Acoustic Absorption in Porous Materials

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Abstract

An understanding of both the areas of materials science and acoustics is necessary to successfully develop materials for acoustic absorption applications. This paper presents the basic knowledge and approaches for determining the acoustic performance of porous materials in a manner that will help materials researchers new to this area gain the understanding and skills necessary to make meaningful contributions to this field of study. Beginning with the basics and making as few assumptions as possible, this paper reviews relevant topics in the acoustic performance of porous materials, which are often used to make acoustic bulk absorbers, moving from the physics of sound wave interactions with porous materials to measurement techniques for flow resistivity, characteristic impedance, and wavenumber.

Introduction

The frequency range for human hearing, commonly referred to as audio frequencies, is typically cited as approximately 20 Hz –20 KHz (Ref. 1). While sensitivity to particular frequencies can be affected by a number of conditions, including age and physical condition, the most important frequencies for understanding speech are typically in the range 500 to 2048 Hz (Ref. 2). Materials and structures that can absorb or attenuate frequencies in this range are important tools in managing the acoustic environment for the health, safety, and comfort of people exposed to noisy environments. In both fixed and rotary wing aircraft, the move toward lighter structures has resulted in an increase in structural vibration and interior noise (Ref. 3). Engine and gearbox noise can be transmitted into the aircraft cabin through sound and vibration energy propagating though the structure itself and through air to the fuselage walls. In order to address these problems, a basic understanding of the phenomena of noise, materials used for its suppression, and the characterization of those materials in order to predict their acoustic performance is necessary.

Vibration damping and acoustic absorption occur through different mechanisms. Vibration damping occurs when acoustic energy is transmitted directly from contact between two solids. Acoustic absorption occurs when acoustic energy is transmitted through the air and interacts with a solid structure. The morphology of the material is a critical aspect in how it interacts with the air. A material that performs well as a vibration damping material will not necessarily be a good acoustic absorber. This paper will focus on acoustic absorption rather than vibration damping.

Physics of Sound Wave Interaction

While electromagnetic radiation does not require a medium to travel through, sound is a disturbance in the medium itself, and is, therefore, classified as a mechanical wave. Acoustic energy repetitively displaces air through a mechanism that causes a periodic change in air pressure. A pressure wave is a region of pressure change moving through a medium. For a pure tone, these pressure waves can be characterized by their frequency (time between pressure maxima) and amplitude (intensity of the pressure change); they will have a repeating pattern in time. Pressure waves travel through air at the speed of sound, which is related to the frequency and wavelength (length of a single cycle) through the equation:
\[ c = \lambda f \]  

where:  
\( c \) = speed of sound in air (m/s)  
\( \lambda \) = wavelength (m)  
\( f \) = frequency in (Hz)  

The speed of sound of an ideal gas may be obtained from (Ref. 4):  
\[ c = \sqrt{\gamma RT} \]  

where:  
\( c \) = speed of sound (m/s)  
\( \gamma \) = specific heat ratio (dimensionless)  
\( R \) = gas constant (287 J/kg·K)  
\( T \) = temperature (K)  

The specific heat ratio, \( \gamma \), is given by (Ref. 4):  
\[ \gamma = \frac{c_p}{c_v} \]  

where:  
\( c_p \) = specific heat at constant pressure (J/kg·K)  
\( c_v \) = specific heat at constant volume (J/kg·K)  

The speed of sound in dry air at standard conditions (0 \(^\circ\)C, one atmosphere of pressure) is 331.5 m/s (Ref. 5). At these conditions, an acoustic wave whose frequency is 1 kHz will have a wavelength of 0.3315 m. Comparison of this acoustic wavelength to the mean free path of air (the average distance an “air molecule” can move before colliding with another), which is 0.066 \( \mu \)m at standard conditions (Ref. 6), shows that molecules in air can move only a short distance before colliding with another molecule. These collisions, not the overall movement of the molecules themselves, pass the energy along and allow pressure waves in air to move long distances without individual molecules moving very far. Since sound waves travel in the same direction as the collisions between molecules, they are, by definition, longitudinal waves. Broadband noise is simply the superposition of individual waves of many different frequencies and amplitudes and, therefore, will not have a repeating pattern in time. The frequencies and amplitudes of the individual component waves are typically not obvious by inspection of the waveform, but can be extracted through a function such as the Fourier Transform.  

When a pressure wave encounters another medium with the surface normal to the propagation direction of the wave, a portion of that wave is reflected at the interface and some is transmitted into the new medium. A portion of the transmitted energy is absorbed and the rest is transmitted through the material. For a wave traveling through medium 1 and encountering medium 2, the ratio of the intensity of the reflected wave to the incident wave is (Ref. 7):  
\[ \frac{I_r}{I_i} = \left( \frac{\rho_1 c_1 - \rho_2 c_2}{\rho_1 c_1 + \rho_2 c_2} \right)^2 \]  

where:  
\( I_r \) = intensity of the reflected wave (W/m\(^2\))  
\( I_i \) = intensity of the incident wave (W/m\(^2\))  
\( \rho \) = density of the medium (kg/m\(^3\))  
\( c \) = speed of sound in the medium (m/s)
The quantity $\rho_c$ is the characteristic resistance of the material and has units of mks rayls (Pa·s/m). Characteristic resistance is the real part of the complex characteristic impedance, with characteristic reactance being the imaginary part. In the literature, when the characteristic reactance is zero, such as for some fabrics at high frequencies (Refs. 8 and 9), the term “characteristic impedance” may more commonly be used than “characteristic resistance”. The larger the difference in the characteristic resistance between the two media, the more energy is reflected. The density of dry air at standard conditions is 1.34 kg/m³, so $\rho_c$ for air is 444.2 Pa·s/m. The density of fused silica is 2210 kg/m³ and the speed of sound is 5968 m/sec, yielding a $\rho_c$ value of $1.32 \times 10^7$ Pa·s/m. Using these values in Equation (4) yields a value for $\frac{I_r}{I_i}$ of 0.9999, which means that 99.99 percent of the energy is reflected from the silica surface; only 0.01 percent is transmitted. Lower density materials will reflect less energy, as will materials with lower sound speeds. Since a solid has much higher density than a gas, the efficiency of sound transmission into a solid material from air is unlikely to be high unless the speed of sound in the solid can be decreased dramatically.

Acoustic energy can be converted into heat through a number of mechanisms. For instance, viscous stresses caused by shearing of the fluid convert fluid kinetic energy into heat. This heat can be transferred from the fluid into a solid structure through heat conduction. Friction on the fluid across the solid also produces heat that can be transferred from the solid to the fluid. Impact of an acoustic pressure wave on a solid structure can dissipate energy through flexing of the solid frame. If the solid surface is nonporous, incident energy reflects back into the environment and is lost. However, if that surface is highly porous, a substantial portion of the pressure wave penetrates the material before encountering a solid surface. The same efficient reflection occurs, but in a structure where the chances are great that the reflected energy will encounter another part of the solid structure before being lost to the environment. A high number of internal reflections can transfer energy to the solid structure through frictional losses and efficiently absorb sound. This scattering is essential to the performance of acoustic absorbers fashioned from materials such as glass fibers, polymers, and metal foams. In order to achieve a large number of interactions, the pressure wave must penetrate deeply enough into the material so as to not immediately be reflected back into the surrounding air. As the pore size decreases, less energy is transferred into the solid structure and more is reflected from the surface, making the material less useful as an acoustic absorber. This is illustrated in Figure 1.

![Figure 1.—Decrease in energy transfer potential as pore size decreases from large (left) to small (right).](image-url)
It is important to note that not all highly porous materials are suitable as acoustic absorbers. If the mean free path of air is on the order of that of the mean distance between pore walls, pressure waves will not be able to efficiently penetrate the material. In this case, even if the material has an open-cell structure, it will perform like a closed-cell material, with conventional aerogels being a prime example. Their porosity is extremely high, with typical void volumes of 80 to 90 percent, but the largest pore diameters average about 2 to 20 nm, on a similar scale to the mean free path of air.

**Flow in Porous Solids**

The flow regime in a porous solid is directly related to the pore size of the material, and can thus give an indication as to how easily pressure waves can penetrate the material. The Knudsen number can characterize the flow regime in porous solids:

\[ Kn = \frac{l_{mfp}}{l_{char}} \]  

where:  
- \( Kn \) = Knudsen number (dimensionless)  
- \( l_{mfp} \) = mean free path of air molecules (m)  
- \( l_{char} \) = characteristic length (m)

The characteristic length for this application is often taken to be the mean distance between pore walls (Ref. 10). For the case of a spherical pore, this would equate to the pore diameter. When \( l_{mfp} \) is much smaller than \( l_{char} \), the gas molecules are essentially moving in a free space. The mean free path of gas molecules in free space is given by (Ref. 11):

\[ \lambda_{mfp_{free}} = \frac{1}{\sqrt{2n_g \pi d_g^2}} \]  

where:  
- \( \lambda_{mfp_{free}} \) = mean free path of gas molecules in free space (m)  
- \( n_g \) = number density of gas molecules (molecules/m³)  
- \( k_B \) = Boltzmann’s constant (1.38049×10⁻²³ J/K)  
- \( T \) = temperature (K)  
- \( d_g \) = diameter of gas molecule (m)  
- \( P \) = pressure (Pa)

For an ideal gas, the number density may be expressed as:

\[ n_g = \frac{nN_a}{V} \]  

where:  
- \( n \) = number of moles (moles)  
- \( N_a \) = Avogadro’s number (6.023×10²³ molecules/mole)  
- \( V \) = volume (m³)
Combining this with the ideal gas law,

\[ PV = nRT \]  

(8)

where:  
\( P \) = pressure (Pa)  
\( V \) = volume (m³)  
\( n \) = number of moles (moles)  
\( R \) = gas constant (8.314 J/mole·K)  
\( T \) = temperature (K)

Equation (6) can be expressed as:

\[ \lambda_{\text{mfp,free}} = \frac{k_B T}{\sqrt{2} \pi d_g^2 P} \]  

(9)

However, when the mean free path of gas molecules is on the same order as the mean distance between pore walls, gas molecules collide both with each other and the solid portion of the porous material. Assuming spherical solid particles, the mean free path can be determined from (Ref. 11):

\[ \lambda_{\text{mfp,porous}} = \frac{1}{\sqrt{2} n_s \pi d_g^2 + \frac{S_s \rho_s}{\phi}} \]  

(10)

where:  
\( n_s \) = number density of gas molecules (molecules/m³)  
\( d_g \) = diameter of gas molecule (m)  
\( S_s \) = specific area of solid (surface area/mass, m²/g)  
\( \rho_s \) = solid density (kg/m³)  
\( \phi \) = porosity (dimensionless)

The dominant flow regime through the material may be characterized as (Ref. 10):

\( Kn < 1 \): viscous flow dominates  
\( Kn \approx 1 \): both viscous and molecular flow are important  
\( Kn >> 1 \): molecular flow dominates

When viscous flow dominates, pressure waves should be able to penetrate the material to a degree that allows enough internal reflections to make the material useful as an acoustic absorber. However, when molecular flow dominates, pressure waves will not be able to penetrate significantly into the material. Most of the acoustic energy will reflect off the surface, and limit its usefulness as an acoustic absorber. Table 1 shows the Knudsen number for some materials using representative properties found in the literature and assuming that the characteristic length, \( l_{\text{char}} \), is the same as a pore diameter (Refs. 10 to 17).

<table>
<thead>
<tr>
<th>Material</th>
<th>( l_{\text{mfp}} ), m</th>
<th>( l_{\text{char}} ), m</th>
<th>( Kn )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ceramic foam</td>
<td>7\times10^{-8}</td>
<td>1\times10^{-8}</td>
<td>7\times10^{-5}</td>
</tr>
<tr>
<td>Metal foam</td>
<td>7\times10^{-8}</td>
<td>2\times10^{-8}</td>
<td>4\times10^{-4}</td>
</tr>
<tr>
<td>Aerogel</td>
<td>9\times10^{-9}</td>
<td>5\times10^{-8}</td>
<td>2\times10^{-1}</td>
</tr>
</tbody>
</table>
Porous Acoustic Absorbers

The solid phases in acoustic absorbers can be described in relation to their mechanical properties. Materials are considered rigid when their solid-phase motion is negligible compared to the fluid phase. This can occur when the density or stiffness of the solid phase is high, requiring a substantial amount of energy to cause movement, or when the coupling between the solid and fluid phases is so poor that little energy is transferred. Limp materials, on the other hand, have very low stiffness in their solid phase in the absence of the fluid phase. A common latex balloon is an example of such a material. The envelope has very little stiffness unless it is filled with air. A force on the surface of the balloon tends to compress the air; it is this compression that resists the force rather than the stiffness of the envelope.

Poroelasticity is also seen in some absorbers. In a poroelastic material, the solid matrix material is both porous and elastic, while the fluid filling it is viscous (Refs. 18 and 19). In poroelastic materials, the solid matrix material contributes to the energy transfer so these materials can support both transverse waves and two kinds of longitudinal (compressional) waves. In contrast, in both rigid and limp materials, the primary energy transport takes place in the fluid phase; they can be considered to support only a single longitudinal wave type, such as pressure waves in air. In transverse waves, the vibrations are perpendicular to the direction of travel. Ocean waves and electromagnetic waves are examples of transverse waves. Longitudinal waves, such as sound waves, oscillate parallel to the direction of travel. The performance of an acoustic absorber may be tailored by balancing the material density, speed of sound in that material, and the porosity, $\phi$ (Ref. 20):

\[
\phi = \frac{V_{\text{pore}}}{V_{\text{total}}} = 1 - \frac{\rho_a}{\rho_m}
\]

where:
- $V_{\text{pore}}$ = pore volume (m$^3$)
- $V_{\text{total}}$ = total volume (m$^3$)
- $\rho_a$ = density of porous material (kg/m$^3$)
- $\rho_m$ = density of solid portion of material (kg/m$^3$)

The speed of sound in the matrix material depends on its elasticity and the overall morphology. The energy damping mechanisms in the solid and the frictional and viscous losses between the air in the pores and the solid ligaments will convert the acoustic energy into heat. Any characteristic of the solid matrix material that enhances the transfer of energy from the air into the structure and the dissipation of that energy in the structure can enhance its performance as an acoustic absorber. An increase in the density of the absorber material must be offset by a decrease in the speed of sound or an increase in the porosity to maintain the same performance. Efficient absorption of sound from the air also depends on an open pore structure—there must be interconnected paths for the air that reach the surface (Ref. 21). As an example, porous fiber mats have a complex tangle of fibers that result in a highly porous, open pore structure.

Most absorbers are both homogenous and isotropic, meaning that their properties are both uniform in space and independent of direction. The uniformity of homogenous and isotropic materials allows their performance to be more easily predicted. Inhomogeneity can arise from a change in composition through space or the inclusion of regions of a second material. This change in the composition often generates a change in the properties, leading to anisotropic behavior. Many fiber-reinforced materials show some anisotropy in their properties, which are dependent upon the orientation of the fibers. Carbon fiber, for example, is well known to have different thermal and electrical properties along the length of the fiber versus across the diameter. However, the assumption of isotropy is often good for fibrous acoustic absorbers.
Foams are an example of a porous material used as acoustic absorbers. Foams are solid materials with a large number of gas bubbles trapped throughout the volume. The foam density depends upon the porosity, the skeletal density of the solid matrix material, and the density of the gas in the pores. Foams are considered open-cell when the pores formed by the gas bubbles interconnect without barriers between them; the solid structure between the individual bubbles is in the form of ligaments. In closed-cell foams, each bubble has intact walls; the internal volumes of the individual bubbles do not communicate. It is not uncommon to have a small volume of closed-cells in foams that are considered to be open-cell. When determining the acoustic properties of foam, the volume of closed pores should not be included in the total pore volume, since air cannot penetrate into a closed pore. Closed-cell foams are not typically useful as acoustic absorbers. Instead of penetrating into the foam, the acoustic pressure wave reflects from the outer surface. Syntactic foams are a special case of closed-cell foams where the bubbles are hollow particles dispersed through the solid matrix. These particles are typically added to and dispersed in the matrix material. Again, these types of foams are not useful as acoustic absorbers.

To summarize, in order for a material to be an efficient acoustic absorber, there must be a structure to transfer the energy into and an acceptable range of porosity so that the sound waves can penetrate far enough into the structure to allow multiple interactions with that structure. If the material pores are too coarse, the pressure wave will pass through it with minimal interaction with the structure. If the porosity is too fine, the majority of the energy will reflect back into the environment from a region in the immediate vicinity of the surface, never entering deeply enough to undergo the multiple interactions with the structure to absorb a substantial fraction of the energy. For materials with rigid or limp frames, the primary energy transfer will be in the fluid phase, while for materials with elastic frames, the solid matrix will also contribute to energy transfer.

**Parameters for Evaluating Acoustic Performance**

**Flow Resistivity**

The measurement of air flow through a material is a physical property useful in evaluating its performance as an acoustic absorber. The following equations describe quantities useful in characterizing such flow (Ref. 22):

\[
R_f = \frac{\Delta p}{q_v} \quad \text{(12)}
\]

\[
R_s = R_f A \quad \text{(13)}
\]

\[
r_f = \frac{R_s}{d} \quad \text{(14)}
\]

where:  
- \( R_f \) = flow resistance (Pa·s/m³)  
- \( \Delta p \) = pressure difference across the test sample (Pa)  
- \( q_v \) = volumetric flow rate through material (m³/s)  
- \( R_s \) = specific flow resistance (Pa·s/m)  
- \( A \) = cross-sectional area of material perpendicular to flow (m²)  
- \( r_f \) = flow resistivity (Pa·s/m²)  
- \( d \) = thickness of material (m)

The specific flow resistance is one of the properties that determines both the sound absorbing and sound transmitting properties of a material. It measures how easily air can enter a porous structure, as well as the resistance that flow meets within the structure. Flow resistivity is independent of the area or thickness of the tested material.
Figure 2 shows a diagram of a flow resistivity test. Figure 3 shows a typical experimental apparatus for determining flow resistivity. Flow resistance requires a measurement of the flow rate through the sample and the pressure drop across it. Standards using a unidirectional, open-tube technique and a bidirectional technique that uses a piston to force air through the sample in alternating directions have both been defined (Refs. 22 and 23). In the open-tube techniques, the air flow through the sample is either measured with a flow element or controlled with a mass flow controller; the pressure across the sample is measured directly. The flow resistance is measured at several flow rates. The intent is to maintain laminar flow through the sample, with the upper limit of the air velocity being set when the flow through the sample begins to transition to turbulent flow.
For a number of applications using homogeneous fiber and open cell foam products, flow resistance information can be sufficient to characterize acoustic performance (Ref. 24). Flow resistivity and porosity can be used in empirical laws to calculate the impedance and wavenumber of the material (Ref. 21). These quantities then completely specify the acoustic properties of the material (Ref. 25). Absorbers need to be about a minimum of a tenth of a wavelength thick to significantly absorb incident sound, and about a quarter of a wavelength to absorb all sound (Ref. 21). Significant absorption occurs furthest from the backing surface of an absorber (Ref. 21). Low frequency absorption usually increases with the thickness of the porous absorber (Ref. 21).

Some materials may not exhibit the same behavior for all frequencies of sound that they encounter. Flow resistance, since it is measured with a steady, laminar flow of air, does not provide any direct information about the frequency dependent behavior of the sample. The frequency dependent characteristics of a material are generally obtained from an experimental measurement of its acoustic impedance. The flow resistivity is analogous to electrical resistance as a DC (non-frequency dependent) property and the acoustic impedance is analogous to electrical impedance as an AC (frequency dependent) property. The difference between the information content of the data collected in the flow resistance and impedance experiments is depicted in Figure 4.

![Figure 4.—Depiction of flow resistance and impedance experimental data.](image)
Characteristic Impedance and Wavenumber

Fluid-saturated, homogeneous, isotropic porous materials that can be approximated as either rigid or limp may be completely described by the characteristic impedance, $Z_c$, and wavenumber, $k$, along with quantities derived from these two quantities (Ref. 25). The complex reflection coefficient is often used to determine these quantities:

$$R = \frac{P_r}{P_i}$$ (15)

where: $P_r =$ reflected complex pressure (Pa)

$P_i =$ incident complex pressure (Pa)

To measure the acoustic impedance, a material is exposed to sound at various frequencies and the reflection and attenuation of that sound is measured. Impedance measurements typically use a sample mounted in a tube, similar to that used in a flow resistance experiment. Instead of a steady air flow entering at one end, a speaker, driven by either a noise signal or a single audio tone, is used to provide a source of pressure waves. The dimensions of the tube are selected to insure that planar pressure waves impinge on the sample face. In place of the pressure transducers used for the flow resistance experiment, one or more microphones are used to measure the sound pressure levels at various locations in the vicinity of the sample faces. These pressures are complex, having both magnitude and phase components, and are generally frequency dependent (Ref. 26). There are a number of techniques for measuring the acoustic impedance, differing in the number and placement of the microphones, and the termination behind the sample. In some cases, a rigid wall is used behind and in direct contact with the sample (Ref. 27), while in others, two samples with different thicknesses (L and 2L) are used, each backed by a rigid wall in contact with them (Ref. 28). In other cases, a rigid wall is used, but with an intervening air space. Some examples (Ref. 29) adjust the gap behind the sample to be one quarter of the excitation wavelength. This quarter wave gap with the hard wall at the end makes the impedance zero at the excitation wavelength. This technique suffers from the necessity to adjust the gap and repeat the experiment for every wavelength of interest. Other examples (Ref. 30) use an air gap with a terminating hard wall, but can handle arbitrary termination impedance, relieving the necessity of adjusting the gap depth as a function of frequency. The technique described by Bolton, et al. (Ref. 31) uses an anechoic termination (a termination that absorbs sound, preventing it from being reflected back to the microphones), Song and Bolton (Ref. 25) used approximately anechoic termination, while Olivieri, et al. (Ref. 32) imposed no conditions on the properties of the termination of the tube.

Since a number of techniques for determining the characteristic impedance use transfer functions, a brief discussion is in order. Measured sound pressures can be transformed to a frequency response function, $H(f)$, using a Fourier transform. The auto power spectrum, $G_{11}$, and cross power spectrum $G_{12}$, are given by the averaged quantities (Ref. 33):

$$G_{11} = P_1^*(f) \cdot P_1(f)$$ (16)

$$G_{12} = P_1^*(f) \cdot P_2(f)$$ (17)

$$G_{21} = P_2^*(f) \cdot P_1(f)$$ (18)

$$G_{22} = P_2^*(f) \cdot P_2(f)$$ (19)
where:  
$P_1$ = complex sound pressure at location 1 (Pa)  
$P_2$ = complex sound pressure at location 2 (Pa)  
$P_i^*$ = complex conjugate of the sound pressure at location $i$ (Pa)

In practice, the microphone will measure a voltage, which will scale to a pressure in Pascals. The complex conjugate of the complex number $r + j(im)$ is $r - j(im)$. The auto power spectrum is real and positive, while the cross power spectrum is complex. The transfer function calculation between two locations depends on where noise is encountered during the test. When noise is encountered at the input:

$$H_{12} = \frac{G_{12}}{G_{11}}$$  \hspace{1cm} (20)$$

When noise is encountered at the output:

$$H_{12} = \frac{G_{22}}{G_{21}}$$  \hspace{1cm} (21)$$

If there is noise at both the input and output, Equation (20) tends to underestimate, while Equation (21) tends to overestimate (Ref. 34). A more accurate expression if there is noise at both the input and the output, is given by (Ref. 33):

$$H_{12} = \left[ \frac{G_{12} \cdot G_{22}}{G_{11} \cdot G_{21}} \right]^{1/2}$$  \hspace{1cm} (22)$$

Another expression that can be used is (Ref. 34):

$$H_{12} = \sqrt{\frac{G_{22}}{G_{11}} \cdot \frac{G_{12}}{G_{21}}}$$  \hspace{1cm} (23)$$

In these experiments the “input” and “output” signals refer to the two signals being sampled, a reference signal from a signal generator versus a microphone signal or two different microphone locations. It is likely that a signal from a microphone would be noisier than one directly connected to a signal generator. In that case, the noise would be encountered on the output. When sampling two microphones, both the input and output signals would likely be noisy.

A number of techniques have been developed to determine the characteristic impedance and wave number. ASTM Standard C384-04 (Ref. 35) uses a single moving microphone to explore a standing wave in a tube. A plot of microphone voltage versus microphone distance from test sample is used to generate a plot of the standing wave in the tube. According to the standard, the locus of the voltage maxima describes an essentially horizontal line, but the locus of the minima typically has a noticeable slope due to attenuation in the tube. As a result, the standing wave ratio is position dependent,

$$\text{SWR}(x) = \frac{V_{\text{max}}(x)}{V_{\text{min}}(x)}$$  \hspace{1cm} (24)$$

where:  
$V_{\text{max}}(x)$ = maximum voltage function (volts)  
$V_{\text{min}}(x)$ = minimum voltage function (volts)
which is then used to determine the components of the complex pressure reflection coefficient:

\[ |R| = \frac{\text{SWR}(0) - 1}{\text{SWR}(0) + 1} \]  

(25)

\[ \theta = \left( \frac{720x_1}{\lambda} \right) - 180 \]  

(26)

where:

- \( |R| \) = complex reflection coefficient magnitude (dimensionless)
- \( \text{SWR}(0) \) = standing wave ratio at the face of the sample (dimensionless)
- \( \theta \) = complex pressure reflection coefficient phase angle (dimensionless)
- \( x_1 \) = distance from sample face to first pressure minimum (m)
- \( \lambda \) = wavelength (m)

The normal incidence sound absorption coefficient and impedance ratio are calculated from:

\[ \alpha = 1 - |R|^2 \]  

(27)

\[ \frac{z}{\rho c} = \frac{1 + R}{1 - R} \]  

(28)

where:

- \( \alpha \) = normal incidence sound absorption coefficient (dimensionless)
- \( R \) = reflection coefficient (dimensionless)
- \( z \) = specific normal acoustic impedance at the surface (Pa·s/m, or mks rayls)
- \( \rho \) = density of material (kg/m\(^3\))
- \( c \) = speed of sound in the material (m/s)

A similar method is discussed in (Ref. 36).

Both the normal incidence sound absorption coefficient and the impedance ratio are functions of frequency. Measurements are made with pure tones at a number of frequencies. ASTM Standard E1050-08 (Ref. 27) uses two microphones to determine the normal incidence sound absorption coefficient and impedance ratio. A noise source produces a broad band signal that generates plane waves. Sound pressure is measured simultaneously at the locations of the two microphones, which are separated by a known distance. The real and imaginary parts of the reflection coefficient are given by:

\[ R_r = \frac{\left\{2H_r \cos(k(2l + s)) - \cos(2kl) - (H_r^2 + H_i^2)\cos[2k(l + s)]\right\}}{D} \]  

(29)

\[ R_i = \frac{\left\{2H_r \sin(k(2l + s)) - \sin(2kl) - (H_r^2 + H_i^2)\sin[2k(l + s)]\right\}}{D} \]  

(30)

\[ D = 1 + H_r^2 + H_i^2 - 2\left[H_r \cos(ks) + H_i \sin(ks)\right] \]  

(31)
where: $H_r = \text{real part of the transfer function}$

$H_i = \text{imaginary part of the transfer function}$

$k = \frac{2\pi f}{c} = \text{real component of the complex wavenumber (m}^{-1})$

$f = \text{frequency (s}^{-1})$

$l = \text{distance from sample to center of closest microphone (m)}$

$s = \text{center-to-center spacing between microphones (m)}$

The transfer matrix approach has been used to determine the characteristic impedance and wave number for homogeneous, isotropic, porous materials with a limp or rigid solid phase in a number of studies (Refs. 31 to 33, 37, and 38). Song and Bolton (Ref. 25) used a single microphone to measure the transfer functions between the signal to a loudspeaker and the sound pressure at four locations. Figure 5 shows a typical apparatus for the four-location measurements. The complex amplitudes of the pressure waves are used to calculate the elements of the transfer matrix, which are then used to determine the characteristic impedance and wavenumber of the material. The transfer matrix elements are properties of the material and do not depend on the measurement environment (Ref. 39) or sample depth (Ref. 25).

As an example of how this method works, assume reference signal $r$ and a transfer function representing the complex pressure at location $i$ of the form:

$$H_{ir} = \frac{G_{ir}}{G_{rr}}$$  \hspace{1cm} (32)

where: $G_{ir} = \text{cross-spectrum of the complex sound pressures}$

$G_{rr} = \text{auto-spectrum of the complex sound pressures}$

![Figure 5.—Schematic representing two- and four-microphone methods of determining acoustic impedance.](image-url)
The cross-spectra at each of the four locations may be expressed as:

\[ H_{1r} = (Ae^{-jkx_1} + Be^{jkx_1})e^{jot} \] (33)

\[ H_{2r} = (Ae^{-jkx_2} + Be^{jkx_2})e^{jot} \] (34)

\[ H_{3r} = (Ae^{-jkx_3} + Be^{jkx_3})e^{jot} \] (35)

\[ H_{4r} = (Ae^{-jkx_4} + Be^{jkx_4})e^{jot} \] (36)

The complex amplitudes may be determined from Equations (33) to (36):

\[ A = \frac{G_{rr} (H_{1r}e^{jkx_2} - H_{2r}e^{jkx_1})}{-2je^{jot} \sin k(x_1 - x_2)} \] (37)

\[ B = \frac{G_{rr} (H_{2r}e^{jkx_1} - H_{1r}e^{jkx_2})}{-2je^{jot} \sin k(x_1 - x_2)} \] (38)

\[ C = \frac{G_{rr} (H_{3r}e^{jkx_4} - H_{4r}e^{jkx_3})}{-2je^{jot} \sin k(x_3 - x_4)} \] (39)

\[ D = \frac{G_{rr} (H_{4r}e^{jkx_3} - H_{3r}e^{jkx_4})}{-2je^{jot} \sin k(x_3 - x_4)} \] (40)

The transfer matrix formulation for a sample extending from \( x = 0 \) to \( x = d \), where \( d \) is the thickness of the sample, is given by:

\[
\begin{bmatrix}
P_{x=0} \\
V_{x=0}
\end{bmatrix} =
\begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix}
\begin{bmatrix}
P_{x=d} \\
V_{x=d}
\end{bmatrix}
\] (41)

where:

\[ P\big|_{x=0} = A + B = 1 + R \] (42)

\[ V\big|_{x=0} = \frac{A - B}{\rho_0c} = \frac{1 - R}{\rho_0c} \] (43)

\[ P\big|_{x=d} = Ce^{-jkd} + De^{jkd} = Te^{-jkd} \] (44)

\[ V\big|_{x=d} = \frac{Ce^{-jkd} - De^{jkd}}{\rho_0c} = \frac{Te^{-jkd}}{\rho_0c} \] (45)
In these equations, the complex amplitudes are used to determine the normal incidence reflection coefficient, \( R \),

\[
R = \frac{B}{A}
\]  

(46)

and the normal incidence transmission coefficient, \( T \):

\[
T = \frac{C}{A}
\]  

(47)

The elements of the transfer matrix are given by:

\[
T_{11} = T_{22} = \frac{P|_{x=d} V|_{x=d} + P|_{x=0} V|_{x=0}}{P|_{x=0} V|_{x=d} + P|_{x=d} V|_{x=0}}
\]  

(48)

\[
T_{12} = \frac{P|_{x=0}^2 V|_{x=d}^2 - P|_{x=d} V|_{x=0}}{P|_{x=d} V|_{x=d} + P|_{x=d} V|_{x=0}}
\]  

(49)

\[
T_{21} = \frac{V|_{x=0}^2 V|_{x=d}^2 - V|_{x=d} V|_{x=0}}{P|_{x=d} V|_{x=d} + P|_{x=d} V|_{x=0}}
\]  

(50)

These expressions result from reciprocity, which requires that:

\[
T_{11} T_{22} - T_{12} T_{21} = 1
\]  

(51)

and symmetry, which requires that:

\[
T_{11} = T_{22}
\]  

(52)

The normal incidence transfer matrix becomes:

\[
\begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix} = \begin{bmatrix}
\cos(k_p d) & j \rho_p c_p \sin(k_p d) \\
j \rho_p c_p \sin(k_p d) & \cos(k_p d)
\end{bmatrix}
\]  

(53)

The characteristic impedance and wavenumber of the material are then calculated as follows:

\[
z = \rho_p c_p = \frac{T_{12}}{\sqrt{T_{21}}}
\]  

(54)

\[
k_p = \frac{\cos^{-1} T_{11}}{d} = \frac{\sin^{-1} \sqrt{-T_{12} T_{21}}}{d}
\]  

(55)
where:  \( \rho_p \) = complex density (kg/m\(^3\))
\( c_p \) = complex speed of sound (m/s)

The complex speed of sound may be expressed as:

\[
c_p = \frac{\omega}{k_p}
\]  \( (56) \)

The angular frequency may be expressed as:

\[
\omega = 2\pi f
\]  \( (57) \)

where:  \( f \) = frequency (s\(^{-1}\))

The normal incidence transmission loss, \( TL \), may be calculated from (Ref. 31):

\[
TL = 10\log \frac{1}{|T|^2} \quad \text{(dB)}
\]  \( (58) \)

This quantity is useful in studying the transmission of sound through a solid.

Equations (37) to (40) contain the term \( e^{j\omega t} \) in the denominator. When these equations are used to determine the elements of the transfer matrix, Equations (48) to (50), this term cancels out and is not present in the final expressions. For this reason, it is often omitted in the above derivations in the literature.

The transfer matrix method discussed above assumes reciprocity in the system, which means that the acoustic response remains the same if the source and the receiver are interchanged. It also assumes symmetry, which is present when the effects of viscosity and thermal conductivity are negligible. When viscothermal effects become important, the impedance matrix is no longer symmetric, and the system is no longer reciprocal (Ref. 40). The above approach may then no longer yield accurate results.

Once the relevant parameters for acoustic absorption have been determined for a material, sound absorbing charts can be constructed to aid in performance prediction for these materials. These charts are typically constant values of \( \alpha \), the normal incidence absorption coefficient, plotted on a log-log chart of \( \frac{fd}{c_0} \) versus \( \frac{R_f d}{z} \). An illustration of a sound absorbing chart is given in Figure 6. An increase in the frequency, \( f \), would result in a horizontal move across the chart. An increase in the flow resistivity, \( R_f \), would result in an upward move. An increase in the thickness of the material, \( d \), would result in an upward diagonal move to the right (Ref. 20). Optimum values of these parameters for a particular application can be determined from these charts.
Suggestions for Further Research

Departing from conditions of homogeneity and isotropy can be useful in tailoring the absorber for a specific task. For example, this can be accomplished through the creation of composite materials, where, for instance, a second component is dispersed throughout the primary matrix material. Hybrid material with distinct regions of different materials can also be tailored to be acoustic absorbers. For example, the low speed of sound and good energy absorbing characteristics of conventional aerogels imply that once energy enters the aerogel structure it can be effectively dissipated, making it a good acoustic damper. However, their extremely small pore size prevents the efficient transfer of acoustic energy from the air, making them inefficient as acoustic absorbers. A hybrid material could potentially be constructed with a layer that efficiently captured energy from the air (acoustic absorption) and transferred it from its solid structure to the solid structure of the aerogel (acoustic damping). Figure 7 shows some concepts for hybrid acoustic absorber structures. The composite example illustrates aerogel particles dispersed in a matrix that efficiently couples the acoustical energy in the pressure wave into the structure. The hybrid example uses a face layer of the coupling material bonded to a layer of aerogel absorber. The layered example uses mechanical porosity (molded in or drilled holes) in thin aerogel sheets separated by air spaces to maximize the penetration of the compression wave into the structure, maximizing the opportunity for interactions with the absorber.

These are just some examples that illustrate how hybrid structures can be used to increase the sound absorbing capabilities over that of the individual materials. Therefore, a material that in itself would not be useful as an acoustic absorber could be incorporated as part of a hybrid structure to yield an efficient acoustic absorber.
Conclusions

This paper has attempted to lay the groundwork for an understanding of porous bulk acoustic absorbers, particularly for materials researchers whose primary background is not in acoustics. Sound travels through air in the form of pressure waves. These waves can travel great distances without the individual molecules traveling very far. This occurs through the molecule to molecule transfer of energy through collisions, allowing the energy to move through the medium. The ability of a material to absorb acoustic energy depends on these pressure waves being able to penetrate a porous solid to a sufficient degree such that energy is dissipated through interaction with the solid portion of the material rather than reflecting back out. The Knudsen number is used as a measure of the flow regime of air through a pore (viscous versus molecular), with Knudsen numbers on the order of one indicating that the mean free path of air is similar to the average pore diameter in the material. In this case it is unlikely that the acoustic waves will be able to penetrate far into the material and dissipate acoustic energy efficiently. Experimental determination of flow resistivity and characteristic impedance allow sound absorbing charts to be constructed that describe the performance of an acoustic absorbing material for various applications.

The understanding of the effect on the acoustic properties of a material as the balance between density, speed of sound, and porosity can provide guidance for the development and evaluation of absorber materials. The understanding of the acoustic testing techniques and the processing of the acquired data provides a basis for the evaluation of new absorbers, particularly hybrid absorber concepts.

References

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Acoustic Absorption in Porous Materials

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Acoustic properties; Porous materials

Abstract:
An understanding of both the areas of materials science and acoustics is necessary to successfully develop materials for acoustic absorption applications. This paper presents the basic knowledge and approaches for determining the acoustic performance of porous materials in a manner that will help materials researchers new to this area gain the understanding and skills necessary to make meaningful contributions to this field of study. Beginning with the basics and making as few assumptions as possible, this paper reviews relevant topics in the acoustic performance of porous materials, which are often used to make acoustic bulk absorbers, moving from the physics of sound wave interactions with porous materials to measurement techniques for flow resistivity, characteristic impedance, and wavenumber.