Detached Eddy Simulations of Hypersonic Transition

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Objective

• Investigate the feasibility of using CFD in general, DES in particular, for prediction of roughness-induced boundary layer transition to turbulence and the resulting increase in heat transfer

› Threat of overheating the delicate RCC wing leading edges was real.
Hardware

- NASA Advanced Supercomputing Facility enables fine-grid unsteady turbulent flow simulations.

Software

- US3D code – Unstructured, parallel, finite-volume Navier-Stokes solver for thermo-chemical non-equilibrium hypersonic flows
- Detached Eddy Simulations (or WMLES) – Hybrid RANS/LES approach
- Optional low-dissipation numerics
### Navier-Stokes equations for a mixture of species in conservation law form

\[
\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \left( \mathbf{F} - \mathbf{F}_i \right) = \mathbf{W}
\]

### Steger-Warming Flux Vector Splitting

\[
\mathbf{F} = \mathbf{F}^+ + \mathbf{F}^- = R^{-1} \Lambda^+ \mathbf{R} \mathbf{U} + R^{-1} \Lambda^- \mathbf{R} \mathbf{U}
\]

\[
F_{i+1/2} = (R^{-1} \Lambda^+ R)_i U_i + (R^{-1} \Lambda^- R)_{i+1} U_{i+1}
\]

At a cell face

### Modified Steger-Warming Flux Vector Splitting

\[
F_{i+1/2} = (R^{-1} \Lambda^+ R)_{i+1/2} U_i + (R^{-1} \Lambda^- R)_{i+1/2} U_{i+1}
\]

\[
\Lambda^+ = (\Lambda \pm |\Lambda|)/2
\]

### Modified Steger-Warming Flux

\[
F_{i+1/2} = \frac{1}{2} (R^{-1} \Lambda R)_{i+1/2} (U_{i+1} + U_i) + \frac{1}{2} (R^{-1} |\Lambda| R)_{i+1/2} (U_{i+1} - U_i)
\]

\[\wedge\]

convection

\[\wedge\]

dissipation

#### Baseline flux scheme

- **Original Steger-Warming Flux**
- **Roe Flux**

For shock waves

For smooth flows

Using a pressure dependent weight

\[
W_{i+1/2} = 1 - \frac{1}{2} \left( \frac{1}{(S\rho)^2 + 1} \right)
\]

High-order fluxes are obtained using the MUSCL approach with a weighted least-squares method to compute gradients.
Low Dissipation Flux

- Existing low-dissipation schemes to predict small turbulent eddies:
  - Skew-symmetric form to reduce the amplitude of the aliasing errors
  - Entropy function for secondary conservation law
- Present approach: Kinetic energy
  - Combined with internal energy: \( pE = \rho C_v T + \frac{D}{2} u^2 \)

**Kinetic energy consistent flux**

\[
\sum_i (\rho k u' S) = \sum_i \frac{1}{2} (u_i u_{i+1} + v_i v_{i+1} + w_i w_{i+1}) (\rho u' S)_i
\]

**Dissipative term from the modified Steger-Warming flux**

\[
d_{i+1/2} = \alpha \frac{1}{2} (R^+ |A| R)_{i+1/2} (U_{i+1} - U_i)
\]

\[
\alpha = \frac{(\nabla \cdot \mathbf{u})^2}{(\nabla \cdot \mathbf{u})^2 + ||\mathbf{\omega}||^2}
\]

Implicit Time Integration

**Point Relaxation**

\[
(I + A^+_{i+1/2,j} - A^-_{i-1/2,j} + B^+_{i,j+1/2} - B^-_{i,j-1/2}) \delta U_{i,j} = H_{i,j}
\]

\[
+ A^+_{i+1/2,j} \delta U_{i+1,j} - A^-_{i+1/2,j} \delta U_{i-1,j} + B^+_{i,j+1/2} \delta U_{i,j+1} - B^-_{i,j+1/2} \delta U_{i,j-1} - B^-_{i,j+1/2} \delta U_{i,j-1}
\]

**Line Relaxation**

\[
B^+_{i,j+1/2} \delta U_{i,j+1} + (I + A^+_{i+1/2,j} - A^-_{i-1/2,j} + B^+_{i,j+1/2} - B^-_{i,j-1/2}) \delta U_{i,j} = H_{i,j}
\]

\[- A^+_{i+1/2,j} \delta U_{i+1,j} + A^-_{i+1/2,j} \delta U_{i,j} + B^+_{i,j+1/2} \delta U_{i,j+1} - B^-_{i,j+1/2} \delta U_{i,j-1} = H_{i,j}
\]

**Data-Parallel Relaxation**

\[
(\tilde{A} - C) \delta U^k = H - \tilde{C} \delta U^{k-1}
\]

\[
\delta U^{k+1} = \delta U^{k\text{ max}}
\]

**Baseline Scheme**

- Original Gauss-Seidel not parallelizable
- Replace the G-S sweeps with a series of point Jacobi-like iterations or a series of line relaxations.
- Second order dual time stepping (KEC flux)

**For laminar sublayers only**

(Modified Steger-Warming flux)
Detached Eddy Simulation

Spalart-Allmaras 1-eq. model with compressibility correction

\[
\frac{Dp\tilde{v}}{Dt} = \nabla \cdot (\frac{1}{\sigma} \mu \nabla \tilde{v}) + \nabla \cdot (\frac{1}{\sigma \rho} \nabla \tilde{\rho} \nabla \tilde{v}) + \frac{c_{s2}}{\sigma} \nabla \frac{\tilde{\rho}}{\rho} \cdot \nabla \tilde{v} + c_{s1} \tilde{S} \tilde{v} - c_{w1} f_{w} \tilde{p} (\frac{\tilde{v}}{d})^2
\]

diffusive transport production destruction

• RANS approach over-predicts the turbulent dissipation levels in separated flow regions.
• DES introduces a new length scale \( \tilde{d} = \min(d \, C_{DES} \Delta) \)
  - Model behaves like a subgrid scale model for LES away from the wall.
  - Wall-Modeled LES

Test Case

NASA LaRC Mach 10 wind tunnel experiment by Danehy et al.

Flow conditions: \( M_{\infty} = 9.93, \, R_{e_k} \sim 6,000, \, T_{\infty} = 51.3K, \, T_w = 308K \)
Grid Refinement Study

Coarse grid (15 M cells) solution using the modified S-W flux

Medium grid (40 M cells) solution using the modified S-W flux

Fine grid (44 M cells) solution using the modified S-W flux (temperature contours)

Grid requirements for DNS: 1B cells to resolve Kolmogorov dissipative scales

Grid points redistributed (grid density 2x)

Thank You.

Laminar Flow
BLT Trip
Formation of Vortices
Transition
Vortex Breakdown/Turbulence

Temperature Contours: Blue Planes (at 30h, 60h, and 85h from the trip) are perpendicular to the horizontal plane (at 0.5h from the wall; h = trip height)

Flow

Fine grid solution using the KEC flux
KEC Flux Solutions

- Fine grid solution using KEC flux with DES turbulence model
- Fine grid solution using KEC flux without a turbulence model (ILES?)

The accuracy of the turbulent flow solutions is driven primarily by the grid resolution.
Comparison

Experiment

Computation

0.5 mm above the wall

Formation of Upstream Vortices

4 mm from the center of the trip (near the leading edge)

3 mm from the center of the trip

At the center of the trip
Conclusions

- DES/ WMLES/ Hybrid RANS-LES can be a useful tool for predictions of hypersonic boundary layer transition to turbulence triggered by an isolated roughness element.
- It is necessary to use the low-dissipation kinetic energy consistent scheme on a sufficiently fine grid for an accurate simulation of transition.
- Accuracy of the turbulent flow solutions is driven primarily by the grid resolution.
- Interaction of vortices leads to vortex breakdown and hence turbulent flow.
- Computational results agree well qualitatively with the experimental observations.

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