5 K ?

- $\beta < 0 \Rightarrow$ What does this mean ?

- Effect of mixing $^3\text{He}$ and $^4\text{He}$ ?
$\beta$ of $^3$He and $^4$He

![Graph showing the volume expansivity as a function of temperature for $^3$He and $^4$He at different pressures.](image)

- $^3$He: Black line for 0.1 MPa, red line for 0.5 MPa, orange line for 1.0 MPa, green line for 2.0 MPa.
- $^4$He: Black line for 0.1 MPa, red line for 0.5 MPa, orange line for 1.0 MPa, green line for 2.0 MPa.

Temperature [K] vs. Volume Expansivity [K$^{-1}$].
\[\beta\] of \(^3\text{He}\) and \(^4\text{He}\)

- \(\beta < 0\): reverse enthalpy flow or \(\Delta \Phi\) between \(\dot{m}\) and \(P\) by \(\sim 180^\circ\)
- \(\beta_3 < 0, T < 1\text{ K}: \beta\) is small
- \(\beta_4 > 0, T < 1\text{ K}: \beta\) is very small

\begin{align*}
\text{Volume Expansivity [K}^{-1}] & \quad \text{Volume Expansivity [K}^{-1}] \\
0.1 \text{ MPa} & \quad 0.1 \text{ MPa} \\
0.5 \text{ MPa} & \quad 0.5 \text{ MPa} \\
1.0 \text{ MPa} & \quad 1.0 \text{ MPa} \\
2.0 \text{ MPa} & \quad 2.0 \text{ MPa} \\
\end{align*}
$\beta = 0$; Mixed $^3\text{He} / ^4\text{He}$

- all data at SVP
- limited data in He-I region
  $\approx$ straight line fit
- more data in He-II region
- lower $\beta > 0$ region over estimated

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Esel'son, B.N., et.al.,
*Solutions of He$^3$ - He$^4$ Quantum Liquids*,

Ebner, C. and Edwards, D.O.,
"The Low Temperature Thermodynamic Properties of Superfluid Solutions of $^3\text{He}$ in $^4\text{He}$,"
Physics Reports 2, pp. 77-154 (1971)
Los Alamos superfluid pulse tube

3He in Superfluid 4He

- 2-fluid behavior
  - $T < 1$ K: 3He low-density gas moving in fixed 4He background
  - Los Alamos pulse tube cooler
    - filled with 17% 3He and operated between 1 K and 0.6 K
  - Compressor does not cause pressure oscillations
    - causes the 3He concentration, $x_3$, and the osmotic pressure, $\Pi_3$, to oscillate.
    - in regenerator, heat exchangers, and orifices
      - $\nabla P$ replaced by $\nabla \Pi_3$
    - In the pulse tube
      - constraint that $\nabla P = 0$ is replaced by $\nabla \mu_4 = 0$
      - $\mu_4$ is the chemical potential of the 4He.
Loci of Constant $\mu_4$

- Dashed line: approx operation of Los Alamos cooler

- $T_{\text{min}}$ = phase separation

- Lowest $T \Rightarrow$
  $x_3 < 6.4$ % @ cold hx
  $x_3 < 1$ % @ 0.6 K

  - Low density of $^3$He limits the mass flow and cooling in practical cooler

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Radebaugh, R., “Thermodynamic Properties of He$^3$-He$^4$ Solutions with Applications to the He$^3$-He$^4$ Dilution Refrigerator,” NBS TN 362 (1967)
Summary

- Below $\approx 1$ K, $^3$He concentration driven pulse tube
  - possible with no known ultimate limiting temperature
  - lack of thermodynamic data at very low $T$

- Limit of conventional pulse tube cryocoolers: $\beta = 0$
  - $^4$He limit $T > \approx 2.2$ K
  - $^3$He limit $T > \approx 1$ K.
  - mixture of $^4$He and $^3$He:
    - limit adjustable
    - mixing ratio not constant throughout the cooler
      - because $\mu_4 (T, P)$
    - $T_{\text{min}}$ depends on the mixing ratio at the cold hx