Optimal Feedback Control of Thermal Networks

A systematic approach to design has been devised.

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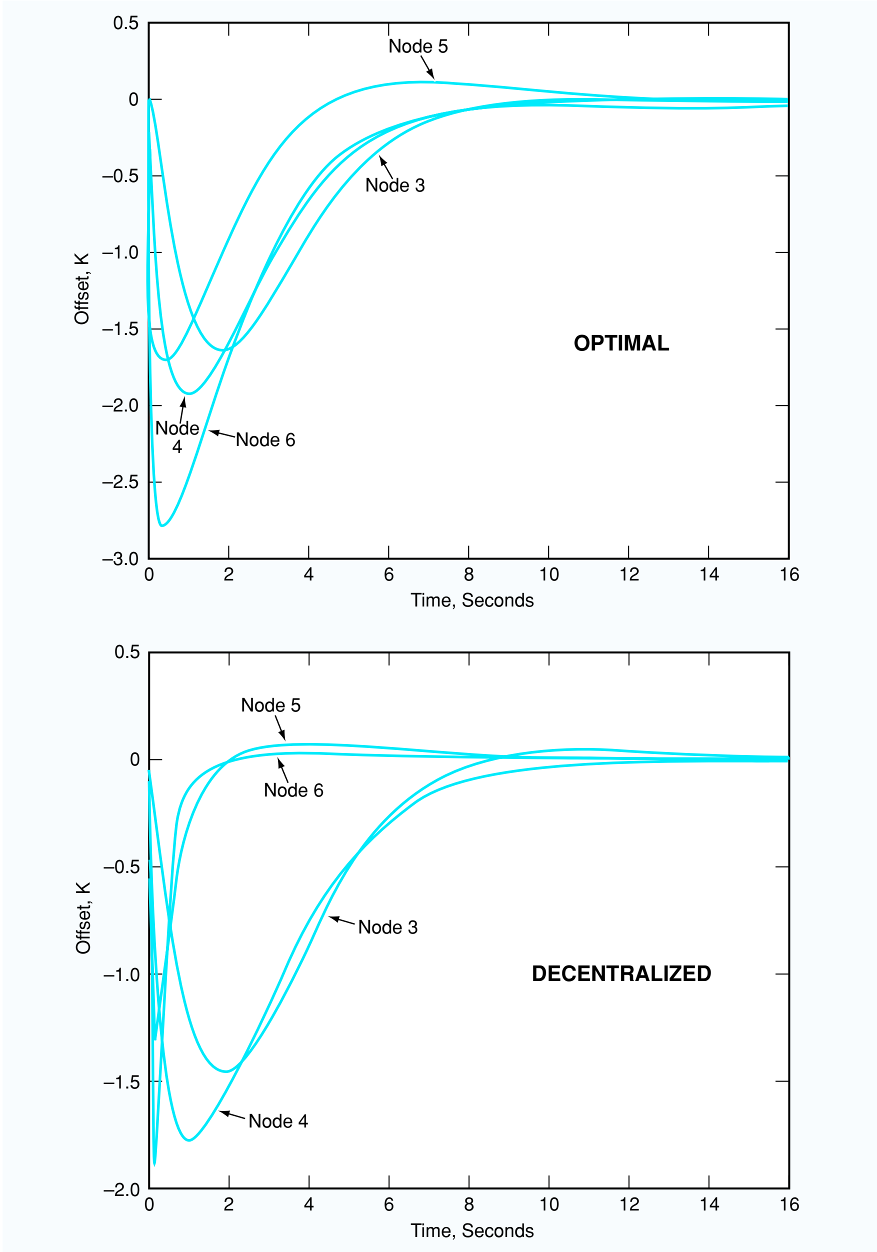
An improved approach to the mathematical modeling of feedback control of thermal networks has been devised. Heretofore software for feedback control of thermal networks has been developed by time-consuming trial-and-error methods that depend on engineers' expertise. In contrast, the present approach is a systematic means of developing algorithms for feedback control that is optimal in the sense that it combines performance with low cost of implementation. An additional advantage of the present approach is that a thermal engineer need not be expert in control theory.

Thermal networks are lumped-parameter approximations used to represent complex thermal systems. Thermal networks are closely related to electrical networks commonly represented by lumped-parameter circuit diagrams. Like such electrical circuits, thermal networks are mathematically modeled by systems of differential-algebraic equations (DAEs) — that is, ordinary differential equations subject to a set of algebraic constraints. In the present approach, emphasis is placed on applications in which thermal networks are subject to constant disturbances and, therefore, integral control action is necessary to obtain steady-state responses.

The mathematical development of the present approach begins with the derivation of optimal integral-control laws via minimization of an appropriate cost functional that involves augmented state vectors. Subsequently, classical variational arguments provide optimality conditions in the form of the Hamiltonian equations for the standard linear-quadratic-regulator (LQR) problem. These equations are reduced to an algebraic Riccati equation (ARE) with respect to the augmented state vector. The solution of the ARE leads to the direct computation of the optimal proportional- and integral-feedback control gains.

In cases of very complex networks, large numbers of state variables make it difficult to implement optimal controllers in the manner described in the preceding paragraph. Therefore, another important element of the present approach is consideration of decentralized control (that is, the use of nominally suboptimal controllers, each affecting only part of the network). Numerical tests of an algorithm that computes feedback gains for decentralized control have shown that the performances of the decentralized controllers are comparable to the performances of the corresponding optimal controllers (see figure). In particular, it was observed that decentralized controllers might require a little more energy than their optimal counterparts; however, this is a small price to pay for the simplification of controller structures that can be achieved. Further, the lower cost of implementation of much simpler feedback loops in decentralized control outweighs the extra amount of energy that decentralized controllers might require.



The **Time-Dependent Offsets of Four Nodes** of a 9-node network were computed in a numerical test of optimal- and decentralized-control laws. Both control laws yield smooth temperature histories, without oscillations or large overshoots, and both require about the same amount of time to achieve nearly zero offset (where offset as used here signifies the difference between the actual and desired temperatures of a given node).

This work was done by Miltiadis Papalexandris of Caltech for **NASA's Jet Propulsion Laboratory**. *Further information is contained in a TSP [see page 1].*

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