Cost Modeling for Space Optical Telescope Assemblies

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ABSTRACT

Parametric cost models are used to plan missions, compare concepts and justify technology investments. This paper reviews an on-going effort to develop cost models for space telescopes. This paper summarizes the methodology used to develop cost models and documents how changes to the database have changed previously published preliminary cost models. While the cost models are evolving, the previously published findings remain valid: it costs less per square meter of collecting aperture to build a large telescope than a small telescope; technology development as a function of time reduces cost; and lower areal density telescopes cost more than more massive telescopes.

Keywords: Space Telescope Cost Model, Parametric Cost Model

1. INTRODUCTION

Parametric cost models for space telescopes provide several benefits to designers and space system project managers. They identify major architectural cost drivers and allow high-level design trades. They enable cost-benefit analysis for technology development investment. And, they provide a basis for estimating total project cost. A survey of historical models found that there was no definitive space telescope cost model [1]. Thus, there is a need for parametric space telescopes cost models. An effort is underway to develop single variable [2] and multi-variable [3] parametric space telescope cost models based on the latest available data and applying rigorous analytical techniques. Since the publication of the single and multi-variable parametric models, the database has changed. New telescopes were added to the database; data for other telescopes was revised; and, other telescopes were removed from the modeling analysis. As a result of these changes, the cost models have changed. But the general findings remain unchanged: aperture diameter is the primary cost driver for large space telescopes; technology development as a function of time reduces cost; it costs less per square meter of collecting aperture to build a large telescope than a small telescope; and it costs more per kg to build a low areal density telescope than a massive telescope.

2. METHODOLOGY

Cost and engineering data has been collected on 59 different parameters for 39 x-ray, UV, optical, infrared, microwave and radio space telescopes. But to date, only the 32 normal-incidence UV, Optical, Infrared (UVOIR) missions have been studied for cost modeling. And, of these 32, sufficient data exists for only 15 with which to develop an Optical Telescope Assembly (OTA) cost model (Table 1). Data was collected from multiple sources, including: NAFCOM (NASA/ Air Force Cost Model) database, RSIC (Redstone Scientific Information Center), REDSTAR (Resource Data Storage and Retrieval System), project websites, and interviews.

Table 1: UV/OIR Cost Model Missions Database

<table>
<thead>
<tr>
<th>UV/Optical Telescopes</th>
<th>Infrared Telescopes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commercial #1</td>
<td>Herschel</td>
</tr>
<tr>
<td>Commercial #2</td>
<td>IRAS</td>
</tr>
<tr>
<td>GALEX</td>
<td>JWST</td>
</tr>
<tr>
<td>HiRISE</td>
<td>Planck</td>
</tr>
<tr>
<td>HST</td>
<td>SOFIA</td>
</tr>
<tr>
<td>IUE</td>
<td>Spitzer</td>
</tr>
<tr>
<td>Kepler</td>
<td>WIRE</td>
</tr>
<tr>
<td></td>
<td>WISE</td>
</tr>
</tbody>
</table>

For our study, Optical Telescope Assembly (OTA) is defined as the space observatory subsystem which collects electromagnetic radiation and focuses it (focal) or concentrates it (afocal). An OTA consists of the primary mirror, secondary mirror, auxiliary optics and support structure (such as optical bench or truss structure, primary support structure, secondary support structure or spiders, etc.). An OTA does not include science instruments or spacecraft subsystems. And, cost is defined as prime contract cost without any NASA labor or overhead. Total mission cost is defined as Phase A-D cost, excluding: launch cost; costs associated with NASA labor (civil servant or support contractors) for program management, technical insight/oversight; or any NASA provided ground support equipment, e.g. test facilities. Accounting for NASA overheads would increase the cost by at least 10% and maybe as much as 33%.
Statistical correlations have been evaluated between 18 of 59 variables. And, these parameters have been used to develop single variable cost estimating relationships (CERs) which are evaluated for their ‘goodness’. The variables are divided between technical (Aperture Diameter, PM Focal Length, System Focal Length, Field of View, Pointing Stability, OTA Mass, Total Mass, Spectral Range Minimum, Wavelength of Diffraction Limit, Operating Temperature, Average Input Power, Data Rate, Design Life, and Orbit) and programmatic (Technology Readiness Level, Year of Development, Development Period, and Launch Year). Additionally, insight is gained by combining independent variables to form collector variables (F/#, Volume, Areal Cost or Cost Density).

Two single variable cost estimating relationships (CERs) are reported in this paper. These CERs estimate OTA cost as a function of OTA diameter and OTA mass.

3. MODEL CREATION

The first step in creating a statistical cost model is to start with the Cross Correlation Matrix (Figure 1) and look for variables which are highly correlated with cost. When using a cross-correlation matrix, there are several things to consider. First, the higher the correlation value, the greater the cost variation explained by that variable. Second, the sign of correlation is important. It must be consistent with known engineering design principals and manufacturing processes. Third, for multi-variable models, we want variables which independently effect cost. Variables which ‘cross-talk’ with each other are multicollinear.

![Cross-Correlation Matrix of data base for 15 Space Telescope Systems. Correlations which are at least 95% significant are Bolded, e.g. for 12 data points a correlation of greater than 60% is significant to better than 95%.](image)

Figure 1: Cross-Correlation Matrix of data base for 15 Space Telescope Systems. Correlations which are at least 95% significant are Bolded, e.g. for 12 data points a correlation of greater than 60% is significant to better than 95%.

A careful study of the cross-correlation matrix shows that OTA Cost is highly correlated with Aperture Diameter, Primary Mirror Focal Length, System Focal Length, Pointing Stability, Total Mass, OTA Mass, Design Life and Development Period. But, caution is required because not all of these variables are independent. Aperture Diameter correlates with PM Focal Length and System Focal Length, simply because larger aperture telescopes tend to have longer focal lengths. Also, because larger aperture telescopes have smaller point spread functions (or plate scales) they need to have smaller pointing stabilities, hence, the inverse correlation between aperture size and pointing. Obviously,
the larger the telescope aperture, the more massive the telescope will be. Additionally, the accompanying science instruments will undoubtedly be larger and more massive as well as the spacecraft. It is interesting to note that there does not appear to be a correlation between cost and operating temperature or diffraction limited performance. One explanation might be that they tend to cancel each other out. While the mirrors for longer wavelength systems are easier to manufacture, their cryogenic operating temperature increases cost. And, while visible systems operate at room temperature, their mirrors and structures are more difficult.

The second step is to select candidate CER variables and perform a regression analysis (Figure 2). The statistical indicators of ‘Goodness of Fit’ and ‘Significance’ are used to evaluate the results of this analysis. Goodness of Fit is tested via a range of statistical measures, including Pearson’s r² coefficient, Student T-Test p-value and standard percent error (SPE). Pearson’s r² (typically denoted as just r²) describes the percentage of agreement between the model and the actual cost. For multi-variable models, we use Adjusted Pearson’s r² (or r² adj) which accounts for the number of data points and the number of variables. In general, the closer r² (or r² adj) is to 1.0 or 100%, the better the model. SPE is a normalized standard deviation of the fit residual (difference between data and fit) to the fit. The closer SPE is to 0, the better the fit. Please note that since SPE is normalized, a small variation divided by a very small parameter coefficient can yield a very large SPE. The p-value is the probability that a fit or correlation would occur if the variables are independent of each other. The closer the p-value is to 0, the more significant the fit or correlation. The closer it is to 1, the less significant. If the p-value for a given variable is small, then removing it from the model would cause a large change to the model. If it is large, then removing the variable will have a negligible effect. Also, it is important to consider how many data points are included in a given correlation, fit or regression.

![Table 1: Single Variable Regression Analysis for OTA Cost](image1)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Aperture Diameter</th>
<th>PM F Lcm.</th>
<th>PM F Fit</th>
<th>OTA Volume</th>
<th>FOV</th>
<th>Pointing Stability</th>
<th>OTA Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-value</td>
<td>1.75</td>
<td>0.06</td>
<td>1.74</td>
<td>0.00</td>
<td>1.08</td>
<td>0.05</td>
<td>0.65</td>
</tr>
<tr>
<td>Adjusted r²</td>
<td>67%</td>
<td>75%</td>
<td>2%</td>
<td>72%</td>
<td>4%</td>
<td>33%</td>
<td>56%</td>
</tr>
<tr>
<td>SPE</td>
<td>146%</td>
<td>117%</td>
<td>729%</td>
<td>116%</td>
<td>719%</td>
<td>205%</td>
<td>68%</td>
</tr>
<tr>
<td>n</td>
<td>15</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>14</td>
<td>8</td>
<td>12</td>
</tr>
</tbody>
</table>

![Table 2: Statistical Results for OTA Cost](image2)

<table>
<thead>
<tr>
<th>Variable</th>
<th>OTA Area Density</th>
<th>Spectral Range Limitation</th>
<th>Diff. Lim. λ</th>
<th>Operating Temp.</th>
<th>Year of Dev.</th>
<th>Date of Launch (exp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-value</td>
<td>0.16</td>
<td>-0.16</td>
<td>-0.19</td>
<td>0.32</td>
<td>0.27</td>
<td>0.43</td>
</tr>
<tr>
<td>Adjusted r²</td>
<td>-9%</td>
<td>2%</td>
<td>-1%</td>
<td>-7%</td>
<td>-8%</td>
<td>-3%</td>
</tr>
<tr>
<td>SPE</td>
<td>1084%</td>
<td>900%</td>
<td>845%</td>
<td>1162%</td>
<td>1225%</td>
<td>1183%</td>
</tr>
<tr>
<td>n</td>
<td>12</td>
<td>15</td>
<td>13</td>
<td>14</td>
<td>14</td>
<td>15</td>
</tr>
</tbody>
</table>

**Figure 2: Single Variable Regression Analysis for OTA Cost**

### 4. SINGLE-VARIABLE MODELS

From an engineering and a scientific perspective, aperture is the best parameter with which to build a space telescope cost model. Aperture defines the observatory’s science performance (sensitivity and resolution) and determines the payload’s size and mass. As discussed in Section 3, Aperture Diameter correlates with all of the other variables which significantly correlate with cost. From Figure 2, OTA cost varies as a function of diameter according to:

\[
\text{OTA Cost} \sim \text{Diameter}^{1.75}\quad (N = 15; \ r^2 = 67\%; \ SPE = 146)
\]

This CER is based on 15 data points, one of which (SOFIA) is not actually a space telescope. But as shown in Figure 3, SOFIA’s OTA cost is ‘in-family’ for its aperture. And, removing SOFIA from the regression has a negligible effect:

\[
\text{OTA Cost} \sim \text{Diameter}^{1.8}\quad (N = 14; \ r^2 = 67\%; \ SPE = 142)
\]

However, both CERs only account for 67% of the cost variation and both are noisy. Therefore, a single variable aperture diameter model is not a good CER. Other variables are needed to account for the remaining 33% of cost variation.
One concern about cost versus diameter is that JWST drives the fit. As a simple sanity check, the data was normalized by collecting area to define Areal Cost (Figure 4). By eliminating the diameter influence, data spread associated with second order factors such as wavelength or operational temperature can be identified. The key point of Figure 4 is that areal cost decreases as a function of aperture diameter. Thus, given that the number of collected photons is proportional to collecting area, larger aperture telescopes have a greater return on investment (ROI) than smaller aperture telescopes.

While an Aperture based CER may be most logical for an optical engineer, many believe that Mass is the more important CER. Total system mass determines what vehicle can be used to launch the mission. And, significant engineering costs are expended to keep a given payload inside of its allocated mass budget, for example: light-weighting mirrors and structure. It is factual to assert that space telescopes are designed to meet a specific mass budget. From Figure 2, OTA cost varies as a function of mass according to:

\[ \text{OTA Cost} \sim \text{OTA Mass}^{0.97} \quad (N = 12; r^2 = 56\%; \text{SPE} = 88) \]

Compared to the Aperture Diameter CER, OTA Mass is less noisy. But, it still only accounts for 56% of the cost variation. One problem is that this CER is based on 12 data points, one of which (SOFIA) is not actually a space telescope. SOFIA is actually an ‘attached’ telescope. It flies attached to a 747 aircraft. And, by flying on a 747 aircraft, it can be designed to an entirely different mass budget paradigm. But, as shown in Figures 3 and 5, while SOFIA has the approximately the same aperture size and OTA mass as HST; it has a significantly different cost than HST. One explanation might be because SOFIA has a longer diffraction limited wavelength but still operates close to ambient temperature. Another explanation might be that SOFIA does not have the challenges of operating in space, i.e. in vacuum or micro-gravity. Removing SOFIA from the regression has a significant effect:

\[ \text{OTA Cost} \sim \text{OTA Mass}^{1.1} \quad (N = 11; r^2 = 96\%; \text{SPE} = 78\%) \]

Without SOFIA, the OTA Mass CER accounts for 96% of the cost variation and has a smaller standard percent error.

Another potential wavelength story is Herschel and Kepler. Herschel and Kepler have essentially the same mass and cost, but vastly different apertures, diffraction limits and operating temperatures. Based only on aperture, Herschel should be more expensive than Kepler, but it has a significantly longer diffraction limit and lower operating temperature.
A very interesting tool for analyzing the role of mass on cost is cost density (cost per kg). Figure 6 plots OTA cost per kg versus OTA aperture diameter. Several obvious conclusions can be drawn from Figure 6. First, all free flying space telescopes have approximately the same cost per kg – independent of aperture diameter. And, all ground telescopes also have approximately the same cost per kg – independent of aperture diameter. And, space telescopes cost about 1000X per kg more than ground telescopes – independent of aperture diameter. Second, while the conclusion regarding attached telescopes might appear to be equally obvious (that attached telescopes are approximately 5X less expensive than free-flying), it is not. For the three shuttle missions (UIT, WUPPE and HUT), the data base only has ‘instrument’ data, i.e. cost and mass of the telescope and science instruments (detectors, electronics, etc.). More research is required. Thus, the only conclusion which can be drawn from Figure 6 is that it costs more per kg to make low areal density flight telescopes than it costs to make massive ground telescopes. One explanation for this data might be that each of these mission ‘types’ are built to different design rules. While all three types need similar wavefront shape and pointing stabilities as a function of aperture diameter, they have different static gravity and dynamic jitter environments; and different mass budgets for achieving the required wavefront shape and pointing stability.

A final caution about using mass as a CER can be found by considering HST vs JWST. While the HST and JWST OTAs have similar mass and cost (although JWST is a bit more expensive), this relationship does not hold at the system level. HST system mass is nearly 2X more than JWST, yet JWST is slightly more expensive than HST.

5. CONCLUSIONS

Cost models are invaluable for system designers. They identify major architectural cost drivers and allow high-level design trades. They enable cost-benefit analysis for technology development investment. And, they provide a basis for estimating total project cost. Cost and engineering data has been collected on 59 different parameters for 39 x-ray, UV, optical, infrared, microwave and radio space telescopes. But to date, only the 29 normal-incidence UV, Optical, Infrared (UVOIR) missions have been studied for cost modeling. And, of these 29, sufficient data exists for only 15 with which to develop an Optical Telescope Assembly (OTA) cost model. Statistical correlations have been evaluated between 18 of 59 variables. And, these parameters have been used to develop single variable cost estimating relationships (CERs) which are evaluated for their ‘goodness’.

From an engineering and a scientific perspective, aperture is the best parameter to build a space telescope cost model. Aperture defines the observatory’s science performance (sensitivity and resolution) and determines the payload’s size and mass. OTA cost varies as a function of diameter according to:

$$\text{OTA Cost} \sim \text{Diameter}^{1.75} \quad (N = 15; r^2 = 67\%; \text{SPE} = 146)$$

This CER is based on 15 data points, one of which (SOFIA) is not actually a space telescope. If SOFIA is removed from the regression it has a negligible effect:

$$\text{OTA Cost} \sim \text{Diameter}^{1.8} \quad (N = 14; r^2 = 67\%; \text{SPE} = 142)$$

However, both CERs only account for 67% of the cost variation and both are noisy. Therefore, a single variable aperture diameter model is not a good CER. Other variables are needed to account for the remaining 33% of cost variation.

A key point of the aperture model is that the diameter coefficient is less than 2. Therefore, areal cost decreases as a function of aperture diameter. Hence, given that the number of photons collected is proportional to collecting area, larger aperture telescopes have a greater return on investment (ROI) than smaller aperture telescopes.

While an Aperture based CER may be most logical for an optical engineer, Mass may be a more important CER. Total system mass determines what vehicle can be used to launch a mission. And, significant engineering costs are expended to keep a given payload inside of its allocated mass budget. OTA cost varies as a function of mass according to:

$$\text{OTA Cost} \sim \text{OTA Mass}^{0.97} \quad (N = 12; r^2 = 56\%; \text{SPE} = 88)$$

Compared to the Aperture Diameter CER, OTA Mass is less noisy. But, it still only accounts for 56% of the cost variation. However, this CER is based on 12 data points, one of which (SOFIA) is not actually a space telescope. Removing SOFIA from the regression has a significant effect:

$$\text{OTA Cost} \sim \text{OTA Mass}^{1.1} \quad (N = 11; r^2 = 96\%; \text{SPE} = 78\%)$$

Without SOFIA, the OTA Mass CER accounts for 96% of the cost variation and has a smaller standard percent error.
Finally, cost density (cost per kg) as a function of aperture diameter indicates that for given ‘classes’ of telescopes of missions, the cost per kg is independent of aperture diameter. Thus, it costs more per kg to make low areal density flight telescopes than it costs to make massive ground telescopes. One explanation might be that it requires significantly more ‘engineering’ effort to design a low areal density telescope with the required wavefront shape and pointing stability for its operational (static gravity load and dynamic jitter) environment than it does for a more massive telescope.

REFERENCES


Cost Modeling for Space Optical Telescope Assemblies

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University of Texas at Austin;
Agenda

• Introduction and Summary
• Data Collection Methodology
• Statistical Analysis Methodology
• What to Model?: OTA or Total Mission Cost
• Single Variable Modes: Mass and Diameter
• Multi-Variable Models
• Conclusions
Agenda

• Introduction and Summary
  • Data Collection Methodology
  • Statistical Analysis Methodology
  • What to Model?: OTA or Total Mission Cost
  • Single Variable Modes: Mass and Diameter
  • Multi-Variable Models
  • Conclusions
Parametric Cost Models

Parametric cost models have several uses:

- high level mission concept design studies,
- identify major architectural cost drivers,
- allow high-level design trades,
- enable cost-benefit analysis for technology development investment, and
- provide a basis for estimating total project cost.
However

All Cost Models are Wrong!

But Some are Useful.

The Rest will get you into Trouble.
DISCLAIMER

Cost Models are only as good as their Data Base

This is a work in progress.

The results evolve as we add new missions to the Database, add data to or correct data in the Database.
Findings

Aperture Diameter is principle cost driver for space telescopes.

OTA Cost $\sim$ Diameter $^{1.8}$

OTA Cost $\sim$ Dia$^{1.6}$ $\lambda^{-0.25}$

Larger diameter OTAs cost less per square meter of aperture.

Longer wavelength OTAs cost less.

If all parameters are held constant, adding mass reduces cost & reducing mass increases cost.

Still examining Year of Development
Agenda

• Introduction and Summary

• **Data Collection Methodology**
  • Statistical Analysis Methodology
  • What to Model?: OTA or Total Mission Cost
  • Single Variable Modes: Mass and Diameter
  • Multi-Variable Models
  • Conclusions
Methodology

Data accumulated on 59 engineering and programmatic variables

18 Variables studied for Cost Estimating Relationships (CERs)

Data sources:
NAFCOM (NASA/ Air Force Cost Model) database,
NICM (NASA Instrument Cost Model),
NSCKN (NASA Safety Center Knowledge Now),
RSIC (Redstone Scientific Information Center),
REDSTAR (Resource Data Storage and Retrieval System),
SICM (Scientific Instrument Cost Model),
project websites, and interviews.
Cost & Mass Definitions

**Total Mission:**
- Spacecraft
- Science Instruments
- Telescope

**Instrument:**
- Entire payload or experiment including telescope

**Optical Telescope Assembly (OTA):**
- Primary mirror
- Secondary (and tertiary if appropriate) mirror(s)
- Support structure
- Mechanisms (actuators, etc.), Electronics, Software, etc.
- Assembly, Integration & Test
Cost & Mass Definitions (2)

**Cost includes:**
- Phase A-D (design, development, integration and test)

**Cost excludes:**
- Pre-phase A (formulation)
- Phase E (launch/post-launch)
- Government labor costs (NASA employees: CS or support contractors)
- Government Furnished Equipment (GFE)
- Existing Contractor infrastructure which is not ‘billed’ to contract.
- These are ‘First Unit’ Costs only – no HST Servicing & there are no 2nd Systems.

**Mass includes:**
- Dry mass only (no propellant)
Technical Variables

Aperture Diameter
PM Focal Length
System Focal Length
Field of View
Pointing Stability
OTA Mass
Total Mass
Spectral Range Minimum
Wavelength of Diffraction Limit
Operating Temperature
Average Input Power
Data Rate
Design Life
Orbit
Programmatic Variables

TRL (Technology Readiness Level)
Year of Development (or Start of Development)
Development Period
Launch Year
Missions (8.6.11 Database)

Currently 45 missions in data base

33 ‘normal-incidence’ UVOIR and Infrared telescopes
5 grazing incidence X-Ray
7 Radio/Microwave

Data for microwave, radio wave & grazing incidence X-Ray/EUV provides wavelength diversity

To date only normal-incidence UVOIR and Microwave telescopes used for cost modeling
Of 37 ‘normal-incidence’ UVOIR and Microwave telescopes

- 27 are ‘Free Flying’
- 4 are ‘Attached’ and
- 5 are ‘Planetary/Other’

Additionally, some of these are Imaging and others are Spectroscopic.

We have not yet investigated the impact of this distinction, but expect spectroscopic to be lower cost.

<table>
<thead>
<tr>
<th>Normal Incidence Database (8.6.11)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Free Flying Telescope</strong></td>
</tr>
<tr>
<td>ACTS</td>
</tr>
<tr>
<td>CALIPSO</td>
</tr>
<tr>
<td>Cloud SAT</td>
</tr>
<tr>
<td>Commercial #1</td>
</tr>
<tr>
<td>Commercial #2</td>
</tr>
<tr>
<td>Copernicus (OAO-3/PEP)</td>
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<td>HST</td>
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<td>ICESat</td>
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<td>IRAS</td>
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<td>IUE</td>
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<tr>
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<td>SOHO/EIT</td>
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<td>TRACE</td>
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<td>WIRE</td>
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<th>Attached Telescopes</th>
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<tr>
<td>SOFIA</td>
</tr>
<tr>
<td>UIT</td>
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<td>WUPPE</td>
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<thead>
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<th>Planetary Telescopes</th>
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<tr>
<td>LRO/LROC NAC</td>
</tr>
<tr>
<td>MO/MOC</td>
</tr>
<tr>
<td>MO/MOLA</td>
</tr>
<tr>
<td>MRO/HiRISE</td>
</tr>
<tr>
<td>STEREO/SECCHI</td>
</tr>
</tbody>
</table>
**Missions (8.6.11 Database)**

For some have only Mission data and for others have both OTA and Mission data.

We have OTA Cost:

- & Diameter data for 15
- & Mass data for 13

---

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% of data</th>
</tr>
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<tbody>
<tr>
<td>Total Cost</td>
<td>89%</td>
</tr>
<tr>
<td>OTA Cost</td>
<td>46%</td>
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<tr>
<td>Total Cost &amp; OTA Cost</td>
<td>68%</td>
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<td>Aperture Diameter</td>
<td>100%</td>
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<td>PM F Len.</td>
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<td>System F Len.</td>
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<td>FOV</td>
<td>86%</td>
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<tr>
<td>Pointing Stability</td>
<td>39%</td>
</tr>
<tr>
<td>Total Mass</td>
<td>93%</td>
</tr>
<tr>
<td>OTA Mass</td>
<td>86%</td>
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<td>Spectral Range minimum</td>
<td>96%</td>
</tr>
<tr>
<td>Diffraction Limited Wavelength</td>
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<tr>
<td>Operating Temperature</td>
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<tr>
<td><strong>Average</strong></td>
<td><strong>78%</strong></td>
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</table>
These are the missions used in our cost model analysis:

15 are ‘Free Flying’
4 is ‘Attached’ and
1 is ‘Planetary’

Of these 8 are spectroscopic or non-imaging.

Missions (8.6.11 Database)

<table>
<thead>
<tr>
<th>Normal Incidence Database (8.6.11)</th>
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<td>Free Flying Telescope</td>
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<th>Planetary Telescopes</th>
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<td>MRO/HiRISE</td>
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Agenda

• Introduction and Summary
• Data Collection Methodology

• **Statistical Analysis Methodology**
  • What to Model?: OTA or Total Mission Cost
  • Single Variable Modes: Mass and Diameter
  • Multi-Variable Models
  • Conclusions
Model Creation

Start with Correlation Matrix.

Look for Variables which are Highly Correlated with Cost.

The higher the correlation the greater the Cost Variation which is explained by a given Variable.

Sign of correlation is important and must be consistent with Engineering Judgment.

Important for Multi-Variable Models:

We want Variables which Independently effect Cost.

When Variables ‘cross-talk’ with each other it is called Multi-Collinearity.

Thus, avoid Variables which are highly correlated with each other.
‘Correlation’ between variables and ‘Goodness’ of single variable models is evaluated via Pearson’s $r^2$ standard percent error (SPE), and Student’s T-Test p-value.

‘Goodness’ of multivariable fits are evaluated via Pearson’s Adjusted $r^2$ which accounts for number of data points and number of variables.

Pearson’s $r^2$ coefficient describes the percentage of agreement between the fitted values and the actual data.

The closer $r^2$ is to 1, the better the fit.

SPE is a normalized standard deviation of the fit residual (difference between data and fit) to the fit.

The closer SPE is to 0, the better the fit.
Significance

The final issue is whether or not a correlation or fit is significant.

*p*-value is the probability that the fit or correlation would occur if the variables are independent of each other.

- The closer *p*-value is to 0, the more significant the fit or correlation.
- The closer *p*-value is to 1, the less significant.

If the *p*-value for a given variable is small, then removing it from the model would cause a large change to the model.

If *p*-value is large, then removing the variable will have a negligible effect.

It is only possible to ‘test’ if the correlation between two variables is significant.

It is not possible to ‘test’ if two variables are independent.
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</table>
Correlations which are at least 95% significant are **Bolded**, e.g. for 12 data points a correlation of greater than 60% is significant to better than 95%.
OTA Cost has significant correlations with:
- Aperture Diameter
- Primary Mirror & System Focal Length (Volume)
- Pointing Stability (inverse correlation)
- OTA Mass
- Design Life

Total Cost has significant correlations with:
- Aperture Diameter
- Primary Mirror & System Focal Length (Volume)
- Pointing Stability (inverse correlation)
- OTA & Total Mass
- Average Power
- Design Life
- Development Period

No correlation for wavelength or temperature

TRL correlation is ‘weak’
Not all Correlated Variables are Independent

<table>
<thead>
<tr>
<th>Aperture Diameter</th>
<th>1.00</th>
<th>0.36</th>
<th>0.72</th>
<th>-0.04</th>
<th>-0.71</th>
<th>0.59</th>
<th>0.84</th>
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<th>-0.28</th>
<th>-0.06</th>
<th>0.45</th>
<th>0.05</th>
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Larger Diameter OTAs:
- have longer Focal Lengths
- have smaller Pointing Stability Requirements
- are just bigger and thus more Massive
- have larger instruments with are more Massive & require Power
- require bigger spacecraft which are more Massive & require Power
- need a long Design Life
- take longer to Develop
- are more Recent – older OTAs were smaller

All these variables are dependent on Aperture Diameter (co-linear).
Variable Linkages

Correlation Matrix can be used to identify variable cross-linkages which should be reconciled with Engineering Judgment.

Aperture Diameter and Pointing Stability have a large negative correlation: Larger Diameter OTAs required smaller Pointing Stability.

Pointing Stability and OTA Mass have a large negative correlation: Small Pointing Stability requires a very stiff, i.e. Massive, OTA.
As expected Spectral Range and Diffraction Limit are highly correlated. Operating Temperature are inversely correlated. But neither are significantly correlated with Cost – probably because they cancel each other out.
Agenda

• Introduction and Summary
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  • Single Variable Modes: Mass and Diameter
  • Multi-Variable Models
• Conclusions
OTA Cost or Total Cost

Engineering judgment says that OTA cost is most closely related to OTA engineering parameters.

But, managers and mission planners are really more interested in total Phase A-D cost.
OTA Cost or Total Cost

Given that Total Cost tracks closely with OTA Cost, and that I’m an optics person and have accumulated mostly OTA data.

Our primary emphasis is to develop an OTA cost model.
OTA Cost as a % of Total Mission Cost

OTA Cost varies from approximately 1% to 25% of the Total. OTA’s cost as % of Total depends upon need to develop custom tooling or infrastructure – or use existing.

WIRE is clearly questionable & under review. Also, have asked GALEX to clarify their CADRe cost (missing Structure cost)
OTA Cost as a % of Total Mission Cost

We have detailed WBS data for 7 of the 14 free flying missions. Mapping (5.3.11) database on common WBS gives OTA ~10% of Total. Some say that Power System is 20% of total mission Cost and Mass. For 1960/1970 mission, electronics costs are greater than OTA costs.

Typical Space Telescope Cost Breakdown

- Optical Telescope Assembly: 11%
- Spacecraft: 34%
- Instruments: 28%
- Other (Mission Specific): 8%
- Program Management: 6%
- Systems Engineering: 6%
- Integration & Testing: 4%
- Ground Support: 3%
- Optical Telescope Assembly: 11%
- Ground Support: 3%
- Spacecraft: 34%
- Instruments: 28%
- Other (Mission Specific): 8%
- Program Management: 6%
- Systems Engineering: 6%
- Integration & Testing: 4%
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OTA Cost Regression

Regressing on 15 normal incidence, ‘free-flying’ UVOIR OTAs

Significant Variables: Diameter, Focal Length, Volume, Pointing & Mass

FL has the highest $R^2_{adj}$ and Mass has the lowest SPE

Volume, FL & Diameter have acceptable $R^2_{adj}$ & SPE (but they all Dia)

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>OTA Cost vs V1</th>
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<tbody>
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<td>Aperture Diameter</td>
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<td>PM F Len.</td>
<td>1.55 0.00</td>
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<td>PM f/#</td>
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<td>OTA Volume</td>
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<td>Pointing Stability</td>
<td>-0.76 0.02</td>
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<tr>
<td>OTA Mass</td>
<td>1.08 0.00</td>
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<table>
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<tr>
<th>Var.</th>
<th>p-value</th>
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<td>Adjusted $r^2$</td>
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<td>SPE</td>
<td>123% 92% 707% 80% 400% 242% 58%</td>
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<td>n</td>
<td>15 11 11 11 12 8 13</td>
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<th>OTA Areal Density</th>
<th>Spectral Range</th>
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<th>Operating Temp.</th>
<th>Year of Dev. (exp)</th>
<th>Date of Launch (exp)</th>
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Mass Model
Mass Model

As an optical engineer, my preference is to develop a model based on an optical parameter, i.e. Aperture Diameter.

Aperture Diameter interests ‘users’ of space telescopes because it is directly proportional to sensitivity and resolution.

But, many believe that Mass is the most important CER. Total system mass determines what vehicle can be used to launch. Significant engineering costs are expended to keep a given payload inside of its allocated mass budget.

Such as light-weighting mirrors and structure.

Space telescopes are designed to mass
OTA Cost Mass Model #1

Regressing on all OTAs in the data base:

OTA Cost ~ OTA Mass $^{0.8}$ \( (N = 17; \ r^2 = 42\%; \ SPE = 142\%) \)

Mass accounts for only 42\% of the cost variation & is noisy
OTA Cost Mass Model #2

Regressing on only Free-Flyer (excluding ‘attached’ and SOFIA):

\[ \text{OTA Cost} \sim \text{OTA Mass}^{1.1} \quad (N = 13; \ r^2 = 87\%; \ SPE = 58\%) \]

Mass accounts for 87% of the cost variation with less noise.
OTA Cost Mass Model #2

The 3 ‘attached’ missions & SOFIA clearly are a different ‘class’

They have a different set of design rules which allow them to have a lower cost for a given mass.

![OTA Cost vs OTA Mass (rev. 8.1.11)](image)
OTA Cost Density

It costs more to design & build a low mass OTA than a high mass OTA.

Cost per kg depends on mission ‘type’; is independent of aperture size.

Free-Flying OTAs are ~2X more expensive per kg than Attached OTAs.

Free-Flying OTAs are ~15X more expensive per kg than SOFIA.

Free-Flying OTAs are 1000X more expensive per kg than Ground.
Mass is not a Good CER

It may appear that Mass is a good CER, but it is not.

JWST & HST have same OTA mass, but JWST OTA costs is 2X HST

HST Total mass is 2X JWST, but JWST Total cost is 2X HST

The reason is complexity – JWST is more complex than HST
Problem with Mass

Mass may have a high correlation to Cost.

And, Mass may be convenient to quantify.

But, Mass is not an independent variable.

Mass depends upon the size of the telescope.

Bigger telescopes have more mass and Aperture drives size.

And, bigger telescopes typically require bigger spacecraft.

The correlation matrix says that Mass is highly correlated with:

- Aperture Diameter, Focal Length and Pointing

But in reality it is all Aperture, the others depend on aperture.
Aperture Model
OTA Cost vs Aperture Model #1

Regressing OTA Cost vs Aperture for all missions in database:

**OTA Cost ~ Diameter**$^{1.6}$  \( (N = 20; r^2 = 80\%; SPE = 142) \)

Diameter accounts for 80% of the cost variation, but is noisy
Regressing OTA Cost vs Aperture for just Free-Flyer missions (and excluding WIRE):

**OTA Cost ~ Diameter**^{1.4} \quad (N = 15; r^2 = 82\%; \text{SPE} = 123)

Diameter accounts for 82% of the cost variation, is less noisy
OTA Areal Cost

Because coefficient for diameter is less than ‘2’, the areal cost (cost per area) decreases as telescopes become larger.

Larger OTAs provide a higher ROI, less $ per photon.

Also, more massive ‘attached’ and ‘ground’ have lower areal cost.
Agenda

• Introduction and Summary
• Data Collection Methodology
• Statistical Analysis Methodology
• What to Model?: OTA or Total Mission Cost
• Single Variable Modes: Mass and Diameter

• Multi-Variable Models
• Conclusions
Need for a second variable

Assuming that Mass is not the right CER and that Aperture is Aperture Model only accounts for 70% of the cost variation.

Therefore, other variables must account for the remaining 30% of the cost variation.

Thus, a multi-variable model is required.

First step is a residual analysis.
How to develop a Multi-Variable Model

Perform multi-variable regression to add a second variable.

Select two variable model based on:

- Change in Significance of Diameter to Fit
- Significance of Variable #2 to Fit
- Increase in $r^2_{adj}$
- Decrease in SPE
- Multi-Collinearity

Some variables may increase $r^2_{adj}$ and/or decrease SPE, but they are not significant or their coefficients are not consistent with engineering judgment or they are multi-collinear.
OTA Cost versus Diameter and V2

<table>
<thead>
<tr>
<th></th>
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<tbody>
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<td>Diam. p-value</td>
<td>1.42 0.00</td>
<td>0.73 0.19</td>
<td>-1.28 0.38</td>
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<td>1.00 0.06</td>
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<td>93% 4%</td>
<td>95%</td>
<td>85% 84%</td>
<td>123% 84%</td>
<td>66% 58%</td>
<td>54%</td>
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<tr>
<td>SPE</td>
<td>123% 84%</td>
<td>84% 142%</td>
<td>66%</td>
<td>58% 54%</td>
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<td>8 13</td>
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<td>Yes No</td>
<td>No No Yes</td>
<td>No</td>
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Diffraction Limit & Spectral Min are most significant, both increase R2 & decrease SPE. OTA Mass increases R2 to 85%, but is multi-collinear with Aperture Diameter. Other multi-collinear variables are FL and Volume. Don’t understand impact of Design Life on Diameter.
Aperture Residual Error Analysis

Divide data by Diameter Model (normalize data) and plot as a function of Variables.

$R^2$ indicates how % of residual error explained by a 2$^{nd}$ Variable

For example, as expected diameter explains ‘zero’ variation
Aperture Residual Error Analysis: Wavelength

Diffraction Limit Wavelength explains 97% of residual variation

A -0.2 coefficient implies that an OTA with a 10X longer wavelength will cost 40% less.
Aperture Residual Error Analysis: Temperature

Operating Temperature does not significantly explain residual aperture variation

But, it might be a good 3rd or 4th CER parameter
Aperture Residual Error Analysis: YOD

Year of Development does not significantly explain residual. But, it might be a good 3\textsuperscript{rd} or 4\textsuperscript{th} CER parameter.

Concern that YOD is correlated with Aperture and Wavelength. Also, what is role of spectroscopic vs imaging.
Two Variable Aperture Model

Two second variables best meet all the criteria:

Wavelength Diffraction Limit and
Spectral Minimum

Diffraction Limited Wavelength yields the best model:

\[
\text{OTA Cost} \sim \text{Dia}^{1.6} \cdot \lambda^{-0.25} \quad (N = 12, \ r^2 = 98\%; \ SPE= 60\%)
\]
OTA Cost versus Diameter, Wavelength and V3

Operating Temperature is the only significant 3\textsuperscript{rd} variable

\textbf{OTA Cost} \sim D^{1.7} \lambda^{-0.3} T^{-0.25}

\((N = 11, r^2 = 96\%; \text{SPE} = 54\%)\)

More effort is required to understand issues related to:

Design Life

Year of Development

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<td>p-value</td>
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Conclusions: Aperture

Consistent with Engineering Judgment Aperture Diameter is a good CER for OTA Cost:

\[
\text{OTA Cost } \sim \text{ Diameter }^{1.4} \quad (N = 15; r^2 = 82\%; \ SPE = 123)
\]

1 variable only explains 82%, thus a 2 variable model is needed

Two variable model using Wavelength Diffraction Limit explains 98% of data variation with a low SPE.

\[
\text{OTA Cost } \sim \text{ Dia}^{1.6} \lambda^{-0.25} \quad (N = 12, r^2 = 98\%; \ SPE = 60\%)
\]

In all cases, Areal Cost ($/m^2) is less for larger telescopes
Three Variable Aperture Model

Temperature gives a statistically significant 3 Variable model:

\[ \text{OTA Cost} \sim D^{1.7} \lambda^{-0.3} T^{-0.25} \]

\((N = 11, r^2 = 96\%; \ SPE = 54\%)\)

More effort is required to understand issues related to:

- Design Life
- Year of Development
Comparison with Historical Models

This study has identified a potential 3 variable model

\[
\text{OTA Cost} \sim D^{1.7} \lambda^{-0.3} T^{-0.25}
\]

Bely Model (corrected):

\[
\text{OTA Cost} \sim D^{1.6} \lambda^{-0.18} T^{-0.2} e^{-0.033(YOD - 1960)}
\]

Horak Model:

\[
\text{OTA Cost} \sim D^{0.7} \lambda^{-0.18} T^{-0.2} e^{-0.033(YOD - 1960)}
\]

But Horak had a different data base.
Conclusions: Mass

OTA mass is not a good CER

OTA mass is multi-collinear with diameter, and
more massive telescopes actually cost less to make.

For a given aperture diameter,

Free-Flying OTAs are ~2X more expensive per kg than Attached OTAs
Free-Flying OTAs are ~15X more expensive per kg than SOFIA
Free-Flying OTAs are 1000X more expensive per kg than Ground

Bottom line: using Mass as an OTA CER could easily lead one to
make inappropriate programmatic decisions.
General Conclusions

Models are only as good as their data bases – we need more data.

Larger OTAs cost more than Smaller, but Larger Diameter OTAs actually cost less per square meter of Collecting Aperture.

Longer Wavelength OTAs cost less than Shorter.

Cryogenic OTAs may cost less than Ambient.

More study is required regarding cost reduction with year.

If all parameters are held constant, adding mass reduces cost & reducing mass increases cost.