Unsteady Aerodynamic Model Tuning for Precise Flutter Prediction

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Keywords: Test validated aeroelastic model, Unsteady aerodynamic model tuning, Object-oriented optimization tool, Flight test data

Abstracts: A simple method for an unsteady aerodynamic model tuning is proposed in this study. This method is based on the direct modification of the aerodynamic influence coefficient matrices. The aerostructures test wing 2 flight-test data is used to demonstrate the proposed model tuning method. The flutter speed margin computed using only the test validated structural dynamic model can be improved using the additional unsteady aerodynamic model tuning, and then the flutter speed margin requirement of 15 % in military specifications can apply towards the test validated aeroelastic model. In this study, unsteady aerodynamic model tunings are performed at two time invariant flight conditions, at Mach numbers of 0.390 and 0.456. When the Mach number for the unsteady model tuning approaches to the measured fluttering Mach number, 0.502, at the flight altitude of 9,837 ft, the estimated flutter speed is approached to the measured flutter speed at this altitude. The minimum flutter speed difference between the estimated and measured flutter speed is -0.14 %.

1 INTRODUCTION

The primary objective of this study is to reduce uncertainties in the unsteady aerodynamic model of an aircraft to increase the safety of flight. To this end, a new flutter analysis procedure using the validated aeroelastic model is proposed, and the block diagram of this new procedure is shown in figure 1.
Significant efforts [1, 2, 3] have been made in developing corrections to linear aerodynamic models to improve correlation with steady-state wind tunnel and flight test data. There has been a limited amount of effort in the correction of unsteady aerodynamics for aeroservoelastic applications, which has been relatively sparse and ad-hoc when compared to the steady-state work that has been performed.
The Aerostructures Test Wing (ATW) 2 test article, shown in figure 2, was developed and flown at the NASA Dryden Flight Research Center (DFRC) on the F15B test bed aircraft on December 15, 2009. To support the envelope expansion, a Test-Validated Finite Element Model (TVFEM) was used for the flutter analysis of the ATW2. Flutter boundaries of the ATW2, before and after the structural dynamic model tuning [4], are compared with the flight envelopes as shown in figure 3 [5]. In this figure, the solid line bounds the ATW2 test envelope that is planned for flight, and the dashed line is the 15% margin of the ATW2 test envelope. This 15% margin line was designed to match the numerical flutter boundaries computed using the TVFEM with the 3% structural damping, the solid line with circular marker. The solid line with the diamond marker represents flutter boundaries using the TVFEM with the measured structural damping [5]. The measured flutter point of the ATW2 is also shown in figure 3, using the x marker.

<table>
<thead>
<tr>
<th>Flutter Boundaries</th>
<th>Flutter Margins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured/1.15 (= V_d)</td>
<td>0 %</td>
</tr>
<tr>
<td>Measured (= 1.15 V_d)</td>
<td>15 %</td>
</tr>
<tr>
<td>TVFEM; after model tuning with measured damping</td>
<td>32 %</td>
</tr>
<tr>
<td>TVFEM; after model tuning with 3% structural damping</td>
<td>41 %</td>
</tr>
</tbody>
</table>

Table 1: Required flutter margins for different types of structural dynamic models

![Figure 3: Flutter boundaries before and after structural dynamic model tuning.](image)
Required flutter margins for the safety of flight were computed and summarized in table 1 and figure 3. It should be noted that the series of Ground Vibration Tests (GVTs) and structural dynamic model tuning has been performed [5, 6, 7] resulting in the computed flutter boundaries, based on the ATW2 configuration in figure 2 and the corresponding TVFEM, shown in figure 3. The GVT for the final ATW2 configuration was performed while the ATW2 was mounted to the Flight Test Fixture (FTF) [8] in the FTF ground handling cart as shown in figure 4. The FTF was sufficiently massive when compared to the ATW2 so that cantilevered boundary conditions were used. It may conclude from table 1 and figure 3 that when only the structural dynamic model is validated with respect to GVT data, the flutter margin required for the flutter certification of the ATW2 should be approximately 40%. In addition to the historically stand-alone structural dynamic model, the unsteady aerodynamic model should also be validated with the test data to use the 15% flutter margin in the military specification [9].

Figure 4: Flight test fixture and ATW2 on ground handling cart.

2 UNSTEADY AERODYNAMIC MODEL TUNING PROCEDURE

A simple technique has been proposed and developed to update unsteady aerodynamic models. The technique is based on matching the measured and numerical aeroelastic frequencies of an aircraft structure. In defining the optimization problem to match the measured aeroelastic frequencies, the variation of the unsteady aerodynamic force was selected as the design parameter. The unsteady aerodynamic force is a function of Mach number, reduced frequency, and dynamic pressure; which can be obtained based on any aerodynamic model. ZAERO [10] code is used in this study. If the Mach number is constant, the reduced frequency and dynamic pressure become variables for changing the unsteady aerodynamic force.

Supporting the Aeronautics Research Mission Directorate guidelines, NASA DFRC has developed an Object-Oriented Optimization (O³) tool [11] which leverages existing tools and
practices, and allows the easy integration and adoption of new state-of-the-art software. Unsteady aerodynamic model tuning used in this study is based on the minimization of the discrepancies in numerical and measured aeroelastic frequencies. A computer code for unsteady aerodynamic model tuning has been developed using the O³ tool together with the pre-processor, ZAERO, and post-processor codes shown in figure 5.

![Figure 5: Unsteady aerodynamic model tuning using object-oriented optimization](image)

### 2.1 Pre-processor code

This code reads in design variables generated by the O³ tool, and then reads modal Aerodynamic Influence Coefficient (AIC) matrices that were computed and saved using ZAERO code. Modified modal AIC matrices are then created as shown in figure 6.

![Figure 6: Flow-chart of the pre-processor](image)
Design variables in this unsteady aerodynamic model tuning are scaling factors for each element in the AIC matrices. The AIC matrix, $A$, at a reduced frequency can be written as:

$$A = \begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mn}
\end{bmatrix} + i \begin{bmatrix}
b_{11} & b_{12} & \cdots & b_{1n} \\
b_{21} & b_{22} & \cdots & b_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
b_{m1} & b_{m2} & \cdots & b_{mn}
\end{bmatrix}.$$  

Where $m$ is number of degrees of freedom, $n$ is number of modes, and $a_{ij}$ and $b_{ij}$ are the $i$-th row and $j$-th column element of the real and imaginary part of the matrix $A$, respectively.

Design variables, $e_{11}, e_{12}, \ldots, e_{21}, e_{22}, \ldots, e_{mn}, f_{11}, f_{12}, \ldots, f_{21}, f_{22}, \ldots, f_{mn}$, are defined as:

$$\begin{bmatrix}e_{11}, e_{12}, \ldots, e_{1n}, e_{21}, e_{22}, \ldots, e_{2n} \\
e_{21}, e_{22}, \ldots, e_{n1}, e_{n2}, \ldots, e_{nn}
\end{bmatrix} + i \begin{bmatrix}f_{11}, f_{12}, \ldots, f_{1n}, f_{21}, f_{22}, \ldots, f_{2n} \\
f_{21}, f_{22}, \ldots, f_{n1}, f_{n2}, \ldots, f_{nn}
\end{bmatrix}.$$  

Where $e_{ij}$ is the design variable for $a_{ij}$ and $f_{ij}$ is the design variable for $b_{ij}$. The following design variable linking options are available for the unsteady aerodynamic model tuning.

**Option 1:** single design variable

$$d = e_{11} = e_{12} = \ldots = e_{mn} = f_{11} = f_{12} = \ldots = f_{mn}$$

**Option 2:** two design variables

$$d_1 = e_{11} = e_{12} = \ldots = e_{mn} \text{; real part}$$

$$d_2 = f_{11} = f_{12} = \ldots = f_{mn} \text{; imaginary part}$$

**Option 3:** columnwise the same design variables (total $n$ design variables)

$$d_1 = e_{11} = e_{21} = \ldots = e_{m1} = f_{11} = f_{21} = \ldots = f_{m1}$$

$$d_2 = e_{12} = e_{22} = \ldots = e_{m2} = f_{12} = f_{22} = \ldots = f_{m2}$$

$$\vdots$$

$$d_n = e_{1n} = e_{2n} = \ldots = e_{mn} = f_{1n} = f_{2n} = \ldots = f_{mn};$$

**Option 4:** columnwise the same design variables (total $2n$ design variables)

$$d_1 = e_{11} = e_{12} = \ldots = e_{m1}$$

$$d_2 = e_{12} = e_{22} = \ldots = e_{m2}$$

$$\vdots$$

$$d_n = e_{1n} = e_{2n} = \ldots = e_{mn}; \text{ real parts}$$

$$d_{n+1} = f_{11} = f_{21} = \ldots = f_{m1}$$

$$d_{n+2} = f_{12} = f_{22} = \ldots = f_{m2}$$

$$\vdots$$

$$d_{2n} = f_{1n} = f_{2n} = \ldots = f_{mn}; \text{ imaginary parts}$$

**Option 5:** No design variable linking; total $2mn$ design variables.
2.2 ZAERO flutter analysis

Flutter analyses in this study are based on ZAERO code. This code acquires the modified modal AIC matrices and performs the matched flutter analysis as shown in figure 7. This computer simulation requires the natural frequencies and mode shapes of the aircraft, and in this study these modal data are computed using MSC/NASTRAN code [12]. The V-g and V-f data are computed and saved for the next post-processing step. Here, V is aircraft speed in Keas, g is structural damping, and f is aeroelastic frequency in Hz.

![Flow-chart of the flutter analysis procedure](image)

2.3 Post-processor code

This program reads in the V-g and V-f data and the target altitude where the flight test was performed. Based on the velocity information V in the V-g and V-f data, corresponding altitudes at fixed Mach numbers are computed. Numerical aeroelastic frequencies are computed from target and computed altitudes using the cubic splining procedure.

This program also computes the frequency difference between the numerical and measured aeroelastic frequencies. Frequency difference will be an objective function, which will be minimized through the use of the O3 tool. The proposed tuning technique is an unconstrained optimization problem that can be solved using a gradient based optimizer [13], a genetic algorithm [14], or a big-bang-big-crunch algorithm [15, 16, 17]. A flow-chart of this post-processor code is shown in figure 8.

![Flow-chart of the post-processor](image)
3 APPLICATION

During the flight test of the ATW2, a classical bending and torsion type of flutter, as shown in figure 9, was observed near a Mach number of 0.502 and a flight altitude of 9,837ft.

Figure 9: Classical bending and torsion flutter during flight test

Measured Mach number, flight altitude, and acceleration at the middle and leading edge of the wing tip boom after take-off, near flutter, and during flutter are shown in figures 10 and 11, respectively. Measured aeroelastic frequencies during the flight test as well as natural frequencies computed using the TVFEM and the measured during GVT are shown in table 2.

Figure 10: Measured Mach number, flight altitude, and acceleration at the middle and the leading edge of the boom after take off (time steps 83720 sec to 83750 sec)
The first model tuning is performed using flight data after take-off, from 83,742 s to 83,745 s as shown in figure 10. The ATW2 in this time period is a time invariant system. The average flight Mach number and altitude during this time period were 0.39 and 9,934 ft, respectively. Unfortunately, measured acceleration data in figure 10 was noisy, and it was quite difficult to estimate the first and third aeroelastic frequencies because of high aerodynamic damping. The second measured aeroelastic frequency is 40.45 Hz as shown in table 2. An initial aeroelastic frequency of 41.12 Hz is computed using ZAERO code as shown in table 3.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Natural Frequencies</th>
<th>Measured Aeroelastic Frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TVFEM</td>
<td>GVT</td>
</tr>
<tr>
<td>1</td>
<td>17.45</td>
<td>17.45(0.623)†</td>
</tr>
<tr>
<td>2</td>
<td>43.48</td>
<td>43.72(0.610)</td>
</tr>
<tr>
<td>3</td>
<td>133.6</td>
<td>N/A</td>
</tr>
</tbody>
</table>

*: Time steps 83,742 sec to 83,745 sec (Mach = 0.39 & altitude = 9,934 ft; time invariant)
**: Time steps 84,668 sec to 84,672 sec (Mach = 0.456 & altitude = 9,858 ft; time invariant)
***: Time steps 84,683 sec to 84,684 sec (Mach = 0.502 & altitude = 9,837 ft; time varying)
( †): Measured damping (%)

Table 2: Numerical and measured frequencies (Hz) of the ATW2 during the flight test

<table>
<thead>
<tr>
<th>Mach Number</th>
<th>Measured (Hz)</th>
<th>Altitude (ft)</th>
<th>Before Tuning (Hz)</th>
<th>Scaling Factor (design variable)</th>
<th>After Tuning (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.390</td>
<td>40.45</td>
<td>9,934</td>
<td>41.12</td>
<td>1.2579</td>
<td>40.45</td>
</tr>
<tr>
<td>0.456</td>
<td>38.99</td>
<td>9,858</td>
<td>40.10</td>
<td>1.2719</td>
<td>38.99</td>
</tr>
</tbody>
</table>

Table 3: The second aeroelastic frequency before and after unsteady aerodynamic model tuning and corresponding scaling factors

In this model tuning procedure, the aeroelastic frequency difference in the second mode is minimized using the design variable linking option 1. The number of target aeroelastic frequencies to be matched is one, and therefore the simplest option is selected. In other words, there is an unconstrained optimization problem with a single design variable. After model tuning, the second aeroelastic frequency of 41.12 Hz becomes 40.45 Hz, and the corresponding scaling factor (single design variable) is 1.2579 as shown in table 3.

<table>
<thead>
<tr>
<th>Comment</th>
<th>Scaling Factor (design variable)</th>
<th>Flutter Speed</th>
<th>Altitude (ft)</th>
<th>Flutter Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td>N/A</td>
<td>276.4</td>
<td>0.00</td>
<td>9,836.9</td>
</tr>
<tr>
<td>Before tuning</td>
<td>1.0</td>
<td>311.3</td>
<td>13.0</td>
<td>3,561.5</td>
</tr>
<tr>
<td>Use M=0.39 Aero</td>
<td>1.2579</td>
<td>277.3</td>
<td>0.33</td>
<td>9,670.0</td>
</tr>
<tr>
<td>Use M=0.456 Aero</td>
<td>1.2719</td>
<td>276.0</td>
<td>-0.14</td>
<td>9,912.5</td>
</tr>
</tbody>
</table>

Table 4: Measured and computed flutter boundaries at Mach = 0.502

The saved AIC matrices at Mach = 0.502 is updated using the scaling factor of 1.2579, and the updated flutter boundary at this Mach number is summarized in table 4 and figure 12. The tuned flutter speed, corresponding altitude, and flutter frequency are 277.3 Keas, 9,670 ft, and
37.69 Hz, respectively. It should be noted in table 4 that flutter speed and frequency difference after the unsteady aerodynamic model tuning are 0.33% and 0.00%, respectively.

The second model tuning is performed using another time invariant system, between time steps of 84,660 s and 84,690 s. Relatively flat time histories of flight Mach numbers and altitude, between time steps of 84,668 s and 84,672 s, are observed as shown in figure 11. In this time period, the average flight Mach number and altitude were 0.456 and 9,858 ft, respectively.

The second measured aeroelastic frequency in these time steps is 38.99 Hz as shown in table 2. The corresponding aeroelastic frequency computed from the ZAERO simulation with Mach 0.456 aerodynamics is 40.10 Hz as shown in table 3. Unsteady aerodynamic model tuning is performed using these two numerical and measured frequencies, and a scaling factor of 1.2719 based on the design variable linking option 1, which is also given in table 3.

Flutter analysis at Mach 0.502 is performed using this new scaling factor, and the flutter speed and frequency are summarized in table 4 and figure 12. Flutter speed difference of 13% before the unsteady aerodynamic model tuning becomes -0.14% after tuning.
Figure 12: Flutter boundaries at Mach=0.502 before and after unsteady aerodynamic model tuning.

In case of the ATW2, computation time required for completing an unsteady aerodynamic model tuning based on option 1 was less than 7 min. Once the scaling factor (design variable) is computed, an additional 1 or 2 min is needed for the Fast Fourier Transformation, one more flutter analysis at a higher Mach number to compute the updated V-g and V-f data, and automatic computations of updated flutter speed and frequency. Therefore, less than 9 min are enough to predict more accurate flutter speed based on the current flight test data.

4 CONCLUSION

A simple unsteady aerodynamic model tuning based on the direct AIC modification is proposed in this study. The value of the unsteady aerodynamic model tuning procedure has been shown with the application to the ATW2 flight test data.

Flutter boundaries and the ATW2 flight test envelope were computed using the TVFEM [5, 6]. The flutter margins required for the safety of flight were approximately 40% when only the structural dynamic model was validated. Excellent flutter speed matching is accomplished when the simple unsteady aerodynamic model tuning is applied resulting in flutter speed differences of 0.33% and -0.14%. The flutter margin requirement of 15% in the military specification can now be used with the test validated aeroelastic model, that is test validated structural dynamic and unsteady aerodynamic models. The modeling uncertainties associated with the unsteady aerodynamics can be easily minimized through the use of the simple model tuning procedure proposed in this study.

Unsteady aerodynamic model tunings are performed at two time invariant flight conditions, at Mach numbers of 0.390 and 0.456. When the Mach number for the unsteady aerodynamic
model tuning approaches to the measured fluttering Mach number, 0.502, at the flight altitude of 9,837 ft, the estimated flutter speed is approached to the measured flutter speed at this altitude. Therefore, we may conclude that the Mach number selected for the unsteady aerodynamic model tuning is closer to the measured fluttering Mach number at the same flight altitude, and we may get a more accurate scaling factor for the precise flutter prediction.

5 REFERENCES


Unsteady Aerodynamic Model Tuning for Precise Flutter Prediction

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NASA Dryden Flight Research Center
Objectives

- The primary objective of this study is to reduce uncertainties in the unsteady aerodynamic model of an aircraft to increase the safety of flight.

- This model tuning technique is applied to improve the flutter prediction of the Aerostructures Test Wing 2.

- This work is supported by the Aeronautics Research Mission Directorate (ARMD) Subsonic Fixed Wing (SFW) and Supersonics (SUP) projects under Fundamental Aeronautics (FA) program.
Flutter Analysis Procedure @ NASA Dryden

- Everyone believes the test data except for the experimentalist, and no one believes the finite element model except for the analyst.
  - Some of the discrepancies come from analytical Finite Element modeling uncertainties, noise in the test results, and/or inadequate sensor and actuator locations. Not the same orientation for each sensor.

- Flutter Analysis
  - Uncertainties in the structural dynamic model are minimized through the use of “model tuning technique”
  - Based on analytical modes

- Validate Structural Dynamic Finite Element Model using Test Data and Update if needed
  - Use MDAO (Multidisciplinary Design, Analysis, and Optimization) tool with Model Tuning Capability or Standalone Model Tuning Code
    - Model tuning is based on optimization.
      - Design Variables
        - Structural sizing information: Thickness, cross sectional area, area moment of inertia, etc.
        - Point properties: lumped mass, spring constant, etc.
        - Material properties: density, Young’s modulus, etc.
      - Constraints
Pre-Test Modal & Flutter Analyses of ATW2
Structural Dynamic & Unsteady Aerodynamic Models

- **Structural Dynamic Finite Element Model**
  - Based on MSC/NASTRAN code
  - Use ATW1 Structural Dynamic Finite Element Model (265 nodes)
    - ATW1 & ATW2: Based on same drawing
  - Use 10 modes for the flutter analysis

- **Unsteady Aerodynamic Model**
  - Based on ZAERO code
  - 416 elements
  - Select 16 reduced frequencies between 0 & 1
  - Mach = .60, .75, .82, and .95
  - Linear Theory
  - Use Matched Flutter Analysis

Cross Sectional Shape: NACA-65A004
Splined Mode Shapes

Mode 1: 17.60 Hz
First Bending

Mode 2: 23.26 Hz
First Torsion

Mode 3: 93.99 Hz
Second Bending
V-g and V-f Curves at Mach = 0.82 Before Model Tuning

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Modal Participation Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.60</td>
<td>75.0 %</td>
</tr>
<tr>
<td>2</td>
<td>23.26</td>
<td>16.8 %</td>
</tr>
<tr>
<td>3</td>
<td>93.99</td>
<td>4.8 %</td>
</tr>
<tr>
<td>4</td>
<td>135.4</td>
<td>0.0 %</td>
</tr>
<tr>
<td>5</td>
<td>163.1</td>
<td>2.6 %</td>
</tr>
<tr>
<td>6</td>
<td>174.5</td>
<td>0.0 %</td>
</tr>
<tr>
<td>7</td>
<td>257.5</td>
<td>0.5 %</td>
</tr>
<tr>
<td>8</td>
<td>391.6</td>
<td>0.0 %</td>
</tr>
<tr>
<td>9</td>
<td>394.3</td>
<td>0.1 %</td>
</tr>
<tr>
<td>10</td>
<td>445.6</td>
<td>0.3 %</td>
</tr>
</tbody>
</table>

Flutter Mode | Speed (Keas) | Frequency (Hz) | Altitude (ft) |
-------------|--------------|----------------|---------------|
1            | 407.4        | 22.86          | 15010         |
Finite Element Model Tuning of the Aerostructures Test Wing 2 Using Ground Vibration Test Data
Structural Dynamic Model Tuning using GVT Data

- MIL-STD-1540C Section 6.2.10
  - Test Requirements for Launch, Upper-Stage, & Space Vehicles
  - Less than 3% and 10% frequency errors for the primary and secondary modes, respectively
  - Less than 10% off-diagonal terms in orthonormalized mass matrix

- AFFTC-TIH-90-001 (Structures Flight Test Handbook)
  - If measured mode shapes are going to be associated with a finite element model of the structure, it will probably need to be adjusted to match the lumped mass modeling of the analysis.
  - Based on the measured mode shape matrix $\Phi$ and the analytical mass matrix $M$, the following operation is performed:

$$\Phi^T M \Phi$$

- The results is near diagonalization of the resulting matrix with values close to 1 on the diagonal and values close to zero in the off-diagonal terms. Experimental reality dictates that the data will not produce exact unity or null values, so 10 percent of these targets are accepted as good orthogonality and the data can be confidently correlated with the finite element model.
Structural Dynamic Model Tuning Procedure

  - Applied to beam finite element model using in-house FEM

  - Using in-house object-oriented optimization tool
  - Structural model tuning is based on MSC/NASTRAN code.
    - Single performance index for frequency differences
    - Single performance index for off-diagonal terms of the orthonormalized mass matrix

  - Extended methods are used
    - Multiple performance indices for frequency differences
    - Multiple performance indices for off-diagonal terms of the orthonormalized mass matrix
Test Setup: #1 GVT (Strong Back Mounting)

- Before installing Teardrop Accelerometers & TAO Pressure Sensors
- Strong Back Mounting @ Flight Loads Lab
- Use Photogrammetry Optical Measuring System
Additional Sensors for Flight Test

Bottom View

Mounting Panel (Not Shown Fully)

Strain Bridges (10)

Piezoelectric Excitation Patch Pairs

Strain Bridges (4)

Teardrop Accelerometers (2: Inside Boom)

Teardrop Accelerometers (2: Epoxied onto Surface)

TAO Pressure Sensors
Test Setup: #2 GVT (FTF Mounting)

- After installing 4 Teardrop Accelerometers & 2 TAO Pressure Sensors
- Flight Test Fixture Mounting @ F-15B Hanger
- Flight Test Fixture was lot heavier than ATW2.
  - FTF ≈ 500 lb vs. ATW2 = 2.66 lb: 500/2.66≈188 >> 10
Test Setup: #3 GVT (FTF Mounting under F-15)

- After installing under the Center Fuselage Pylon
- Add Flexibilities between Flight Test Fixture & Center Fuselage Pylon
## Results (Frequency Comparisons)

<table>
<thead>
<tr>
<th>Mode</th>
<th>GVT (Hz)</th>
<th>Before Tuning</th>
<th>After Tuning (Target #2 GVT data)</th>
<th>MIL-STD (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#1: Strong Back</td>
<td>#2: FTF</td>
<td>#3: FTF &amp; F15B</td>
<td>Freq (Hz)</td>
</tr>
<tr>
<td></td>
<td>Wrt #1</td>
<td>Wrt #2</td>
<td>Wrt #3</td>
<td>Wrt #1</td>
</tr>
<tr>
<td>1</td>
<td>17.24</td>
<td>17.45</td>
<td>17.42</td>
<td>17.60</td>
</tr>
<tr>
<td>2</td>
<td>44.10</td>
<td>43.72</td>
<td>43.73</td>
<td>23.26</td>
</tr>
<tr>
<td>3</td>
<td>84.00</td>
<td>83.66</td>
<td>84.14</td>
<td>93.99</td>
</tr>
<tr>
<td>4</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>135.4</td>
</tr>
<tr>
<td>5</td>
<td>N/A</td>
<td>142.3</td>
<td>143.0</td>
<td>163.1</td>
</tr>
</tbody>
</table>

![Mode 1](image1)
![Mode 2](image2)
![Mode 3](image3)
![Mode 5](image4)

**Legend:**
- **Before**
- **After**
- **#2 GVT**

**Graph:**
- X-axis: Mode Number
- Y-axis: Frequency (Hz)

**Notes:**
- MIL-STD: Military Standard
Results (Total Weight, Orthogonality, & MAC)

<table>
<thead>
<tr>
<th></th>
<th>Measured</th>
<th>Before Tuning</th>
<th>After Tuning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Weight</td>
<td>2.66 lb</td>
<td>1.76 lb (error 34%)</td>
<td>2.85 lb (error 7.1%)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Orthonormalized Mass Matrix</th>
<th></th>
<th>1</th>
<th>2</th>
<th>Violate</th>
<th>3</th>
<th>1</th>
<th>2</th>
<th>Satisfy</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td>-24.9%</td>
<td>38.0%</td>
<td>1</td>
<td></td>
<td>-1.92%</td>
<td>-4.46%</td>
</tr>
<tr>
<td>2</td>
<td>-.249</td>
<td>1</td>
<td></td>
<td>-66.1%</td>
<td></td>
<td></td>
<td>-.0192</td>
<td>1</td>
<td>6.16%</td>
</tr>
<tr>
<td>3</td>
<td>.380</td>
<td></td>
<td>-.661</td>
<td>1</td>
<td>-.0446</td>
<td>.0616</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MAC</th>
<th>Mode 1</th>
<th>.97</th>
<th></th>
<th></th>
<th>.99</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mode 2</td>
<td>.70</td>
<td></td>
<td></td>
<td>.99</td>
</tr>
<tr>
<td></td>
<td>Mode 3</td>
<td>.75</td>
<td></td>
<td></td>
<td>.98</td>
</tr>
</tbody>
</table>

MIL-STD & AFFTC-TIH-90-001 Requirements: 10%
Flutter Analysis Using Validated Structural Dynamic Finite Element Model
V-g and V-f Curves at Mach = 0.82 After Model Tuning

### Table 1: Modal Participation Factors

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Modal Participation Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.45</td>
<td>10.7%</td>
</tr>
<tr>
<td>2</td>
<td>43.72</td>
<td>87.2%</td>
</tr>
<tr>
<td>3</td>
<td>83.66</td>
<td>1.6%</td>
</tr>
</tbody>
</table>

### Table 2: Flutter Mode

<table>
<thead>
<tr>
<th>Mode</th>
<th>Speed (Keas)</th>
<th>Frequency (Hz)</th>
<th>Altitude (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>341.5</td>
<td>34.59</td>
<td>23475</td>
</tr>
</tbody>
</table>
Flutter Boundaries vs. Flight Envelope

Flutter Boundary Before Model Tuning (3% structural damping)
Flutter Boundary After Model Tuning (3% structural damping; FEM based on #1 GVT data)
Flutter Boundary After Model Tuning (3% structural damping; FEM based on #2 GVT data)
Flutter Boundary After Model Tuning (measured structural damping; FEM based on #2 GVT data)
Flight Test & Summary of Flutter Margins
ATW II Flight Test

NASA Dryden Flight Research Center

December 15, 2009
Summary of Flutter Margins

- Purple: Flutter Boundary Before Model Tuning (3% structural damping)
- Green: Flutter Boundary After Model Tuning (3% structural damping; FEM based on #1 GVT data)
- Red: Flutter Boundary After Model Tuning (3% structural damping; FEM based on #2 GVT data)
- Blue: Flutter Boundary After Model Tuning (measured structural damping; FEM based on #2 GVT data)

- O: ATW2 Measured Flutter Boundary
- +: ATW2 Measured Flutter Boundary/1.15

Diagram shows the relationship between altitude (ft) and Mach number, with marked percentages indicating various flutter boundaries.
Summary of Flutter Margins (continued)

<table>
<thead>
<tr>
<th>Flutter Boundaries</th>
<th>Flutter Speed Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured/1.15 = $V_d$</td>
<td>0 %</td>
</tr>
<tr>
<td>Measured = 1.15 $V_d$</td>
<td>15 %</td>
</tr>
<tr>
<td>Test validated FEM; using #2 GVT data; with measured damping</td>
<td>32%</td>
</tr>
<tr>
<td>Test validated FEM; using #2 GVT data; with 3% structural damping</td>
<td>41%</td>
</tr>
<tr>
<td>Test validated FEM; using #1 GVT data; with 3% structural damping</td>
<td>52%</td>
</tr>
<tr>
<td>FEM; before model tuning; with 3% structural damping</td>
<td>74%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Validated Structural Dynamic Model</th>
<th>Validated Unsteady Aerodynamic Model</th>
<th>Recommended* Flutter Margins</th>
<th>ATW2 Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>15%</td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>No</td>
<td>49%</td>
<td>32 – 52 %</td>
</tr>
<tr>
<td>No</td>
<td>No</td>
<td>54%</td>
<td>74%</td>
</tr>
</tbody>
</table>

*: DOD’s JSSG-2006 Guidelines for Flutter Speed Clearance; Faustino Zapata, AFDC May 22-23, 2008 JSSG(Joint Service Specification Guide)

During Design Phase (paper plane phase):
- Can we use the 15% flutter margin??
- If not, then what percentage of flutter margin should be used. 54% ??
Unsteady Aerodynamic Model Tuning
New Flutter Analysis Procedure @ NASA Dryden

Old Procedure:
1. Weight, C.G., Moment of inertia, & GVT data
2. Structural Dynamic Finite Element Model Tuning
3. Validated Structural Dynamic Model
4. Create Unsteady Aerodynamic Model
5. Perform Flutter Analysis

New Procedure:
1. Weight, C.G., Moment of inertia, & GVT data
2. Structural Dynamic Finite Element Model Tuning
3. Validated Structural Dynamic Model
4. Create Unsteady Aerodynamic Model
5. Perform Flutter Analysis
6. Measure Frequencies During Flight Test
Unsteady Aerodynamic Model Tuning

- **Objective Function:**
  
  Minimize \( J = \) measured aeroelastic frequency – computed aeroelastic frequency

- **Design Variables:** \( e_{ij} \) & \( f_{ij} \)

\[
\begin{bmatrix}
e_{11}a_{11} & e_{12}a_{12} & \cdots & e_{1n}a_{1n} \\
e_{21}a_{21} & e_{22}a_{22} & \cdots & e_{2n}a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
e_{m1}a_{m1} & e_{m2}a_{m2} & \cdots & e_{mn}a_{mn}
\end{bmatrix} + \begin{bmatrix}
f_{11}b_{11} & f_{12}b_{12} & \cdots & f_{1n}b_{1n} \\
f_{21}b_{21} & f_{22}b_{22} & \cdots & f_{2n}b_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
f_{m1}b_{m1} & f_{m2}b_{m2} & \cdots & f_{mn}b_{mn}
\end{bmatrix} A = \begin{bmatrix}a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mn}\end{bmatrix} + \begin{bmatrix}b_{11} & b_{12} & \cdots & b_{1n} \\
b_{21} & b_{22} & \cdots & b_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
b_{m1} & b_{m2} & \cdots & b_{mn}\end{bmatrix}
\]

- **Design Variable Linking:**

  - **Option 1:** single design variable
    
    \( d = e_{11} = e_{12} = \cdots = e_{mn} = f_{11} = f_{12} = \cdots = f_{mn} \)

  - **Option 2:** two design variables
    
    \( d_1 = e_{11} = e_{12} = \cdots = e_{mn}; \) real part  \( d_2 = f_{11} = f_{12} = \cdots = f_{mn}; \) imaginary part

  - **Option 3:** columnwise the same design variables (total \( n \) design variables)
    
    \( d_1 = e_{11} = e_{21} = \cdots = e_{m1} = f_{11} = f_{21} = \cdots = f_{m1} \)
    
    \( d_2 = e_{12} = e_{22} = \cdots = e_{m2} = f_{12} = f_{22} = \cdots = f_{m2} \)
    
    \( \cdots \)
    
    \( d_n = e_{1n} = e_{2n} = \cdots = e_{mn} = f_{1n} = f_{2n} = \cdots = f_{mn}; \)

  - **Option 4:** columnwise the same design variables (total \( 2n \) design variables)
    
    \( d_1 = e_{11} = e_{21} = \cdots = e_{m1} \quad d_2 = e_{12} = e_{22} = \cdots = e_{m2} \quad \cdots \quad d_n = e_{1n} = e_{2n} = \cdots = e_{mn}; \) real parts
    
    \( d_{n+1} = f_{11} = f_{21} = \cdots = f_{m1} \quad d_{n+2} = f_{12} = f_{22} = \cdots = f_{m2} \quad \cdots \quad d_{2n} = f_{1n} = f_{2n} = \cdots = f_{mn}; \) imaginary parts

  - **Option 5:** No design variable linking; total \( 2mn \) design variables.
Unsteady Aerodynamic Model Tuning using Object-Oriented Optimization Tool

The NASA Dryden has developed an Object Oriented Optimization ($O^3$) tool.

- The $O^3$ tool leverages existing tools and practices, and allows the easy integration and adoption of new state-of-the-art software.
- Local gradient based optimizer as well as global optimizers are available. Hybrid methods are also available.

- Optimizers:
  - DOT (local), Genetic Algorithm (GA), & Big Bang-Big Crunch (BBBC) algorithm

- Hybrid optimizers:
  - GA(CDV)+DOT(CDV), GA(CDV)+DOT(CDV)+GA(DDV), BBBC(CDV)+DOT(CDV), & BBBC(CDV)+DOT(CDV)+BBBC(DDV)

The $O^3$ tool leverages existing tools and practices, and allows the easy integration and adoption of new state-of-the-art software.
Pre-Processor

Read Design Variables

Read Modal AICs

Read Option Number for Design Variable Linking

not changed during optimization

Compute Modified Modal AICs

Write Modified Modal AICs

Modal AIC

Pre-Processor

Input Data

Modified Modal AIC

NASTRAN ~.f06 file Frequency & Mode Shapes

Post-Processor

V-g & V-f

Frequencies Measured from Flight Test

ZAERO Flutter Analysis

Object Oriented Optimization Tool

Optimizer

Objective Function J & Constraints G(x)

Design Variables

Script Commands

Performance Indices

Frequency Difference

Input Data
Flutter Analysis

- Based on ZAERO code
- Modal analysis is based on NASTRAN

1. Read Modified Modal AICs
2. Read Natural Frequencies & Mode Shapes
3. Read ZAERO Input Data (not changed during optimization)

Compute & Save V-g & V-f Data using ZAERO code

Input Data → Pre-Processor → Modal AIC → ZAERO Flutter Analysis → Post-Processor → V-g & V-f

Input Data → NASTRAN ~.f06 file Frequency & Mode Shapes

Frequency Measured from Flight Test

Input Data → V-g & V-f

Object Oriented Optimization Tool

- Objective Function J & Constraints G(x)
- Design Variables
- Script Commands
- Performance Indices

Frequency Difference

ZAERO Input Data not changed during optimization
Post-Processor

Read V-g & V-f Data

Compute Aeroelastic Frequencies \( f \)

Compute Frequency Difference

Write Frequency Difference

Read Flight Test Altitude & Measured Aeroelastic Frequencies

not changed during optimization

Read

Flight Test Altitude & Measured Aeroelastic Frequencies

Objective Function J & Constraints \( G(x) \)

Optimizer

Design Variables

Object Oriented Optimization Tool

Performance Indices

Script Commands

Frequency Difference

Modal AIC

Pre-Processor

Input Data

Modified Modal AIC

Input Data

NASTRAN ~.f06 file
Frequency & Mode Shapes

V-g & V-f

Input Data

Frequencies Measured from Flight Test

Compute Aeroelastic Frequencies

Frequency Difference

Frequency Difference

Write Frequency Difference
Time-Invariant Flight Conditions

- **First selection:** Mach = 0.390, Altitude = 9934 ft
- **Second selection:** Mach = 0.456, Altitude = 9858 ft
- **Flutter condition:** Mach = 0.502, Altitude = 9837 ft (time varying)
# Results

## Numerical and measured frequencies (Hz) of the ATW2 during flight test

<table>
<thead>
<tr>
<th>Mode</th>
<th>Natural Frequencies (Hz)</th>
<th>Measured Aeroelastic Frequencies (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test Validated FEM</td>
<td>GVT</td>
</tr>
<tr>
<td></td>
<td></td>
<td>After take off M = 0.390</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Near flutter M = 0.456</td>
</tr>
<tr>
<td></td>
<td></td>
<td>During flutter M = 0.502</td>
</tr>
<tr>
<td>1</td>
<td>17.45</td>
<td>17.45(0.623%)</td>
</tr>
<tr>
<td>2</td>
<td>43.48</td>
<td>43.72(0.610%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40.45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>38.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td>37.69</td>
</tr>
<tr>
<td>3</td>
<td>82.98</td>
<td>83.66(0.778%)</td>
</tr>
<tr>
<td>4</td>
<td>133.6</td>
<td>N/A</td>
</tr>
<tr>
<td>5</td>
<td>153.8</td>
<td>142.3(0.674%)</td>
</tr>
</tbody>
</table>

The second aeroelastic frequency before and after unsteady aerodynamic model tuning and corresponding scaling factors

<table>
<thead>
<tr>
<th>Mach Number</th>
<th>Measured (Hz)</th>
<th>Altitude (ft)</th>
<th>Before Tuning (Hz)</th>
<th>Scaling Factor (design variable)</th>
<th>After Tuning (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.390</td>
<td>40.45</td>
<td>9934</td>
<td>41.12</td>
<td>1.2579</td>
<td>40.45</td>
</tr>
<tr>
<td>0.456</td>
<td>38.99</td>
<td>9858</td>
<td>40.10</td>
<td>1.2719</td>
<td>38.99</td>
</tr>
</tbody>
</table>
Flutter boundaries at Mach=0.502 before and after unsteady aerodynamic model tuning

- X: Measured Flutter Point
- Δ: After Unsteady Aerodynamic Model Tuning (based on M = 0.39 aerodynamics)
- ○: After Unsteady Aerodynamic Model Tuning (based on M = 0.456 aerodynamics)
# Measured and computed flutter boundaries at Mach = 0.502

<table>
<thead>
<tr>
<th>Comment</th>
<th>Scaling Factor (design variable)</th>
<th>Flutter Speed</th>
<th>Altitude ft</th>
<th>Flutter Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Keas</td>
<td>% difference</td>
<td>Hz</td>
</tr>
<tr>
<td>Measured</td>
<td>N/A</td>
<td>276.4</td>
<td>0.00</td>
<td>9836.9</td>
</tr>
<tr>
<td>Before tuning</td>
<td>1.0</td>
<td>311.3</td>
<td>13.0</td>
<td>3561.5</td>
</tr>
<tr>
<td>Use M=0.390 Aero</td>
<td>1.2579</td>
<td>277.3</td>
<td>0.33</td>
<td>9670.0</td>
</tr>
<tr>
<td>Use M=0.456 Aero</td>
<td>1.2719</td>
<td>276.0</td>
<td>-0.14</td>
<td>9912.5</td>
</tr>
</tbody>
</table>
Conclusions

- Model tuning based on flight test data is **needed** to minimize uncertainties in the unsteady aerodynamic model and to increase the safety of flight.

- After model tuning (for ATW 2 case)
  - Maximum of 13%, \(0.13 = \frac{311.3 - 276.4}{276.4}\), flutter speed error becomes -0.14%.

<table>
<thead>
<tr>
<th>Flutter Boundaries</th>
<th>Flutter Mach Number</th>
<th>Flutter Speed Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured/1.15 = (V_d)</td>
<td>0.437</td>
<td>0%</td>
</tr>
<tr>
<td>Measured = 1.15 (V_d)</td>
<td>0.502</td>
<td>15%</td>
</tr>
<tr>
<td>Test validated FEM &amp; unsteady aerodynamics; use (M=0.456) aerodynamics</td>
<td>0.5015</td>
<td>14.8%</td>
</tr>
<tr>
<td>Test validated FEM &amp; unsteady aerodynamics; use (M=0.390) aerodynamics</td>
<td>0.5039</td>
<td>15.4%</td>
</tr>
<tr>
<td>Test validated FEM; using #2 GVT data; with measured damping</td>
<td>0.576</td>
<td>32%</td>
</tr>
<tr>
<td>Test validated FEM; using #2 GVT data; with 3% structural damping</td>
<td>0.616</td>
<td>41%</td>
</tr>
<tr>
<td>Test validated FEM; using #1 GVT data; with 3% structural damping</td>
<td>0.665</td>
<td>52%</td>
</tr>
<tr>
<td>FEM; before model tuning; with 3% structural damping</td>
<td>0.762</td>
<td>74%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Validated Structural Dynamic Model</th>
<th>Validated Unsteady Aerodynamic Model</th>
<th>Recommended Flutter Margins</th>
<th>ATW2 Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>15%</td>
<td>14.8 – 15.4 %</td>
</tr>
<tr>
<td>Yes</td>
<td>No</td>
<td>49%</td>
<td>32 – 52 %</td>
</tr>
<tr>
<td>No</td>
<td>No</td>
<td>54%</td>
<td>74%</td>
</tr>
</tbody>
</table>
Questions?
### Summary of the Modal Participation Factors

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
<th>Modal Participation Factor</th>
<th>Mach = 0.60</th>
<th>Mach = 0.75</th>
<th>Mach = 0.82</th>
<th>Mach = 0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Modal Participa<strong>tion</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Factor</td>
<td>Mach = 0.60</td>
<td>Mach = 0.75</td>
<td>Mach = 0.82</td>
<td>Mach = 0.95</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>17.60 Hz</td>
<td>68.1 %</td>
<td>72.9 %</td>
<td>75.0 %</td>
<td>79.7 %</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>23.26 Hz</td>
<td>22.2 %</td>
<td>18.3 %</td>
<td>16.8 %</td>
<td>13.6 %</td>
<td>97.6 %</td>
</tr>
<tr>
<td>3</td>
<td>93.99 Hz</td>
<td>5.2 %</td>
<td>5.0 %</td>
<td>4.8 %</td>
<td>4.3 %</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>135.4 Hz</td>
<td>0.0 %</td>
<td>0.0 %</td>
<td>0.0 %</td>
<td>0.0 %</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>163.1 Hz</td>
<td>3.3 %</td>
<td>2.9 %</td>
<td>2.6 %</td>
<td>1.9 %</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>174.5 Hz</td>
<td>0.0 %</td>
<td>0.0 %</td>
<td>0.0 %</td>
<td>0.0 %</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>257.5 Hz</td>
<td>0.7 %</td>
<td>0.6 %</td>
<td>0.5 %</td>
<td>0.3 %</td>
<td>2.4 %</td>
</tr>
<tr>
<td>8</td>
<td>391.6 Hz</td>
<td>0.0 %</td>
<td>0.0 %</td>
<td>0.0 %</td>
<td>0.0 %</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>394.3 Hz</td>
<td>0.1 %</td>
<td>0.1 %</td>
<td>0.1 %</td>
<td>0.0 %</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>445.6 Hz</td>
<td>0.4 %</td>
<td>0.3 %</td>
<td>0.3 %</td>
<td>0.2 %</td>
<td></td>
</tr>
</tbody>
</table>

- Participation of the first three modes is a function of Mach number.
- In-plane modes do not participate for the first flutter mechanism at all.
  - Modes 4, 6, and 8
- Primary Modes: Modes 1, 2, and 3
  - Frequency error should be less than 3%.
- Secondary Modes: Modes 4 through 10 (higher)
  - Frequency error should be less than 10%.
## Flutter Results Before & After Model Tuning

### Modal Participation Factors before Model Tuning

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
<th>Mach = 0.60</th>
<th>Mach = 0.75</th>
<th>Mach = 0.82</th>
<th>Mach = 0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.60 Hz</td>
<td>68.1 %</td>
<td>72.9 %</td>
<td>75.0 %</td>
<td>79.7 %</td>
</tr>
<tr>
<td>2</td>
<td>23.26 Hz</td>
<td>22.2 %</td>
<td>18.3 %</td>
<td>16.8 %</td>
<td>13.6 %</td>
</tr>
<tr>
<td>3</td>
<td>93.99 Hz</td>
<td>5.2 %</td>
<td>5.0 %</td>
<td>4.8 %</td>
<td>4.3 %</td>
</tr>
</tbody>
</table>

### Modal Participation Factors after Model Tuning

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
<th>Mach = 0.60</th>
<th>Mach = 0.75</th>
<th>Mach = 0.82</th>
<th>Mach = 0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.45 Hz</td>
<td>5.0 %</td>
<td>8.2 %</td>
<td>10.7 %</td>
<td>22.6 %</td>
</tr>
<tr>
<td>2</td>
<td>43.72 Hz</td>
<td>93.8 %</td>
<td>90.0 %</td>
<td>87.2 %</td>
<td>71.4 %</td>
</tr>
<tr>
<td>3</td>
<td>83.66 Hz</td>
<td>0.9 %</td>
<td>1.4 %</td>
<td>1.6 %</td>
<td>2.4 %</td>
</tr>
</tbody>
</table>

### Speed

<table>
<thead>
<tr>
<th>Before Tuning</th>
<th>After Tuning</th>
<th>Mach = 0.60</th>
<th>Mach = 0.75</th>
<th>Mach = 0.82</th>
<th>Mach = 0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed</td>
<td></td>
<td>453.0 Keas</td>
<td>421.5 Keas</td>
<td>407.4 Keas</td>
<td>377.9 Keas</td>
</tr>
<tr>
<td>Frequency</td>
<td></td>
<td>23.18 Hz</td>
<td>22.97 Hz</td>
<td>22.86 Hz</td>
<td>22.53 Hz</td>
</tr>
<tr>
<td>Altitude</td>
<td></td>
<td>-7501 ft</td>
<td>8751 ft</td>
<td>15010 ft</td>
<td>25590 ft</td>
</tr>
</tbody>
</table>

### Speed Difference

<table>
<thead>
<tr>
<th>Speed Difference</th>
<th>Mach = 0.60</th>
<th>Mach = 0.75</th>
<th>Mach = 0.82</th>
<th>Mach = 0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>33.8 %</td>
<td>23.8 %</td>
<td>19.3 %</td>
<td>9.6 %</td>
<td></td>
</tr>
</tbody>
</table>