

Calculating Launch Vehicle Flight Performance Reserve

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Abstract

This paper addresses different methods for determining the amount of extra propellant (flight performance reserve or FPR) that is necessary to reach orbit with a high probability of success. One approach involves assuming that the various influential parameters are independent and that the result behaves as a Gaussian. Alternatively, probabilistic models may be used to determine the vehicle and environmental models that will be available (estimated) for a launch day go/no go decision. High-fidelity closed-loop Monte Carlo simulation determines the amount of propellant used with each random combination of parameters that are still unknown at the time of launch. Using the results of the Monte Carlo simulation, several methods were used to calculate the FPR. The final chosen solution involves determining distributions for the pertinent outputs and running a separate Monte Carlo simulation to obtain a best estimate of the required FPR. This result differs from the result obtained using the other methods sufficiently that the higher fidelity is warranted.

Introduction

This paper focuses on determining how much extra propellant (called flight performance reserve, or FPR) a launch vehicle needs to have available for ensuring it reaches the targeted orbit, in order to compensate for all the uncertainties in vehicle and environmental parameters that impact performance. One of the experiences during Ares I launch vehicle development is that the FPR needed an increase during each design cycle due to increasing fidelity of the system models. Some of the reasons for the increase included simply modeling all the uncertainties for the first time, increasing the fidelity of modeling the propulsion system, modeling low propellant cutoff sensors and their uncertainties, and increasing the uncertainty of propellant tank loading. Therefore, FPR is an area that can eat into margins if there is insufficient conservatism at the start of a program.

The Space Shuttle program uses a procedure that evaluates the impact of each important parameter on the ascent propellant remaining and then assumes the various parameter impacts are independent and that the result behaves as a Gaussian (Ref. 1). Essentially a root sum square (RSS) of the various propellant remaining standard deviations for each input variation is calculated. This approach is tried in this paper for the Ares I vehicle models.

Because a launch vehicle ascent trajectory is highly nonlinear and is influenced by many parameters, running Monte Carlo simulations with all uncertainties included is another potential way to evaluate how much extra fuel and oxidizer is needed to enable the vehicle to reach orbit for the specified percentage of cases. The simulations include high-fidelity models of how the vehicle behaves, including such effects as navigation error, vehicle vibrations, fuel slosh, effect of data delays, dynamics resulting from the engine nozzle movement, and many others. These high-fidelity models would also be used with the RSS approach described above.

The FPR should only cover for the uncertainties that exist on flight day, because any uncertainties that exist when the vehicle is being designed, but are known on flight day can be taken into account in making the go/no go launch decision. Some vehicle parameters that are not well known during design will be better known about a particular launch vehicle prior to launch, so these parameters would be used in designing the trajectory and would

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not be considered launch day uncertainties. This may include, for example, vehicle component weights and certain engine parameters. Developing probabilistic vehicle models for these parameters that are known prior to launch is discussed in Ref. 2. Also the launch day wind and temperature are measured, so all these parameters are known when the decision is made to launch. Assuming the propellant tanks are to be filled to a standard “full” amount (also with uncertainty), FPR covers for the flight day uncertainties, plus any other uncertainties that cannot otherwise be accommodated.

There are vehicle models where plenty of propellant is available, and a precise determination of the FPR needs is not so critical (e.g. a model with lighter masses and higher engine thrust). But during design, the performance-driving cases must be examined closely, and typically during a flight program there are performance-driving launches that make every kilogram precious. This was certainly the case in both the Apollo and Space Shuttle programs, when the lunar rover was added to the payload and when flights to assemble the International Space Station were planned. On the Shuttle, performance shortfalls resulting from lessons learned during initial flights also drove a need for more performance. Other launch vehicles generally have some payloads that are designed after the rocket is already flying and use all the performance available.

Besides the RSS approach, other approaches tried in this paper include fitting a two-dimensional (fuel and oxidizer) Gaussian distribution to the propellant remaining results, using binomial acceptance criteria to obtain the required propellant levels, and the method that was finally adopted that involves determining distributions of the parameters of interest and running a larger Monte Carlo simulation where samples are taken of these distributions.

If it were possible to examine an infinite number of Monte Carlo samples, the statistics in the tail of the probability distribution of propellant remaining would be sufficiently accurate. But computer limitations, and the fact that a number of simulations are needed for varying purposes, means that the tail data have a small number of points. Besides the need to look in the distribution tail for the data of interest (e.g., between the last two data points in a Monte Carlo simulation of 2000 samples), low propellant cutoff sensors become important in the performance-driving cases. If these sensors command the engine shutdown prior to when the normal shutdown would occur (when the guidance system determines the engine should shut down to achieve the nominal orbit injection conditions), any uncertainties associated with these shutdowns will affect the result in only a small number of cases. So the results cannot be considered to be accurate with any reasonable confidence.

The Ares I rocket consists of a First Stage that uses a five-segment solid rocket motor based on Space Shuttle technology, and an Upper Stage that uses a J-2X engine based on updates from Apollo technology. In order to remove any concerns about publishing sensitive data, all liquid oxygen (LO2) remaining values have been multiplied by a factor, and all liquid hydrogen (LH2) remaining values have been multiplied by a different factor. Although this approach will change the results, it will not affect the relative answers. For example, if P is the nominal propellant, and $P_2 = m*P$ is the adjusted propellant, where m is the multiplier, then in the original results $P - FPR$ is the propellant without FPR. $m*(P - FPR) = m*P - m*FPR = P_2 - m*FPR$ and so the propellant without FPR is adjusted by the same factor, with $FPR_2 = m*FPR$. In computing the RSS result, if each variation in propellant is multiplied by m , then m factors out from the square root and the result is that the total RSS estimate is multiplied by m . One additional change made is that some of the uncertainties used are not their true values, so that the sensitivities are not the true ones.

RSS Approach

The FPR should cover for flight day uncertainties (all parameter variations that are uncertain at the time of the launch decision) and for any other uncertainties that must be covered by FPR. An example of the latter is the full engine mixture ratio variation (variation of the ratio of oxidizer used to fuel used) for Ares I. Because the propellant tanks would be filled on launch day regardless of any mixture ratio variation that might be known prior to launch (from testing the engine, for example), FPR sizing during the vehicle design process has to cover for whether that mixture ratio value is high or low, because it could be either for a particular vehicle that is assembled. On launch day, the go/no go decision can be made with this known variation in mind, so the FPR set aside when making that decision could be reduced to reflect the reduction in uncertainty.

Table 1 shows the impact to the liquid oxygen (LO2) remaining and to the liquid hydrogen (LH2) remaining from the set of known influential input variations. The results in the table were derived by running the ascent simulation while varying only the individual parameter being investigated. Assuming that the output variation derived from a 99.865% input variation is a 99.865% value and that all the outputs are independent, one may

compute the RSS (square root of the sum of the squares of all the impacts) to obtain 653.3 kg for the LO2 needed and 540.1 kg for the LH2 needed for FPR.

With this result obtained, the next step is to explore the various approaches that use Monte Carlo simulation.

Table 1. Impact on propellant remaining from known influential input parameters (input variation corresponding to 99.865% value).

| Input Parameter | Impact to LO2 remaining (kg) | Impact to LH2 remaining (kg) |
|----------------------------------|-------------------------------------|-------------------------------------|
| FS burn rate - | -329 | -90 |
| FS Isp - | -366 | -100 |
| PMBT - | -40 | -11 |
| Axial force coefficient + | -20 | -5 |
| J-2X Isp - | -178 | -49 |
| J-2X thrust - | -77 | -21 |
| Mixture ratio + | -275 | 0 |
| Mixture ratio - | 0 | -422 |
| LOX loading + for LH2, - for LO2 | -193 | -202 |
| LH2 loading + for LO2, - for LH2 | -138 | -225 |
| US dry masses | -8 | -2 |
| FS dry masses | -64 | -17 |
| FS propellant load | -72 | -20 |
| Delta staging time | -60 | -16 |
| LO2 inlet conditions | -2 | -3 |
| LH2 inlet conditions | -8 | -10 |
| inlet pressures | -5 | -9 |

Uncertainties and Vehicle Models

When designing launch vehicles, there are several types of uncertainties, divided up by when they become known, i.e. adequate information is available. The three categories are distinguished by the following information:

- Uncertainties that are unknown while the design proceeds, but are known for a particular assembled vehicle. For example, engine thrust and efficiency (specific impulse, a measure of the thrust achieved for each kilogram of propellant) will be better known after the design is complete and an engine is in hand, and the engine may be fired on a test stand and its parameters measured. Components will be weighed prior to flight day. Aerodynamic coefficients will be better known from all the wind tunnel tests and other analysis.
- Environmental parameters that are known prior to launch, particularly the measured wind on flight day and estimated temperatures of solid propellants.
- Parameters that are still uncertain when the vehicle launches. All vehicle and environmental models have uncertainties that remain when the vehicle launches.

Results would not be acceptable if all these uncertainties were simply randomly varied in a Monte Carlo simulation. Figure 1 shows how much the results change if two independent parameters with equal Normal distributions (standard deviation of 1.0) are evaluated separately versus if both are put in the same Monte Carlo simulation. When evaluated separately, the first distribution represents parameters known prior to flight day and the second represents parameters unknown on flight day. If any vehicle model must be able to successfully fly, then flight success must be achieved with a vehicle in the tail of the first distribution. So, for example, a 3-sigma value of the first distribution provides the nominal for the flight. The flight day uncertainties are distributed about this new

mean, with the result that a 3-sigma high value of the parameters has a value of about 6 as compared to less than 5 if the parameters are not addressed separately.

With this in mind, models are developed for a launch vehicle, using a probabilistic combination of the parameters known prior to flight day. Then, using simulation, a particularly bad launch day (headwinds and low temperatures) is chosen for a February launch (lowest performance month), since a successful design will provide for successful launch any time of year and in most temperature and wind situations. Development of these vehicle models and this approach is described in more detail in Ref. 2. Because the engine mixture ratio (ratio of the flow rate of oxidizer to the flow rate of fuel) is one of the parameters that will be better known for a particular engine that is tested and put on the vehicle, vehicle models are developed separately for one that yields a particularly low level of oxidizer (liquid oxygen in this paper) remaining upon arrival in orbit and for one that yields a low level of fuel remaining (liquid hydrogen). Once all these models are defined, the question is then how much liquid oxygen (LO2) must be set aside to cover for the low oxidizer case and how much liquid hydrogen (LH2) must be set aside to cover for the low fuel case, to ensure they reach orbit in the required percentage of simulations. There is a subtle difference between the engine mixture ratio uncertainty and other uncertainties that are partially estimated prior to launch day. For parameters such as engine thrust and specific impulse, having a more precise number available (there is still some uncertainty at the time of launch) allows for its use in trajectory design and in determining the propellant used for nominal flight at nominal mixture ratio, so the effects of improved information on these parameters prior to flight day would not be part of the FPR. As mentioned before, since the tanks will be filled to the nominal fill levels (with uncertainty), mixture ratio variations are included in FPR even though they are partially known prior to launch. The trajectory design and nominal performance are unaffected by mixture ratio variation, unlike the other parameters.

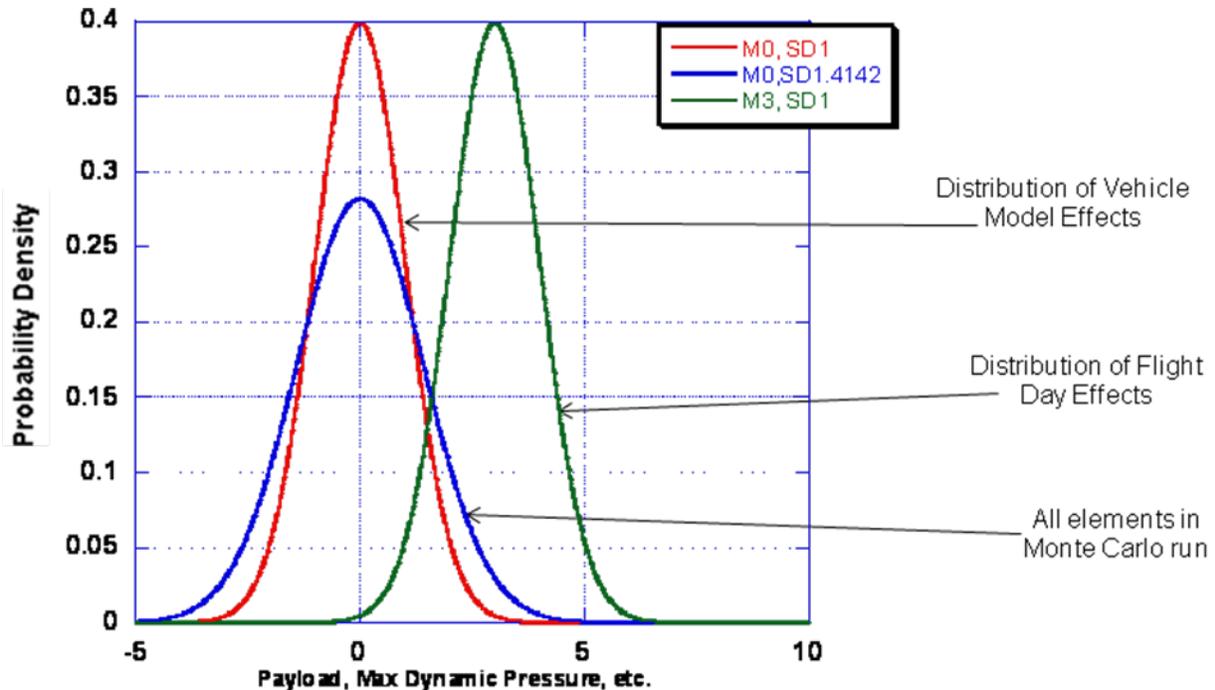


Figure 1. Effect of separating vehicle models and flight day uncertainties as compared to not separating them.

Number of Monte Carlo Samples

A typical requirement might state that orbit must be successfully achieved in 99.86% of Monte Carlo samples with 10% “consumer risk”. 99.86% is approximately a one-tailed 3-sigma level for Normal distributions. The consumer risk statement basically means that, if 10,000 Monte Carlo runs (of, say, 2000 random samples each) were to be made, 10% of them would show the requirement not being met and 90% would show the requirement is met. That is, consumer risk is the risk that the design is accepted as meeting the requirement when in fact it does not meet the requirement.

If running out of propellant (oxygen or hydrogen) before arriving successfully in orbit is considered to be a failure, and arriving in orbit is a success, then the binomial probability of k failures in an N -sample Monte Carlo run may be written as

$$P_{\text{BIN}}(k|p, N) = \binom{N}{k} p^k (1-p)^{N-k} \quad (1)$$

where p is the actual probability of failure for each Monte Carlo sample. Summing this probability from 0 to k (where $k = 5$) provides the decreasing probability curve in Figure 2 (called the “operating characteristic”). So for a very low actual failure probability, the chance is near 100% that 5 or fewer failures will occur in 2000 samples. At the intuitive failure probability of 0.25% (5/2000), the chance is >60% that the real failure probability is larger given that 5 failures are observed. In order to achieve a chance of less than 10% of the actual failure probability being greater than the estimated value, one must allow for a failure probability of about 0.47%. These concepts are developed in more detail in Ref. 2.

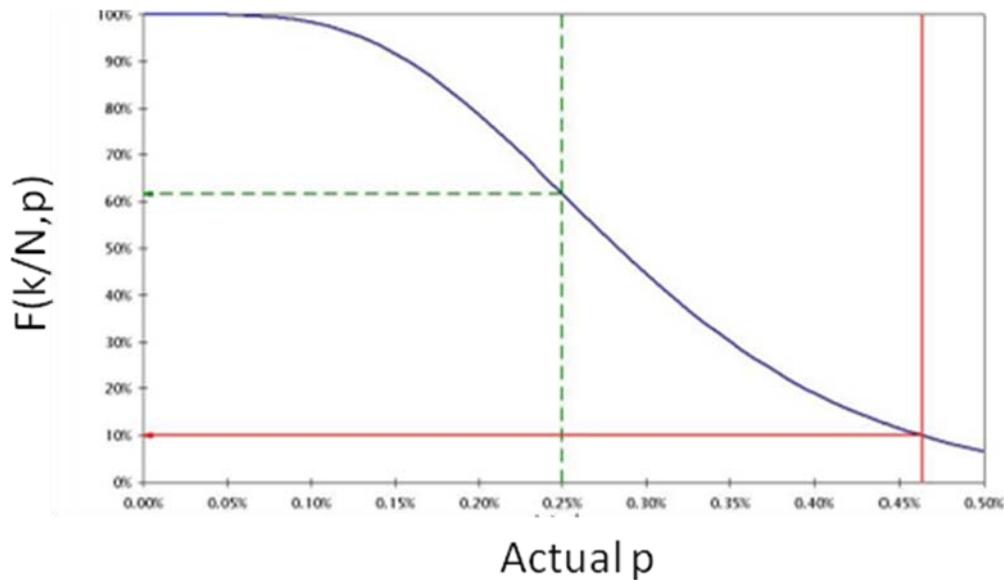


Figure 2. Probability of a sum of failures in 2000 samples as a function of the actual failure probability

One can interpolate between data points in the tail of the distribution (Ref. 2) to accurately meet the specified success level (for the particular Monte Carlo result being examined). To meet the 99.86% with 10% consumer risk (90% confidence) requirement with 2000 Monte Carlo samples, point 1999.69 out of 2000 provides the appropriate setting for parameters of interest. This point is very close to the last point, illustrating how much conservatism must be added to account for the consumer risk part of the requirement: 0.9986 corresponds to 2.8 failures ($0.9986 = 1 - 2.8 / 2000$).

Definition of Success

Success in reaching orbit means that the desired semi-major axis has been achieved to within a specified accuracy, taken here to be +/- 15 km. The launch vehicle's guidance system commands engine shutdown when the energy has reached a sufficient level so that the expected additional energy supplied by the engine during the shutdown process will provide just enough to yield an orbit that is exactly at the desired semi-major axis. Uncertainties in the amount of impulse achieved during engine shutdown, navigation errors, and other variations, lead to an actual semi-major axis that is distributed about the nominal value and is within 15 km of the desired value. If either LO2 or LH2 is running low, there are low propellant cutoff sensors that shut the engine down in order to ensure the engine does not run dry (which can be catastrophic). In these situations, the engine is shutting down before the guidance believes the appropriate point has been reached. However, if the semi-major axis is within 15 km of the target, these cases would be considered to be successes.

FPR Calculation: Some Candidate Approaches and Comparisons

Generating 2000 Monte Carlo samples for the low oxidizer and low fuel scenarios resulted in the situation shown in Figures 3 and 4 for a sample analysis. These results are for simulations that began with what was thought to be sufficient FPR (extra LO2 and LH2), based on previous analyses. Twelve of the low LO2 cases were shut down by the low propellant sensor and three of the low LH2 cases were shut down early. Some propellant remains in each case since the low propellant sensors are placed in order to ensure that the tanks don't run completely dry.

All of the sample simulations in the Monte Carlo runs satisfied the semi-major axis limits if they were not shut down by the low fuel sensors. Most of the ones shut down by the low fuel sensors also met the insertion accuracy specification. In fact, only four of the low LO2 cases were not within 15 km of the target, and only one of the low LH2 cases was not within 15 km of the target (Figure 4). In the case of LH2, the one case that does not meet the required insertion is close enough that the 1999.69 value (interpolating between point 1999 and this last point) barely meets the 15 km value. It turns out that if about 91 kg of LO2 is added (taken from vehicle payload), then the LO2 results in Figure 4 will improve to the point that the requirement is met for both LO2 and LH2.

Several methods may now be compared for determining the required FPR. Besides the previously described RSS approach and the approach of taking point 1999.69 from the just-described Monte Carlo simulation, two more methods were tried. The first is to fit a two-dimensional Gaussian distribution to the propellant remaining results and to compute a FPR that will satisfy the 99.86% with 10% CR requirement assuming the distribution is correct. The data appear to be fairly close to Gaussian (see for example the left side of Fig. 5). This method minimizes the sum of the masses of the two propellants for each of the two Monte Carlo simulations; the low LO2 case and the low LH2 case. The FPR must then include the LO2 value resulting from the low LO2 case and the LH2 value resulting from the low LH2 case. This method is developed in Ref. 3.

The second additional method works just like the first, except that prior to doing the optimization, each propellant remaining number is adjusted to obtain the value that would just meet the minimum semi-major axis requirement. If there are cases that fall below the semi-major axis requirement, they are adjusted upwards (the method for doing this is described later in the paper) and more propellant is assumed to be used to get there. The reverse is done for cases that exceed the requirement. The effect of this is to define propellant numbers that will just meet the accuracy requirement, so that the lowest propellant remaining case that is needed just meets the insertion accuracy. The rest of the successful cases will have plenty of propellant. This will reduce the FPR as compared to the method discussed in the last paragraph, since achieving the minimum semi-major axis target uses less fuel in general than achieving a nominal cutoff before running out of propellant would use.

These various methods result in Table 2, which also gives the number of failures that would likely result if these changes were in fact made. This estimate of the number of failures comes from looking at the additional number of cases that would fall below the line with this change in FPR, using Fig. 3. A higher-fidelity answer for the number of failures could be obtained by re-running the Monte Carlo simulation with these adjusted FPR values. It is possible that this RSS approach did not capture all the appropriate varying parameters, but the result is clearly too small.

It should be noted that the number of failures in Table 2 correspond to the vehicle model that is driving low propellant, launched on a particularly challenging day. Any other vehicle model would yield better results (fewer or no failures). Of course, if a less challenging vehicle model were chosen to drive the requirements (assuming that the

engine would be swapped out if it produced such a challenging case as this one), this same analysis could be performed with the less challenging model.

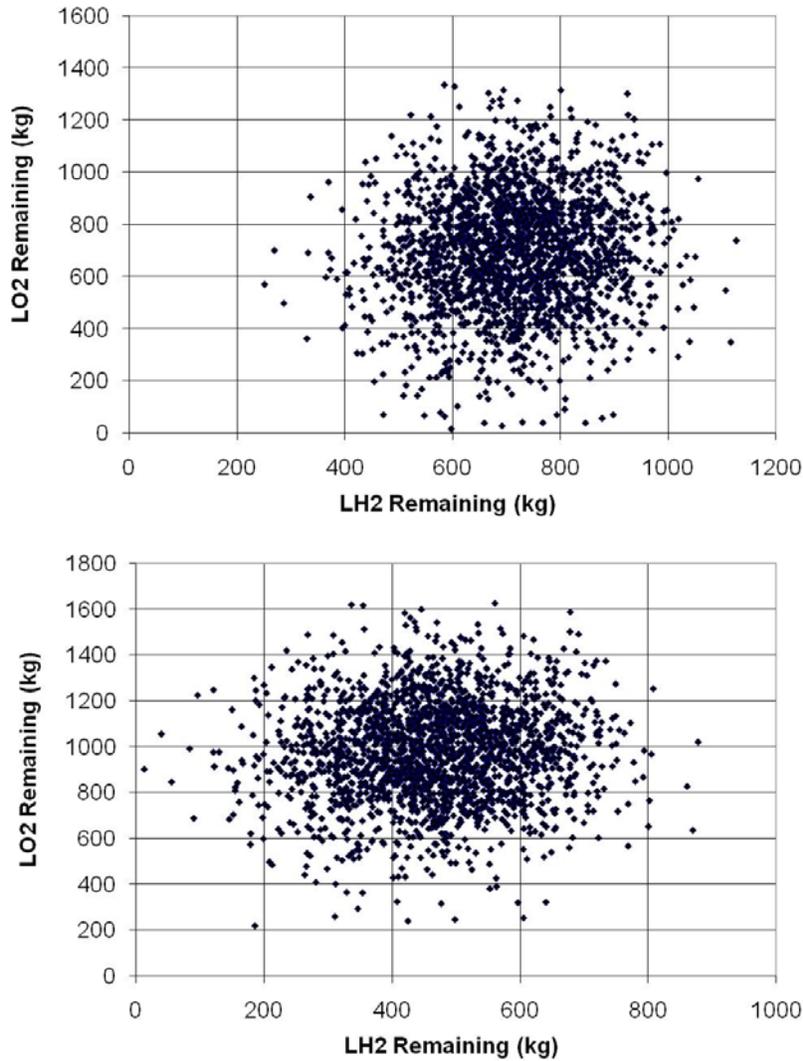


Figure 3. Usable propellant remaining for (a) low LO2 remaining case and (b) low LH2 remaining case.

Issues with the Approaches

All the results in Table 2 are only accurate if all the input models and uncertainty values are correct, but examination of those is beyond the scope of this paper. The last result in Table 2 is the only one that can be shown (in the particular Monte Carlo simulation that was run) to satisfy the requirement. In the other cases, too many failures occur to allow that FPR to satisfy the requirement. Even though it meets the numerical requirement, there are several problems with the last result in Table 2. Issues that will be addressed include:

- The low propellant cutoff sensors measure propellant level in the tanks, essentially measuring volume remaining. There is uncertainty in the volume remaining when the sensors sense that they have gone dry, uncertainty in the propellant density (and thus the mass of propellant remaining), and uncertainty in the

engine impulse and propellant usage during engine shutdown. With only four to twelve shutdown data points contributing to the analysis, this does not lead to good statistical results.

- Obtaining the results from the last two data points in the tail of the distribution means that the points are not that accurate. The variations in the simulation that led to these two furthest outliers are somewhat random and lie at random points in the tail of the true distribution that is being sampled. (This is covered mathematically by conservatism in the value chosen that yields 10% consumer risk; see next bullet.)
- The small number of failures allowed (using point 1999.69) means that there is some probabilistic conservatism in the result (in order to achieve 10% consumer risk). So it may be that the true distribution requires less propellant.

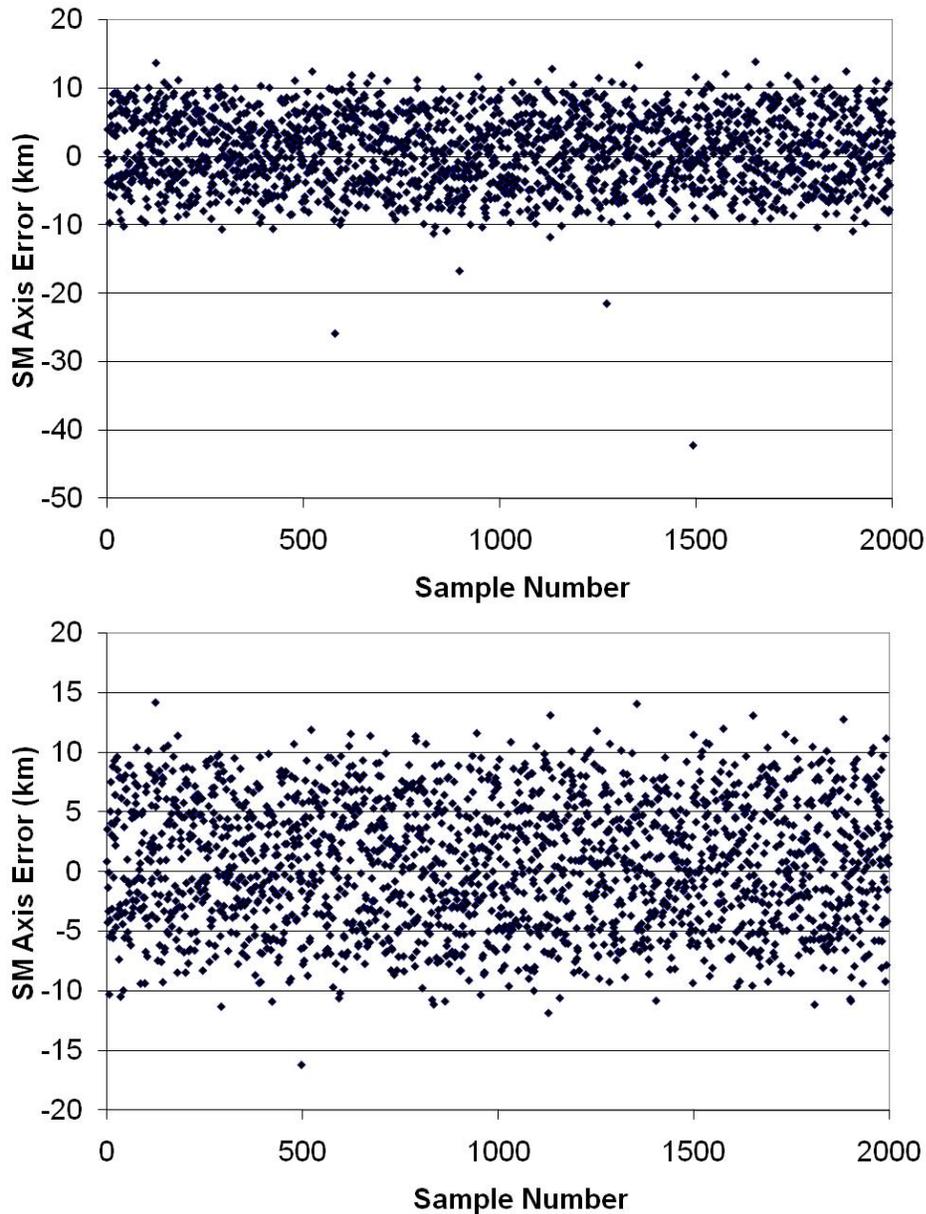


Figure 4. Semi-major axis error for (a) low LO2 remaining case and (b) low LH2 remaining case.

Table 2. FPR results using various approaches.

| Approach | LO2 FPR (kg) | LH2 FPR (kg) | Approx. number of LO2 failed cases | Approx. number of LH2 failed cases |
|--|--------------|--------------|------------------------------------|------------------------------------|
| Nominal FPR used for Fig 3 | 912.2 | 663.5 | 4 | 1* |
| RSS Approach | 653.3 | 540.1 | 48 | 7 |
| Fit distribution to MC results for propellant remaining | 897.9 | 681.2 | 5 | 0 |
| Adjust propellant remaining to hit minimum semi-major axis, fit distribution to MC results | 841.7 | 665.9 | 11 | 0 |
| Assure that 1999.69 value reaches semi-major axis | 1003.2 | 663.5 | 1** | 1* |

*1999.69 value is very close to meeting the requirement.

**1999.69 value defines the boundary; point 2000 just fails to meet the requirement.

Is there really enough LH2? And is 91 kg the right amount of LO2 that needs to be added? Maybe the right value is 50, or 150. If 50 kg can be saved, that is 50 kg that can be used for more payload. Ideally a simulation would be run that contains enough samples so that good statistics would be obtained for the low propellant cutoff sensors (1000 or more samples where the low fuel cutoff sensors commanded shutdown), and so that the extra conservatism would be removed. With a large number of Monte Carlo samples, the number of allowed failures with 10% consumer risk more closely approximates the overall allowed number of failures. For example, if 400,000 samples are run, 529 failures are allowed. This represents a 99.868% success rate, much lower than the 99.98% rate represented by the 1999.69 point. Also, with the low propellant cutoff sensor results, 12 and 4 cases out of 2000 corresponds roughly to 2400 and 800 cases for 400,000 samples, respectively. Unfortunately, much larger samples than the 2000 already run are not feasible within the computer resources available. Some slightly larger runs are possible in special cases.

Figure 5 compares the experimental distribution with no low propellant cutoff sensor to a Gaussian distribution with the same mean and standard deviation. Although the fit appears to be quite good in the left graph, the fit at the low end of the tail in the Gaussian approximation shows better propellant remaining than what the experimental data show. So a Gaussian approximation is inadequate, even without the issue of the low propellant cutoff sensors.

Refined Approach

In order to generate proper statistical results, the following approach was used (once each for both the LO2 and LH2 cases):

- Perform a Monte Carlo simulation (2000 samples) in which performance was artificially reduced so that all cases were shut down by the low propellant cutoff sensors. Using this, generate a distribution for the propellant remaining at the cutoff command when these sensors command the shutdown.
- Perform a 10,000-sample Monte Carlo simulation where the low fuel sensors are disabled (assuming that 10,000 samples represents a reasonable maximum with computer limitations taken into account). This will generate a distribution for the fuel remaining when guidance commands the shutdown that is better than the previous distribution (of 2000 samples). More samples would be desirable, but computer limitations drive the limit.
- Also from the 10,000-sample run, generate distributions for the semi-major axis error at the time of guidance command and for the propellant used during shutdown. As seen in Figure 6, the change in semi-major axis during the engine shutdown is nearly completely correlated with the propellant used during shutdown, so that if one knows the propellant used, then the semi-major axis gain during shutdown is determined.
- With the above distributions available, run a 400,000-sample Monte Carlo simulation that picks random values from the distributions (and does not run the ascent simulation). This 400,000-sample simulation runs very quickly on a computer. This simulation will yield a sizable distribution tail and will allow for

many failures (529 for the 99.86% with 10% consumer risk success level), so that good statistics of the required propellant should result.

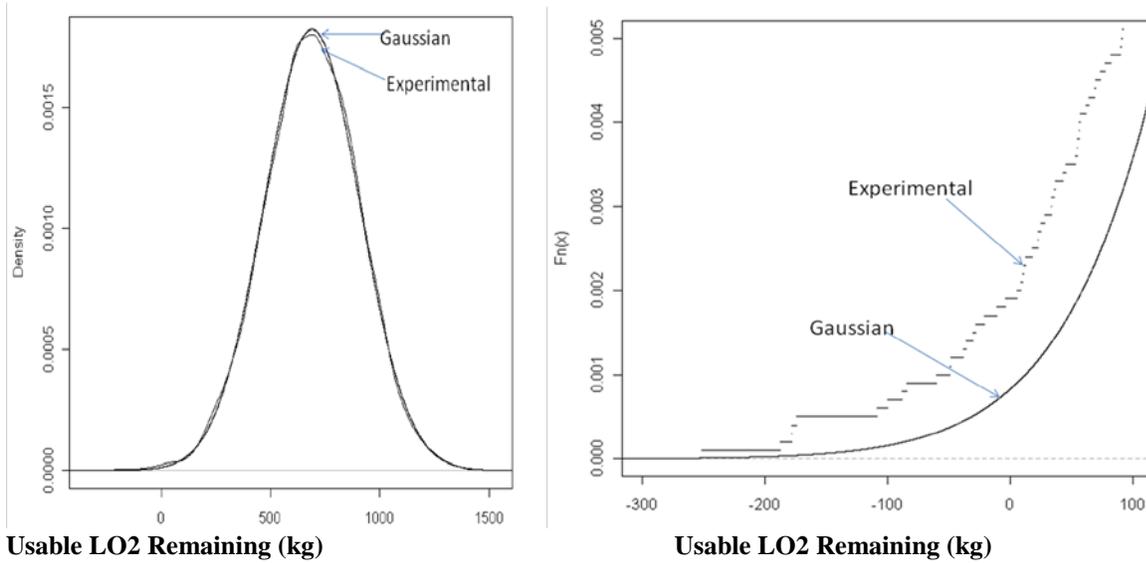


Figure 5. LO2 remaining after shutdown if there is no low propellant cutoff sensor (10,000 samples). The graph on the left is probability density, comparing the experimental distribution to a Gaussian distribution with the same mean (689.2812) and standard deviation (219.0123). The graph on the right shows the cumulative distribution, focusing on the lower tail. Individual results are visible in the experimental distribution.

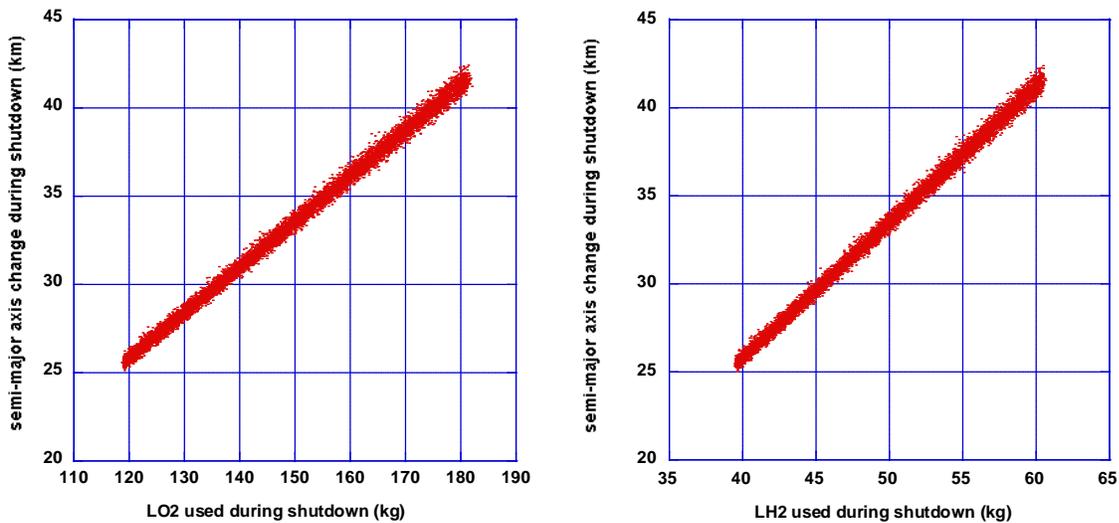


Figure 6. Demonstration that the semi-major axis change during shutdown is highly correlated to propellant used during shutdown, for (a) low LO2 remaining case and (b) low LH2 remaining case.

In each of the 400,000 samples:

- Randomly choose a propellant remaining from the guidance-commanded shutdown distribution and from the low propellant sensor shutdown distribution. If the guidance-commanded value is higher, then the shutdown is due to guidance and the orbit insertion is successful.
- For cases where the low propellant sensor value is higher, the engine shut down early. The first step is to determine what the semi-major axis error would have been if the vehicle had not shutdown early. Choose a random semi-major axis error at guidance shutdown command value from that distribution, and a random propellant used during shutdown value from its distribution, yielding the semi-major axis the vehicle would achieve during shutdown. From these, calculate the semi-major axis error that would have resulted if guidance commanded the shutdown in this case.
- The next step adjusts the final guidance-commanded semi-major axis error in order to yield the semi-major axis error due to the low propellant sensor shutdown. This is based on the remaining propellant level differences from when the low propellant sensor commanded the shutdown as compared to when the guidance wanted to command the shutdown. This difference in thrusting due to differing propellant levels at the commanded shutdown is done at full thrust, so equations will be used rather than using the data for what happens during shutdown in Fig. 6. Start by using the rocket equation (equation 2) to calculate the change in velocity available from the delta propellant. Reduce this change in velocity to account for cosine losses on the nominal trajectory, using the nominal angle between the vehicle thrust and the velocity vector at the time of engine cutoff (equation 3). Then use the energy equation (equation 4) to calculate the change in semi-major axis.

The rocket equation is

$$\Delta V = g * Isp * \ln\left(\frac{m_o}{m_f}\right) \quad (2)$$

where ΔV is the change in speed, g is the gravity acceleration at sea level, Isp is the engine specific impulse (thrust/burn rate/ g), and m_o and m_f are the initial and final masses (final is initial minus propellant used). For propellant cosine losses, use the angle, α , between the vehicle thrust direction and the velocity vector on a nominal flight at the time of engine cutoff:

$$\Delta V_{reduced} = \frac{\Delta V}{\cos(\alpha)} \quad (3)$$

The energy equation can be used to determine the change in semi-major axis by the reduced change in velocity.

$$V = \sqrt{2\left(\frac{\mu}{r} - \frac{\mu}{2a}\right)} \quad (4)$$

where a is the semi-major axis, μ is Earth's gravity constant, and r is the radius at the time of the burn. Equation 4 gives the velocity at the final guidance-commanded semi-major axis. Then $V - \Delta V_{reduced}$ determines the velocity at the end of the burn caused by the early shutdown due to the sensor. This velocity determines the semi-major axis at the end of the burn, using Eq. 4 arranged to solve for a . Compare this semi-major axis with the requirement to determine whether the insertion is successful. A flow chart of the process is shown in Figure 7.

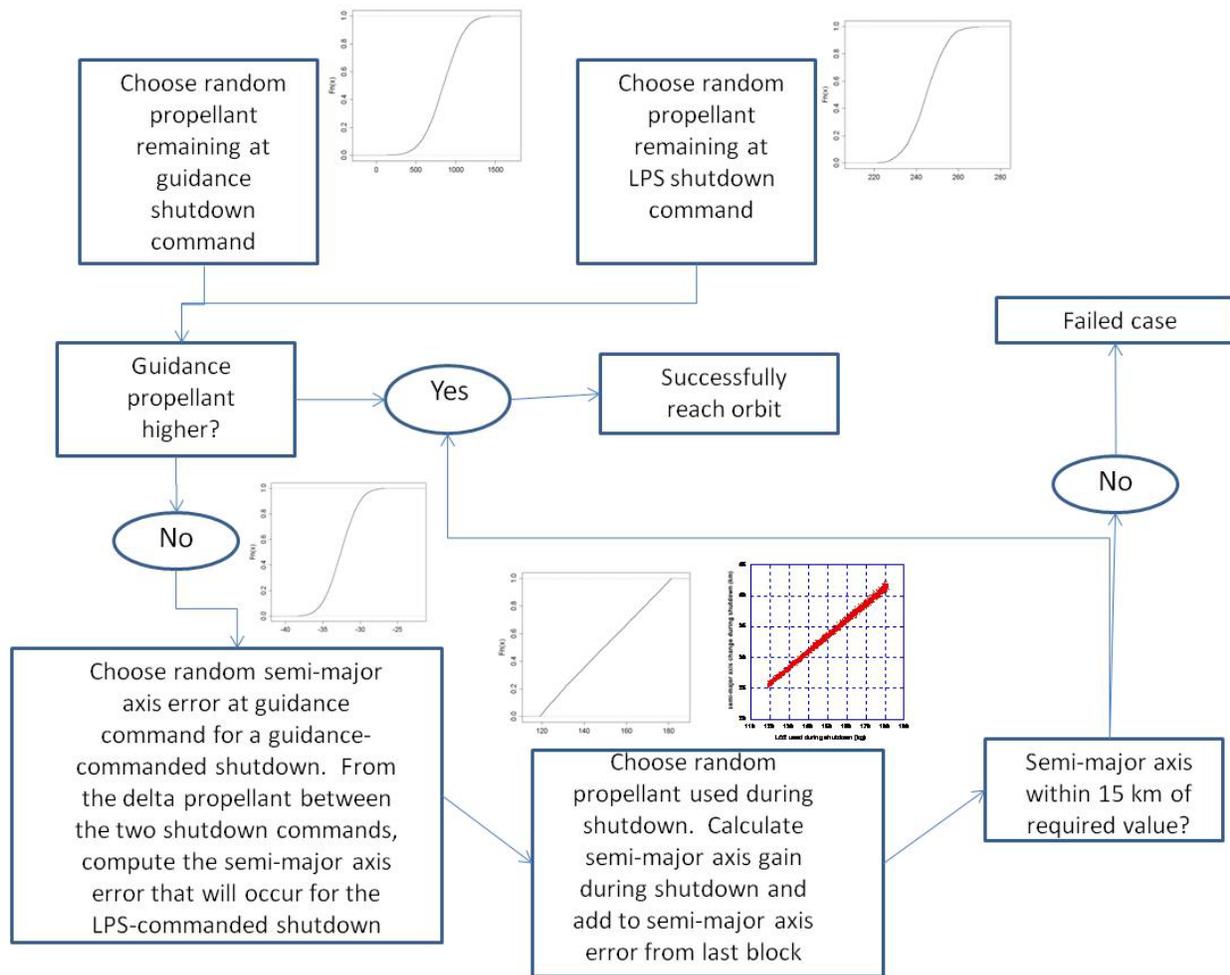


Figure 7. Flowchart of the Monte Carlo process using the various distributions. LPS stands for low propellant sensor.

Required Distributions

Figure 8 shows the cumulative distribution for usable propellant remaining when the low propellant sensors command shutdown. Figure 9 shows the cumulative distributions for the semi-major axis error at the time of guidance command, when the guidance commands the shut down. As expected, it is about the same for both low LO2 and low LH2 runs, because guidance commands the shutdown based on when orbit is about to be achieved, not based on propellant remaining. Figure 10 shows cumulative distributions for the propellant used during shutdown. The propellant used during shutdown is very close to a uniform distribution. Figure 11 shows the cumulative distributions for the propellant remaining when guidance commands the shutdown (low propellant sensor disabled). Note that the distributions for the low propellant cutoff sensors (Fig. 8) overlap with the low tails of the distributions in Fig. 11 (not so visible in the LH2 case, but with some overlap nevertheless).

Notice that the low end of the tail of the distributions in Fig. 11 is still somewhat ragged (e.g. Fig. 12). The 99.86% case for 10,000 samples is at about the 14th data point, which is at about 130 kg. The experimental cumulative distribution appears to be fairly well-behaved in this region. However, since the number of data points is relatively few, three approaches will be tried for the LO2 case when generating the 400,000 samples, in particular focusing on the fit of the low end of the guidance-commanded shutdown distribution in Fig. 12. The first approach has already been described, where random samples are taken from the experimental distribution. The other two will

use a Generalized Pareto Distribution (GPD) fit to the extreme values of the distribution in Fig. 11. A GPD is defined by

$$F(x) = 1 - \left[1 + \xi \left(\frac{x - \mu}{\sigma} \right) \right]^{-1/\xi} \quad (5)$$

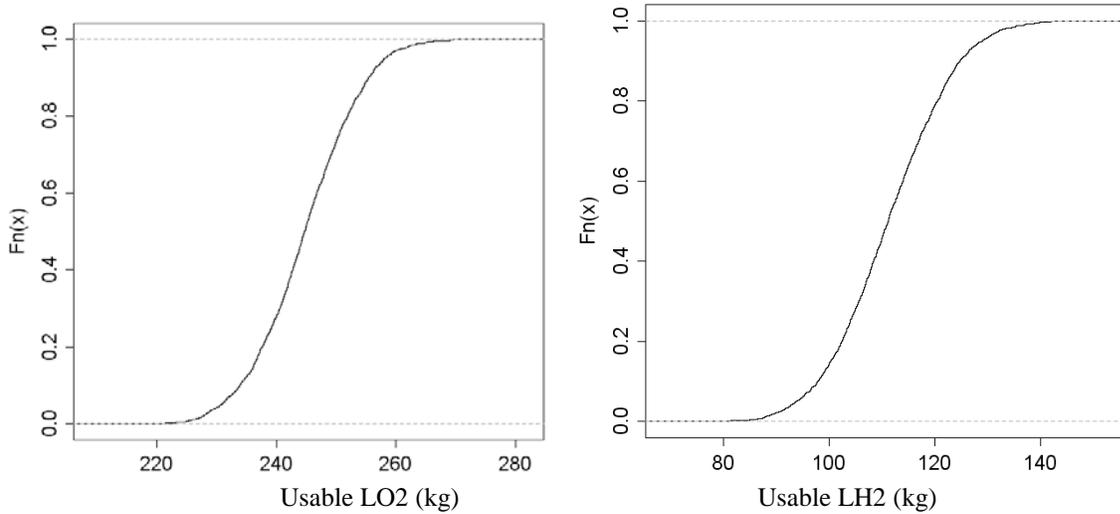


Figure 8. Cumulative distributions for propellant remaining when the low fuel sensors command engine shutdown.

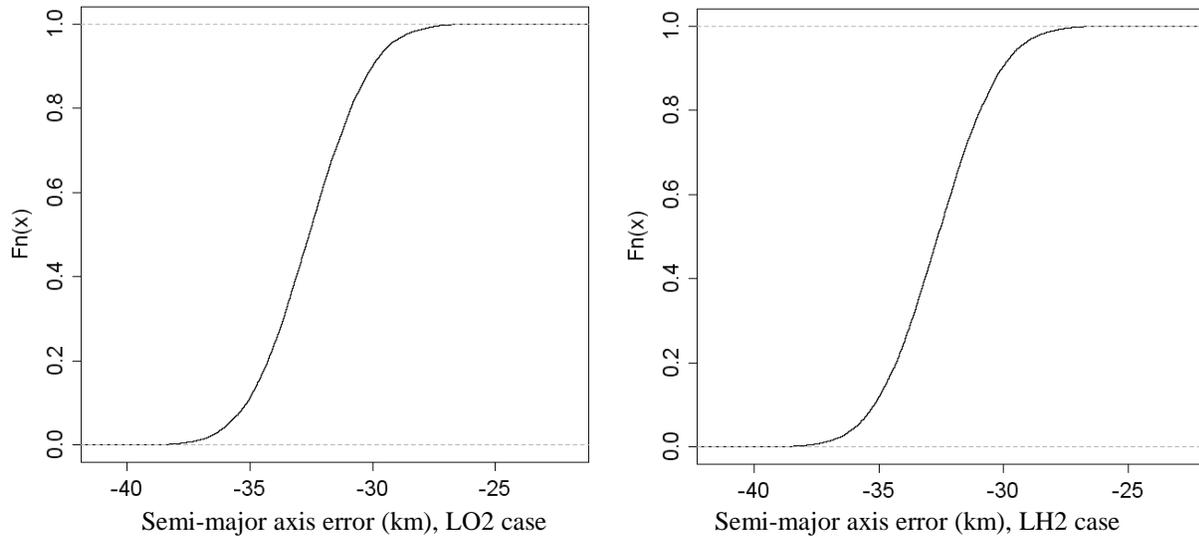


Figure 9. Cumulative distributions for semi-major axis error at the time of guidance shutdown command

where $F(x)$ is the cumulative distribution for the random variable x , μ is the “threshold”, ξ is the “shape parameter”, and σ is the “scale parameter”. The threshold is a value chosen in the tail, where only values beyond this are considered. The free statistics package “R” was used to generate the GPDs (Ref. 4). A GPD fit with a threshold of

225 kg (usable LO2 remaining at the time of guidance command is 225 kg or less) is shown in Figure 13. 43 data points out of the 10,000 are below 225 kg. A GPD fit for a threshold of 450 kg (402 data points) is shown in Figure 14.

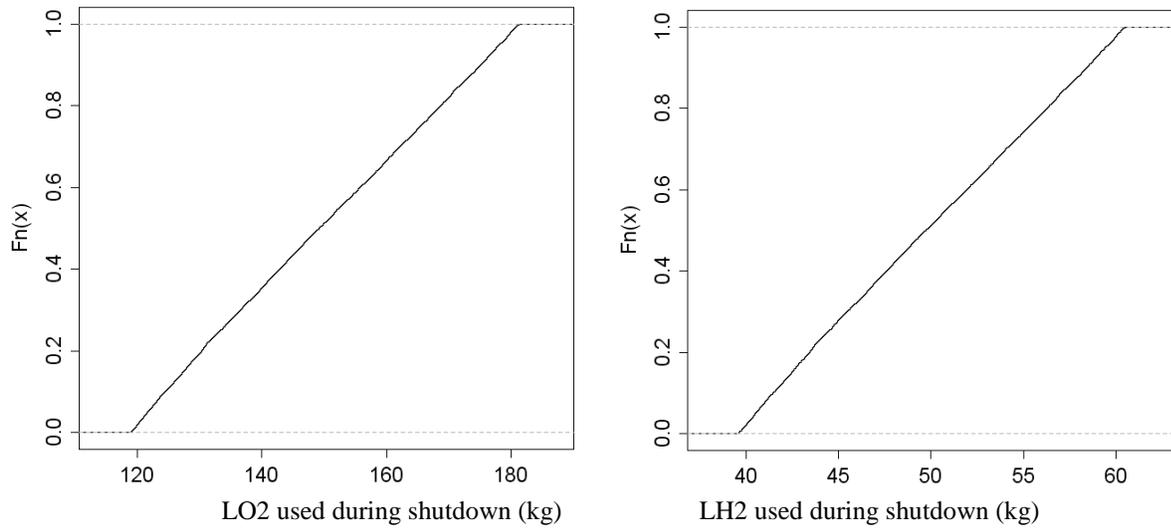


Figure 10. Cumulative distributions for propellant used during shutdown

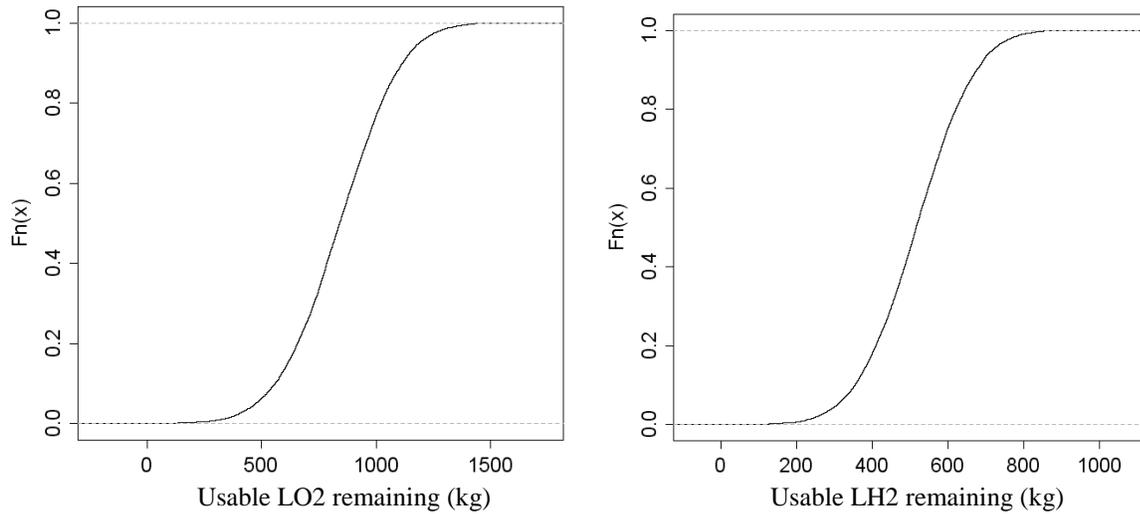


Figure 11. Cumulative distributions for propellant remaining at the time of guidance-commanded shutdown (low fuel sensors disabled).

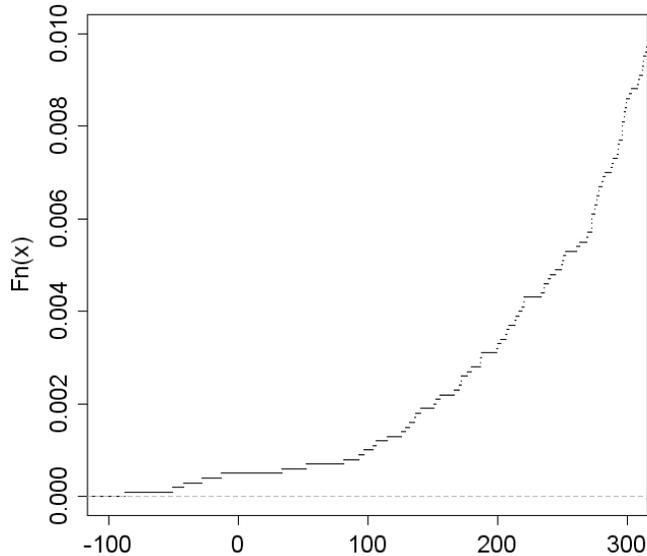


Figure 12. Low tail of cumulative distribution for LO2 from Fig. 11.

Results

Earlier, using 2000 samples each for LO2 and LH2, the result was that 91 kg more LO2 was needed and that the LH2 was just sufficient. Using the distributions shown in Figs. 8-11, and conducting a 400,000-sample Monte Carlo simulation, resulted in a need for 66 kg more LO2 and an excess of 22 kg LH2, a net savings of 47 kg from the earlier result. Use of the GPD fit instead of the direct experimental distribution for the guidance shutdowns (for LO2) resulted in a shortfall of 67 kg for the 225 kg threshold and 61 kg for the 450 kg threshold (compared to 66 kg). The reduced fuel needs with the higher fidelity analysis are a combination of using more accurate distributions and less conservatism due to the large number of Monte Carlo samples.

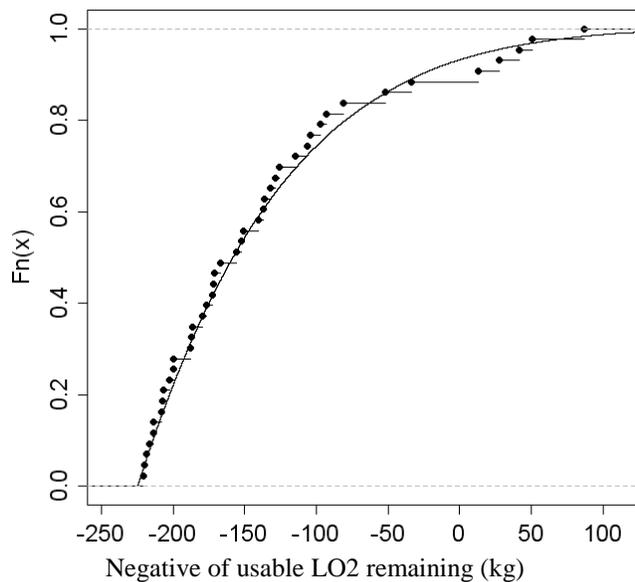


Figure 13. Generalized Pareto Distribution fit for LO2 remaining at guidance command, with a 225 kg threshold. The negative of the propellant remaining is graphed so that the cumulative distribution has the normal sense of increasing with x .

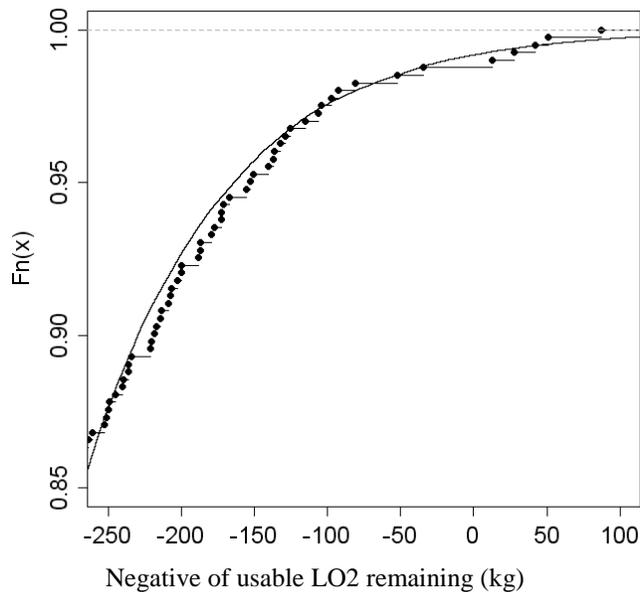


Figure 14. Generalized Pareto Distribution fit for LO2 remaining at guidance command, with a 450 kg threshold. The negative of the propellant remaining is graphed so that the cumulative distribution has the normal sense of increasing with x. Only the part of the distribution for 250 kg and below is shown, since this is the region of interest.

Conclusion

This paper addressed the need for an accurate estimate of the flight performance reserve (both fuel and oxidizer) needed for successful launch vehicle ascent flight. Several approaches were examined. Most methods did not provide sufficient precision to allow for minimization of the total needed while meeting required success percentages. Use of a method of combining the effects of the various input variations (assuming the effects were independent and that the important parameters were sufficiently captured) did not yield a FPR result that was close to covering for the required percentage of cases. Fitting distributions to the propellant remaining was also inadequate. Using binomial calculations of required success provided a better result but did not provide for sufficient statistical information to obtain the desired accuracy, and also included some undesired conservatism.

The final approach involved generating distributions for 1) the propellant remaining when the guidance system thinks engine shutdown should occur, 2) the propellant remaining when low propellant cutoff sensors generate a cutoff command, 3) the propellant used during the engine shutdown, and 4) the semi-major axis error at the time of guidance command. These distributions were used along with the correlation between propellant used during shutdown and the semi-major axis gain during the shutdown, and simplified calculations of the amount of fuel needed to make up a semi-major axis shortfall, in a large Monte Carlo simulation of 400,000 samples with random choices from the various distributions. The net result of the analysis, for the case considered, was a savings of 47 kg in total propellant relative to the lower fidelity (2000 Monte Carlo samples with the low propellant cutoff sensors only active in a small number of samples) answer. The savings is in part because the required level of conservatism is reduced by performing a larger Monte Carlo simulation. There is also more confidence in this answer because the number of cases where the low fuel sensor commands the shutdowns is on the order of one thousand, providing much better statistical information, and also because with 400,000 samples, the random error in the calculation of the final answer is much less (since there are 529 cases in the outlier tail of failures rather than one). The results would change with any change in the vehicle models or in the uncertainties modeled.

The analysis in this paper focused on a vehicle model where the engine mixture ratio was known better for each flight than for the overall ensemble of potential vehicles. Thus vehicle models were simulated that stressed one propellant alternatively over the other. As a result, each propellant could be treated individually. The low LO₂ remaining model drove the amount of LO₂ needed for FPR, and the low LH₂ remaining model drove the amount of LH₂ needed for FPR. Potential vehicle designs may not include this consideration, if nothing beyond the nominal mixture ratio target is known about an engine prior to launch. In this case, the driving vehicle model will stress both propellants at the same time. If an approach like the one developed in this paper is used, and a similar Monte Carlo simulation is used to generate the final result (e.g. 529 failures allowed), a simple iteration could be used to minimize the sum of the two propellants needed for FPR. That is, the amount of LO₂ needed to satisfy each failure case (0 to 529) and the amount of LH₂ needed for each case are identified by the simulation. Requiring the sum of failures to be 529 allows the user to calculate the FPR needed for n LO₂ failures and 529 – n LH₂ failures. The minimum FPR for which the sum of failures is 529 is the FPR that satisfies the requirement.

A similar procedure could be used for a model where the launch day propellant loading is adjusted to reflect the mixture ratio estimate for the engine(s). In this case, there would again be a single model (not a separate one for low fuel and low oxidizer) and the FPR would only cover for that part of mixture ratio uncertainty that still exists on flight day.

References

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2. J. M. Hanson and B. B. Beard, "Applying Monte Carlo Simulation to Launch Vehicle Design and Requirements Verification," AIAA-2010-8433, AIAA Guidance, Navigation, and Control Conference, Toronto, Canada, August 2010.
3. J.M. Hanson and B.B. Beard, "Applying Monte Carlo Simulation to Launch Vehicle Design and Requirements Analysis," NASA TP-2010-216447, NASA Marshall Space Flight Center, Sept. 2010 (Appendix F).
4. R Statistics Package, located at <http://www.r-project.org/>.