Inversion of magnetic measurements of the *CHAMP* satellite over the Pannonian Basin

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Abstract

The Pannonian Basin is a deep intra-continental basin that formed as part of the Alpine orogeny. In order to study the nature of the crustal basement we used the long-wavelength magnetic anomalies acquired by the *CHAMP* satellite. The anomalies were distributed in a spherical shell, some 107,927 data recorded between January 1 and December 31 of 2008. They covered the Pannonian Basin and its vicinity. These anomaly data were interpolated into a spherical grid of 0.5°×0.5°, at the elevation of 324 km by the Gaussian weight function. The vertical gradient of these total magnetic anomalies was also computed and mapped to the surface of a sphere at 324 km elevation. The former spherical anomaly data at 425 km altitude were downward continued to 324 km. To interpret these data at the elevation of 324 km we used an inversion method. A polygonal prism forward model was used for the inversion. The minimum problem was solved numerically by the Simplex and Simulated annealing methods; a \(L_2\) norm in the case of Gaussian distribution parameters and a \(L_1\) norm was used in the case of Laplace distribution parameters. We interpret that the magnetic anomaly was produced by several sources and the effect of the sable magnetization of the exsolution of hemo-ilmenite minerals in the upper crustal metamorphic rocks.

Keywords: *CHAMP*, Pannonian Basin, total and vertical gradient magnetic anomalies, downward continuation, inversion

1. Introduction

The Pannonian Basin extends some 800 by 500 km in a generally east-northeast direction with depths extending to 7 km. Large crustal features produce long wavelength
magnetic anomalies. These anomalies are sufficiently resolved by satellite altitude observations. The German satellite CHAMP was launched on July 15, 2000 (Reigber et al. 2003, 2005) and it finished its mission on September 19, 2010. This satellite measured the gravity and magnetic field of the Earth with high accuracy. The total magnetic data are obtained by a scalar Overhauser magnetometer developed by the Laboratorie d'Electronique de Technologie et d'Instrumentation in Grenoble, France. The accuracy of the scalar magnetic measurements was ±0.5 nT and these magnetic field data was recorded every second (Rother et al. 2003).

CHAMP had a nearly circular, polar orbit its initial elevation of 456 km decreased due to upper atmospheric drag and it was boosted several times. The elevation interval of the orbit was between 319 and 340 km in 2008. The magnetic anomalies used in the present paper had been derived by the NASA by the application of the CHAOS2 model (Olsen et al. 2009).

2. Interpolation and coordinate transformation

The aim of our calculations is the reduction and interpretation of the magnetic anomalies over the Pannonian Basin and its vicinity (latitude, 38° – 52° North; longitude, 14° – 28° East). The magnetic measurements are mapped on a spherical shell at 319–340 km elevation, and these data are given as a function of the latitude, longitude and elevation. The experimental frequencies of the latitude, longitude and elevation distributions of the recorded locations are plotted in Figure 1. As shown, they generally cover our study area. Data whose Kp index was less than or equal to 1. were selected for further processing. Using this criterion we obtained 107,927 data points. These data are interpolated into a spherical grid of 0.5°×0.5° at the elevation of 324 km.

The Gaussian weight function of the 3D interpolation is

\[ w(\Delta_i, k) = \frac{\pi^{3/2}}{k^2} \exp\left(-\frac{\pi^2}{k^2} \Delta_i^2 \right), \]  

where \( k \) is the parameter of the weight function, \( \Delta_i \) is the distance between the \( i \)th data observed and the single reference point of the spherical grid details are given by Végés (1971), Kis and Wittmann (1998), (2002). The interpolated value is normalized by the following:
where \( n \) is the number of data taken into consideration and \( T_i \) is the \( i \)th total magnetic anomaly value. The interpolated total magnetic anomalies are plotted in Figure 2.

For the methods used in this study, the quantitative interpretation of the satellite measured magnetic anomalies requires the transformation from a spherical to Cartesian coordinates. The origin of the Cartesian coordinate system is \( \varphi = 47^\circ \) (latitude) and \( \lambda = 21^\circ \) (longitude) at an elevation of 324 km. In accordance with the general usage in geomagnetism, the coordinate axes \( x \) and \( y \) directed towards the North and East, respectively, while the \( z \)-axis points downwards. The details of these computations are given in the Appendix A.

3. Vertical derivative and downward continuation

Vertical gradient anomalies show good correlation with the probable extension of the geologic body (Blakely 1995). They qualitatively delineate the lateral extension of the magnetic source. The determination of the vertical gradients is a linear transform; its transfer function is given by:

\[
S(f_x, f_y) = 2\pi \left( f_x^2 + f_y^2 \right)^{1/2},
\]

where \( f_x \) and \( f_y \) are the spatial frequencies in the \( x \) and \( y \) axes (Blakely, 1995). It has long been recognized that high frequency amplification is undesirable, since these frequencies possess the lowest signal-to-noise level. In order to eliminate noise this transfer function is multiplied by a two-dimensional Gaussian low-pass window:

\[
S_{LP}(f_x, f_y) = \exp \left( -k^2 \left( f_x^2 + f_y^2 \right) \right).
\]

The parameter \( k \) controls the passed frequency range. The weight function of this transform is:
where $M$ means the confluent hyper-geometric function. The details of this transform are given by Kis and Pusztai (2006). The vertical gradients of the total magnetic anomalies at the altitude of 324 km are plotted in Figure 3. A negative anomaly, with a minimum gradient of 0.01 nT/km, covers the Pannonian Basin. The spatial shape of the vertical gradient anomaly determines the extension of our model in the inversion procedures.

The downward continuation of the magnetic anomalies can also be expressed as a linear transform. Its transfer function is:

$$S_{\text{downward}}(f_x, f_y) = \exp \left( 2\pi h \left( f_x^2 + f_y^2 \right)^{1/2} \right), \quad (6)$$

where $h$ is the downward continuation value. Different authors (Bullard and Cooper, 1948; De Meyer, 1974) have suggested the application of an appropriate window for the computation of this transform. We used, however, Meskó (1984) for our procedure.

$$S_{\text{downward}}(f_x, f_y) = \exp \left( 2\pi h \left( f_x^2 + f_y^2 \right)^{1/2} \right) \quad \text{for} \ (f_x^2 + f_y^2)^{1/2} \leq f_c$$

$$= \exp \left( 2\pi h \left( f_x^2 + f_y^2 \right)^{1/2} - \gamma \left( \left( f_x^2 + f_y^2 \right)^{1/2} - f_c \right) \right) \quad \text{for} \ (f_x^2 + f_y^2)^{1/2} > f_c. \quad (7)$$

This equation is our modified transfer function for the solution of the downward continuation. From an earlier calculation we have the CHAMP total magnetic anomalies on the elevation of 425 km (Taylor et al. 2005). We will use this as our base level for the downward continuation. This transformation is used for the downward continuation plotted in Figure 4. The parameters of the downward continuation are: average sampling interval 41.3 km, $h$ depth of downward continuation. The appropriate value of the parameters of $\gamma$ and $f_c$ is 145 and 0.005, respectively. The anomalies at 425 and 324 km altitudes as well as the downward continued anomalies are presented in Figure 4. The deviation between the calculated magnetic anomalies and the downward continued anomalies is probably caused by the different depths of the complex magnetic sources especially in the North-East part of the downwar ded anomalies, in the territory of the Carpathian Mts.
4. Model of inversion and Bayesian inference

The magnetic anomaly map (324 km) reveals a large NW–SE oriented negative anomaly (-13 nT) in the middle of the Pannonian Basin (Figure 4). The qualitative interpretation of this anomaly was given by Taylor et al. (2005), the reverse magnetization of (-1.5 A/m) of the upper crust was the basis for their interpretation. The anomaly was forward modeled by a triangular polygonal prism using Plouff’s (1976) method. Our calculated magnetic anomaly field (324 km) corresponds to the previous magnetic anomaly field (425 km) (Taylor et al. 2005).

The shape of the forward model and the magnetization are the same as they are in the qualitative interpretation (Taylor et al. 2005). For the solution of the inverse problem the magnetic anomaly of 324 km is used and the direction of the magnetization is $\alpha = -60^\circ$, $\beta = 60^\circ$, $I = 60^\circ$ and $D = 0^\circ$, where $\alpha$ and $\beta$ are the inclination and declination of the magnetization and $I$ and $D$ are the inclination and declination of the Earth magnetic field.

Model parameters are the top and bottom of the polygonal prism, and the three coordinate pairs of the horizontal triangle (Figure 5).

An effective tool of the geophysical inversion is the Bayesian inference. The mathematical basis and theory of the method are summarized, for example, by Box and Tiao (1973) and Tarantola (1987); its geophysical application is given by Duijndam (1988a), (1988b), Menke (1989), Sen and Stoffa (1995). The elements of the measured and model vectors are indicated by $d$ and $m$; they are random variables.

The conditional probabilities in the applied Bayesian equation (Bayes, 1763) is

$$p(m|d) = p(d|m)p(m), \quad (8)$$

where $p(m|d)$ is the $a\ posteriori$ conditional probability density, $p(d|m)$ is the likelihood conditional probability density, and $p(m)$ is the $a\ priori$ probability density.

The multivariate Gaussian $a\ posteriori$ probability can be expressed as the multiplication of the $a\ priori$ and likelihood probability densities. Disregarding the constant multipliers the $a\ posteriori$ probability is given as:
The \textit{a priori} vector expresses the interpreter's decision to select the value of the model parameters, $C_m$ is the \textit{a priori} covariance matrix superscript $T$ indicates the transpose vectors. The variances in the matrix $C_m$ express the uncertainty of the interpreter. $T_{\text{calculated}}(x,y,m)$ represents the calculated magnetic direct problem with the parameters $m$ at the $x,y$ coordinates. $C_D$ is the data covariance matrix. It consists of two parts (1) the measurements uncertainty matrix $C_d$ (the measurements variances), and (2) the model error matrix $C_T$, namely

$$C_D = C_d + C_T. \quad (10)$$

The elements of the model error matrix are also determined by the interpreter. This matrix contains the goodness of the selected model.

The model parameter values of the source can be determined by the solution of an optimum problem. It means maximizing the \textit{a posteriori} probability density as a function of $m$. This is equivalent to minimizing the sum of exponents of the Equation (9). The objective function $E(m)$ has the following form of

$$
E(m) = -(m - m_{\text{a priori}})^T C_m^{-1} (m - m_{\text{a priori}}) +
+ (d_{\text{measured}}(x,y) - T_{\text{calculated}}(x,y,m))^T C_D^{-1} (d_{\text{measured}}(x,y) - T_{\text{calculated}}(x,y,m)). \quad (11)
$$

The minimum of the objective function is determined by a numerical method.

The last step of the interpretation is the calculation of the \textit{a posteriori} covariance matrix $C'_m$. It is given in the form

$$C'_m \approx (G_n^2 C_D^{-1} G_n + C_m^{-1})^{-1} \quad (12)$$
The multivariate Laplace a posteriori probability density distribution is given by the following:

\[ p_{a \text{ posteriori}} \propto \exp \left( - \frac{|m - m^{a \text{ priori}}|}{C_{m}^{1/2}} \right) \cdot \exp \left( - \frac{|d_{\text{measured}}(x, y) - T_{\text{calculated}}(x, y, m)|}{C_{D}^{1/2}} \right), \quad (14) \]

where we disregard the constant multipliers. The objective function is expressed by the equation:

\[ E(m) = \left( \frac{|m - m^{a \text{ priori}}|}{C_{m}^{1/2}} \right) + \left( \frac{|d_{\text{measured}}(x, y) - T_{\text{calculated}}(x, y, m)|}{C_{D}^{1/2}} \right). \quad (15) \]

The minimum problem is solved by the Simplex (Walsh, 1975) and Simulated annealing (Kirkpatrick et al. 1983, Sen and Soffa 1995) methods. The minimum problems are solved by L₂ norm in the case of the Gaussian probability and by L₁ norm in the case of the Laplace probability. Figures 6 and 7 show the objective functions versus iterative steps. These figures illustrate how these two optimum procedures work. The parameters estimated by the former numerical methods are summarized in Table 1. The covariance matrices are diagonal; there are no correlations between the related parameters. The a priori variances are set to \((5 \text{ nT})^2\), variances of the measured data are set to \((0.5 \text{ nT})^2\). Because the complex structure of the direct problem the elements of the a posteriori covariance matrix are approximated by difference quotients of Equation (13).

Figure 8 shows those residual anomalies that determine the application of the parameters given in Table 1. The residuals in the four parts of the figure are calculated for Gaussian and Laplace parameter distributions determined by Simplex and Simulated annealing methods. It can be concluded that the lowest residuals are given by the Laplace distribution parameters obtained by the Simulated annealing method.
5. Possible origin of the magnetization of sources

Large amplitude magnetic anomalies have been mapped by aircraft and satellites over regions of the Earth and by satellite over the Martian crust. Many of these anomalies have negative signs. For our geological interpretation we call upon analogous source regions such as the Mid-Proterozoic granulites in southwestern Sweden (McEnroe et al. 2001); Proterozoic Ána Sira anorthosite in Rogaland, Norway (Robinson et al. 2002, McEnroe et al. 2004, 2005) and the metamorphic complex in Modum district, Southern Norway (Fabian et al. 2008).

The magnetic properties of these rocks have been investigated by the above mentioned authors. They suggest that the stable remanent magnetization is caused by the exsolution of the hematite-ilmenite minerals. This exsolution can produce stable ferrimagnetism which is investigated by model calculation (Robinson et al. 2002) and transmission electron microscopy analysis (McEnroe et al. 2005). These analyses show 1μm to 4nm exsolution of both hematite lamella in ilmenite hosts and ilmenite lamella in hematite hosts. The contact zone of these minerals can produce strong ferromagnetic effect which belongs to neither hematite nor ilmenite source rocks.

According to Kleteschka et al. (2002) stable remanent magnetization can be developed in the hemo-ilmenite minerals. The remanent magnetization is formed in the cooling process. The anti-ferromagnetic hemo-ilmenite lamellas have multi-domain structures. They are able to form intense thermo-remanent magnetization. This process can be more intensive in the exsolution of hematite-ilmenite minerals.

Is this kind of magnetization found in the crust of the Pannonian Basin? The deep magnetic sources are located in the upper crust; they probably belong to the eastern part of Variscan Europe (Szederkényi 1996, Tari and Pamić 1998). The granulite xenoliths and peridotite xenoliths obtained from the Pliocene basaltic rocks located in the Balaton Highlands (Pelso-unit, part of the African Alcapa block). These granulite and peridotite xenoliths discovered in these locations show crustal and upper mantle origins as suggested by Embey-Isztin et al. (2003) and Dobosi et al. (2003). According to the thermo-barometric investigations of these xenoliths they formed at a temperature of 800–900°C and at pressures of 8–15 kbar (0.8 to 15 GPa). These meta-volcanic xenoliths have apparently formed at a depth of 40–50 km. This depth is significantly deeper than the present 25–30 km thick crust. The present crust was developed in the Tertiary due to the NW–SE extension of the Pannonian Basin (Konečný et al. 2002). The alkaline basaltic rocks were formed in the
Pliocene after the intense widespread Miocene calc-alkaline volcanism (Embey-Isztin et al. 2001).

The international literature discusses a few other possible solutions for source of large negative magnetic anomalies. Due to the unknown parameters of the source rock in the upper crust like age or direction of its magnetization, as well as accurate composition of this rock we can not propose proven fact for the source. But the granulite xenoliths and peridotite xenoliths obtained from the Pliocene basaltic rocks located in the Pannonian Basin give light evidence of the significant part of the negative anomaly presented in this study is derived to the exsolution of hematite-ilmenite minerals.

6 Conclusions

The total magnetic field, vertical gradient and downward continued anomalies indicate a magnetic low over the Pannonian Basin. Our inversion method determined that the source region was in the upper crust of the basin. We propose that the strong magnetization can be produced in the crust of the Pannonian Basin. The CHAMP magnetic anomaly can be explained by the exsolution of hemo-ilmenite minerals. It has been previously reported that hematite-ilmenite mineralogy can produce stable remanent magnetization in the crust and in our study we propose that the crustal rocks of the Pannonian Basin display negative remanent magnetization. We further propose that some of the magnetic source bodies were formed during the late Miocene-Pliocene tectonic activity of compression and extension and/or volcanism (Hámor, 2001) and emplaced in the upper crust.

Appendix A

It is often required the transform of the satellite data from the spherical polar coordinates to Cartesian \(xyz\) coordinate system. This transformation can be done in two steps: one translation and rotation.

The origin of the \(XYZ\) coordinate system is in the center of the Earth. The \(X\)-axis is in the plane of the equator and points to the Greenwich meridian. The \(Z\)-axis coincides with the Earth's rotation axis and points upward. The \(Y\)-axis is also in the plane of the equator and perpendicular to the \(XZ\) sheet and points to East. The \(X\), \(Y\), and \(Z\) coordinates of the satellite data are:
\[ X = r \sin \theta \cos \lambda, \quad Y = r \sin \theta \sin \lambda, \quad Z = r \cos \theta, \quad (A.1) \]

where \( r \) is the distance from the center of the Earth, \( \theta \) and \( \lambda \) are the colatitude and longitude, respectively.

Let us translate the origin of the \( x'y'z' \) Cartesian coordinate system in the central point \((r_0, \theta_0, \lambda_0)\), where \( r_0 \) = Earth’s radius + altitude of the satellite. The \( x' \), \( y' \), and \( z' \) axes are parallel to the \( X \), \( Y \), and \( Z \) axes. The \( t_x \), \( t_y \), and \( t_z \) coordinates of this point in the \( X'Y'Z' \) coordinate system are:

\[ t_x = r_0 \sin \theta_0 \cos \lambda_0, \quad t_y = r_0 \sin \theta_0 \sin \lambda_0, \quad t_z = r_0 \cos \theta_0. \quad (A.2) \]

The equations of translation are:

\[ x' = X - t_x, \quad y' = Y - t_y, \quad z' = Z - t_z. \quad (A.3) \]

The origin of the rotated Cartesian \( xyz \) coordinate system is the point \((r_0, \theta_0, \lambda_0)\), where the \( x \)-axis points to the geographic North; the \( y \)-axis to the East; and the \( z \)-axis points downward. The equations of rotation are:

\[ x = -x' \cos \theta_0 \cos \lambda_0 - y' \cos \theta_0 \sin \lambda_0 + z' \sin \theta_0, \]

\[ y = -x' \sin \lambda_0 - y' \cos \lambda_0, \quad (A.4) \]

\[ z = -x' \sin \theta_0 \cos \lambda_0 - y' \sin \theta_0 \sin \lambda_0 + z' \cos \theta_0. \]

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References


Kis, K. I., Wittmann, G., 2002. 3D reduction of satellite magnetic measurements to obtain magnetic anomaly coverage over Europe, J. Geodyn. 33, 117–129.


Captions

Figure 1. Experimental frequency of the CHAMP magnetic anomalies versus latitude, longitude and altitude over the Pannonian Basin and vicinity.

Figure 2. The CHAMP total magnetic anomaly map determined by interpolation of data from the Pannonian Basin region, plotted on an Albers' projection at 324 km altitude; anomalies are in nT with a range of 22 grey levels and a contour interval of 1 nT, inner frame shows the investigated territory.
Figure 3. The vertical gradient map of the CHAMP total magnetic anomaly field, plotted in Albers' projection at the altitude of 324 km; anomalies in nT/km with a 14 grey scale levels and a contour interval of 0.005 nT/km, inner frame shows the investigated territory.

Figure 4. CHAMP magnetic anomaly maps at (a) 425 km and (b) 324 km altitude; (c) downward continued magnetic anomaly map from 425 km to 324 km, the maps are plotted on Albers' projection; grey scale units in nT.

Figure 5. Three dimensional triangular model of the magnetic source body we used in the inverse problem, upper and lower depths are indicated by: $z_T$ and $z_B$, respectively, the triangular base is given by three coordinate pairs: $(x_1,y_1), (x_2,y_2)$ and $(x_3,y_3)$.

Figure 6. Logarithm of the objective functions determined by Simplex method versus iterative steps in the case of the Gaussian and Laplace distribution model parameters.

Figure 7. Logarithm of the objective functions determined by Simulated annealing method versus iterative steps in the case of the Gaussian and Laplace distribution model parameters.

Figure 8. Residual anomalies in the case of the Gauss and Laplace distributed model parameters when the minimum problem is solved by Simplex and Simulated annealing methods; anomalies are in nT unit in a gray scale, horizontal coordinates are given in km.

Table 1. Determined parameters by Simplex and Simulated annealing methods in the case of the Gaussian and Laplace parameter distributions.
Simplex method

Gaussian distribution (solid line)
Laplace distribution (dashed line)
Simulated annealing method

Gaussian distribution (solid line)
Laplace distribution (dashed line)